Expert Knowledge and Prejudice: two Interpretations of Differing Priors

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To Zubi

Acknowledgments

Albert Einstein once said that “the whole of science is nothing more than a refinement of everyday thinking”. This thesis would not exist if I had to refine thinking without sharing everyday thoughts with so many persons I met during the last five years: I enjoyed exchanging ideas, discussing, sometimes moaning, and often laughing with you.

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Introduction

This thesis consists of three chapters that study the introduction of differing priors in classic models. Aumann (1976)'s result that agents who share a common prior cannot agree to disagree triggered a debate on the common prior assumption. For instance, Gul (1998) points out that "the prior view is an inherently dynamic story" and "it amounts to asserting that at some moment in time everyone must have identical beliefs". In the long run, when agents learn or communicate this seems reasonable. In noncooperative one-shot games as analyzed in this thesis, however, it seems conceivable that agents have differing priors. We interpret this assumption as expert knowledge in Hotelling's line of horizontal differentiation and as prejudice in a principal agent framework.

Chapter 1 studies two exemption clauses to the general obligation that public contracts must be awarded by means of tendering procedures. Typically, public procurement policies allow for direct awards to experts if the buyer is uncertain about his specific need or the contract's value is low. In a model of horizontal differentiation with demand uncertainty, we derive the expert's offer for a directly awarded contract and the offers of duopolists with differing priors in a tendering procedure. The expert's price in the former case appropriates the buyer's willingness to pay less a discount that—according to the expert's prior—fully compensates the buyer for unsuitable project specifications. Consistently, the expert minimizes unsuitability by customizing the project's specification to the buyer's need. In the tendering procedure, however, prices equal implementation cost plus a premium that the competitors charge due to horizontal differentiation. Comparisons of the two cases in terms of welfare as well as the buyer's total cost reveal that, concerning both criteria, direct awards are beneficial if expert knowledge is valid and the contract's net value is low. These insights support common policies on public procurement.

Chapter 2 more thoroughly analyzes the specification-then-price game that provides the theoretical foundation of chapter 1. In the case of converging priors, the model is equivalent to Hotelling's line with uniformly distributed demand and quadratic cost. Accordingly, firms offer maximally differentiated projects to mitigate price competition. In case of differing priors, however, we derive other equilibrium types. Their existence depends on the expert's knowledge: if expertise is moderate and includes awareness of unsuitable project specifications that affect the expert's special field, she customizes her offer and launches price competition. If expertise is deep, in contrast, the expert's pricing strategy stifles competition, and her specification is contingent on the perceived unsuitability of her competitor's project. Comparative statics determine conditions for the profitability and market power of expert knowledge.

Chapter 3 suggests prejudice as another interpretation of differing priors in a simple formulation of a static principal agent model with discrete effort choice. The prejudiced principal wants to hire two agents to carry out a common project. Without regulation he would hire two male team members because he wrongly believes that women bear higher cost of performing demanding work than men. However, even if we impose a quota that forces the principal to offer jobs to candidates of both genders, he fails to hire a mixed team consisting of two hard working peers. This is not due to discrimination, but self-selection: the underestimated woman rejects her offer as a peer because her overestimated coworker shirks. In contrast, she accepts the offer for a trivial job even though her coworker shirks. This result constitutes a new rationale for the gender gap in workplace hierarchies. We propose wage equality as a remedy for the free-riding problem, and consequently, the underrepresentation of women in demanding positions.

Chapter 1

Public Procurement with an Expert

1.1 Introduction

"There are as many opinions
as there are experts."
—Franklin D. Roosevelt (1942)

In 2015 the light trucks of the Swiss army (the so-called "Duro I") had been in service for roughly 20 years. After a debate, the parliament directly awarded a conversion contract over CHF 558 millions to GDELS-mowag, the producer of said Duro I. This direct award entailed the rejection of two offers for the procurement of comparable transportation fleets from other suppliers. One of these offers was rejected due to its higher price, the other due to unsuitability concerns.[1]

The latter is a common justification for direct awards to experts if the buyer is uncertain about his specific need, and thus, launches an incomplete catalogue of requirements. It relies on the notion that an expert, who has superior information than potential competitors, is able to customize her offer to the contracting authority's need. In this vein, expert knowledge is especially valuable in complex projects where the buyer typically is uncertain about his specific need. Most policies for public procurement account for such situations by allowing pre-selection of potential suppliers, and even direct awards, if a project calls for "relevant prior experience [...] to meet the requirements of the procurement" (Art. VIII.2.b, GPA, 2012).[2] Empirical evidence suggests that this is often the case in IT projects. In Switzerland, for instance, nearly half the contracts of public IT procurement is directly awarded to external suppliers without any tendering procedure, even though these contracts' values exceed the threshold of currently CHF 230'000 (Art. 6, BöB, 2015; Stürmer et al., 2017).

[1]According, e.g., to the then president of the Security Commission in the decisive session of the Council of States, see Baumann (2016).

Note that general procurement policies typically do not apply for army supply and war material, see e.g., Art. III.1. in the *Agreement on Government Procurement* by the WTO (GPA, 2012). Also the direct award to GDELS-mowag over half a billion CHF was in line with Swiss law, see Art. 3, BöB (2015) and Art. 7ff, MatV (2018).

[2]The GPA is currently (December 2020) signed by 48 parties including the US, all members of the EU as well as Switzerland. In their national directives, most parties of the GPA explicitly list reasons that allow direct awards. In Switzerland, e.g., direct awards are permitted if "due to technical or artistic features of the contract [...] there is a single potential supplier and no suitable alternative" (Art. 13.2.c, VöB, 2018).

Most procurement policies stipulate such financial ceilings for direct awards. That is, the general obligation to conduct a tendering procedure applies only to contracts of high monetary value. Figure 1.1 sketches the rationale for these thresholds: the fixed cost of a direct award is typically lower than that of a tendering procedure because in the latter, potential suppliers bear cost of preparing offers, and the tendering authority of evaluating them. Evidence suggests that the sum of these transaction costs is increasing in the project's value (and its positively correlated complexity) at a decreasing rate (see e.g., Strand et al. (2011) or Jäger et al. (2006)).[3] Under the natural assumption that the value of a project's implementation increases with the underlying contract's monetary volume, the net benefit and net cost of tendering procedures intersect.[4] Below the intersection direct awards are welfare superior to tendering procedures, and thus, intended by procurement policies.

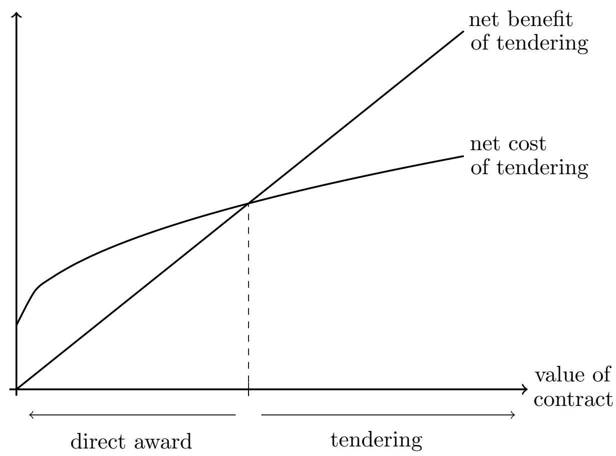

Figure 1.1: (Net) cost and benefit of tendering procedures (compared to direct awards)

This chapter studies both widely used policies, financial ceilings for direct awards and exception clauses for these ceilings due to expert knowledge. It seems

[3]The concavity of total fixed cost of a tendering is mainly driven by the cost on the suppliers side: while the administrative cost (for preparing the call for tenders and evaluating the offers) is rather independent, the cost for preparing an offer strictly increases in contract value (and complexity). However, evidence indicates that the number of offers is decreasing in contract value (and complexity).

[4]At least in the non-trivial case where net cost are neither higher nor lower than the net benefit for all values, and a tendering therefore neither unambiguously costlier nor more efficient than a direct award. Further note that this commonly used rationale for financial ceilings ignores unsuitability cost that might depend on the procurement procedure as well as the contract's value.

intuitive that an expert's customized solution may suit the buyer's need better than the offer from another supplier with less information. However, it is not obvious that the single (monopolistic) offer for a directly awarded project is more suitable than a variety of (competitive) offers a buyer receives if he calls for tenders. We compare unsuitability cost of a direct award and a tendering procedure, and find that welfare crucially depends on the validity of expert knowledge: the buyer's unsuitability cost of a directly awarded project indeed is lower than in a tendering procedure if expert knowledge is valid. We additionally consider prices, and thereby identify another rationale for financial ceilings than the common reasoning described above: while the expert appropriates the buyer's willingness to pay in case of a direct award, competition in a tendering procedure keeps her from doing so. Thus, even if we ignore any transaction cost, the buyer's total cost under direct award is lower than under tendering if the contract's value is rather low due to the suitability gains.

These findings contribute to the policy discussion on procurement procedures in presence of an expert. In contrast to previous literature, our expert is *self-proclaimed*, that is, her ex ante information on the buyer's need may be inaccurate. More precisely, we model a direct award to an expert who is convinced of knowing the buyer's horizontal need better than the latter himself. The expert customizes her offer, which consists of a horizontal specification and a price, according to her potentially inaccurate conviction. In terms of the buyer's unsuitability as well as total cost, we compare this expert offer for a directly awarded contract with the offers of two participants in a tendering procedure.

We determine the tendering offers by introducing differing priors in Hotelling's location-then-price game with demand uncertainty: in our interpretation, there is a single buyer who's preference for horizontal project specifications ex ante is unknown. He asks two potential suppliers for tenders.[5] These suppliers have differing priors: while one of them believes in the buyer's incomplete catalogue of requirements, the expert is convinced to have superior ex ante information about these requirements. Formally, we model the expert's prior on the support of the buyer's need as a strict subinterval of the interval determined by the catalogue of requirements. Note that the expert is self-proclaimed since we allow

[5] In contrast to an open tendering, the model rather captures a so-called *invitation* to or *request* for tenders, where selected suppliers are invited to compete and submit an offer within a particular time frame.

for different degrees of her knowledge's validity (including the limit case where validity converges to 0, i.e., where the expert's competitor's prior is almost surely accurate). Specifically, the suppliers' are both convinced that their own prior is true, while their competitor's is wrong. Because priors are not updated, and moreover, assumed to be common knowledge, the competitors agree to disagree throughout the entire game. Therefore, the following determination of equilibrium offers requires no assumption on the need's true support.

If the contract is awarded directly to the expert, her price appropriates the buyer's willingness to pay up to a discount that compensates him for his unsuitability cost. She thus minimizes the latter by offering a specification equal to her expected value of the buyer's need. The optimal price discount equals the maximum unsuitability cost, i.e., the expert expects the buyer to certainly accepts her direct award offer.[6]

In the tendering game, the optimality of such a "play it safe" strategy depends on two divergent aspects of expert knowledge, its precision and the degree of perceived unsuitability. On the one hand, precise information enables the expert to customize her project to the buyer's need, which increases her expected winning probability. On the other hand, knowledge includes awareness of unsuitable project specifications. If unsuitability affects the expert's own special field she compensates the buyer by granting a lower price. The relative degree of both aspects determines the equilibrium offers: if precision (relative to the expert's perceived unsuitability) is high enough, the expert offers a price that stifles competition, i.e., that ensures her to win the tendering. We call this "play it safe" strategy *Limit pricing.* Otherwise, i.e., if relative precision is moderate, the expert expects both firms to win with strictly positive probability. We refer to this situation as the *Duopoly case.*

In both types of price equilibria, competition prevents the suppliers to appropriate the buyer's willingness to pay. Instead, they offer prices that account for the cost of implementation and charge a differentiation premium. This premium

[6] This result is true for the interesting, because comparable, set of parameters where the expert's pricing power is high enough. It is in line with former literature on monopolists and horizontal differentiation, see e.g., Böckem (1994) who shows that the monopolist's horizontal specification is equal to the median taste of the consumer mass. Böckem (1994) abstains from explicitly stating the monopolist's price. However, by relaxing her assumption that every consumer has unit demand (i.e., a high enough willingness to pay to certainly buy the product) it turns out that her monopolist sets the identical price as our expert under direct award.

is increasing in horizontal differentiation, that is, the latter mitigates price competition (as in the example of Hotelling's line in d'Aspremont et al. (1979)). Consequently, the expert's competitor (holding a prior according to Hotelling's line) opts for maximum differentiation. The expert's awareness of unsuitable project specifications, however, constitutes an additional incentive to customize her offer.

In the Duopoly equilibrium, she customizes her offer if the unsuitability affecting her own special field is high. In this case, the positive effect of customization on the expert's winning probability through a larger home turf (often referred to as "business stealing" effect) dominates the negative effect of customization on the price premiums. Otherwise, that is, if the expert assesses her own special field as suitable for the buyer's need, the latter effect prevails. Consequently, the expert in this case maximally differentiates her project to keep the buyer's price sensitivity low.

In the Limit equilibrium, the expert's pricing strategy guarantees her to win the contract, which renders business stealing obsolete. Therefore, the unsuitability of her own special field is irrelevant. Instead, the expert focuses on the unsuitability of her competitor: if she assesses the latter's specification as rather suitable, she customizes her project in order to keep the buyer's unsuitability cost of her own project at the same (rather low) level. This allows the expert to avoid compensating the buyer by granting an unsuitability discount, which in this case, would be higher than the price reduction due to intensified price competition. Otherwise, i.e., if the expert assesses her competitor's specification as rather unsuitable, competition would induce a higher price reduction than the discount needed for compensating the buyer's unsuitability cost. She thus maximally differentiates to mitigate price competition.

In the policy analysis we compare the equilibrium offers of both procurement procedures, the direct award and the tendering, in terms of welfare and the buyer's total cost. While the equilibrium offers are independent of the need's true distribution[7], welfare considerations (based on expected unsuitability cost) require an additional assumption. For this reason we introduce the probability that the

[7]As outlined above, equilibrium offers are independent of the need's true support because both suppliers' are convinced that their own (commonly known) prior is true, while their competitor's is wrong, and they learn no new information (i.e., do not update their priors).

expert's prior is correct, and call this probability the *reliability* of expert knowledge. Together with the precision of expert knowledge, its reliability determines the density of the buyer's need in the small support the expert beliefs in. Or, in our wording, her knowledge's *validity*, which drives our first insight concerning welfare: the higher the validity of expert knowledge, the lower the unsuitability cost of a direct award compared to a tendering procedure. This seems intuitive since unsuitability cost of a directly awarded contract is minimized in the small support (with high relative density equal to the validity), whereas unsuitability cost in a tendering is minimal in the peripheries of the need's support (where density is low). To see this, recall that the expert under direct award minimizes unsuitability cost by offering her expected value of the buyer's need, which naturally lies in the small support. In a tendering, however, this incentive to customize is dominated by the competition-induced incentive to differentiate. Therefore, in a tendering, both suppliers offer project specifications that lie outside the small interval.

These considerations also yield the second insight concerning welfare: in absolute terms, a direct award is welfare superior to a tendering if the reliability exceeds a threshold. Such a set of parameters exists for all types of tendering offers if we ignore the implementation probability. Specifically, while the buyer certainly implements one of the two tendering offers by assumption of a high enough willingness to pay, he rejects the offer for a directly awarded project if the self-proclaimed expert underestimates maximum unsuitability cost, and thus, not fully compensates the buyer. This is the case if her ex ante information is of moderate reliability. Or conversely, the offer for a directly awarded project is implemented with probability 1 if the reliability exceeds a particular threshold.

In Limit equilibria, this threshold is higher than the threshold that renders direct awards welfare superior to tendering procedures. If we thus focus on offers that are accepted with certainty, direct awards are unambiguously welfare superior to the Limit offers in tendering procedures. Consequently, we conclude that the welfare superiority of tendering procedures in the Limit case relies on the possibility that the buyer rejects the expert's direct award offer. Specifically, if the expert's knowledge is precise enough (such that she offers the Limit price in tendering), and moreover, reliable enough (such that the buyer certainly accepts her direct award offer), unsuitability cost of a direct award is lower than that of a tendering. For the Duopoly offers, however, this is not true: there exists a set of parameters in which unsuitability cost of a direct award exceeds cost of a tendering even if the directly awarded project is implemented with certainty.

Note that these results on welfare support procurement policies allowing for direct awards to experts provided that the latter's knowledge is reliable (or similarly, valid) enough. In our introductory example the validity of GDELS-mowag's superior information (on converting the existing Duro I fleet) seems questionable because the two competitive procurement offers for (the new procurement of) light trucks came from recognized suppliers of such vehicles.[8] Considering that the army's need actually was the modernization of its transportation fleet rather than the conversion of the Duros, it might have been welfare beneficial to procure one of the two offers for new fleets rather than directly award the conversion contract to GDELS-mowag.

Another exception clause from the general obligation to award public contracts by tendering procedures are financial ceilings. As described above procurement policies typically allow direct awards for contracts of low value. Our comparison of the buyer's total cost supports these policies even if we ignore that tendering procedures entail higher transaction cost than direct awards. The buyer's total cost is the sum of his unsuitability cost and the offered price. Recall that the latter in case of a direct award equals the buyer's willingness to pay less an unsuitability discount, whereas prices in tendering procedures equal the implementation cost plus a differentiation premium. We find that these differentiation premiums are asymmetric, such that there exists a set of parameters for which the prices in the introductory example are in line with our results: the price for a directly awarded project indeed may lie within the two prices offered in a tendering. Moreover, we conclude that the buyer's total cost of a directly awarded contract exceeds total cost of a tendering if the probability that the buyer rejects the expert's offer for a directly awarded project is strictly positive. The intensity of the debate whether GDELS-mowag's price exceeded the value of the converted transporters indicates that her offer might have been rejected. In this case, our result suggests that, in terms of the procuring authority's cost, the tendering offers for new fleets would have been preferable to the direct award of the conversion contract. In the other case, i.e., if acceptance of (the valid expert) mowag's offer for the

[8]While the more expensive transporters are used in mountainous terrains by other armies (as well as by the ambulance of the Swiss Army that has to reach every summit), the comparable priced offer was for a prototype of a well-tried light truck's new model. See Baumann (2016).

conversion contract was certain, our result suggests that the direct award indeed was preferable to the tenders if the expert's pricing power was low, or equivalently, if either the contract's net value was low or the army's sensitivity to unsuitable specifications high.

The trade-off between suitability gains of direct awards to the expert and the latter's ability to skim the buyer's rent constitutes an additional rationale for financial ceilings on direct awards as previous theories mainly consider cost. More precisely, one strand of literature models procurement situations, in which the suppliers' production costs ex ante (in the bidding stage) are private information. In the implementation stage, the winning supplier then minimizes cost. Typically, the optimal design is a menu of contracts, and potential suppliers reveal their type by self-selection. For a thorough review of this literature see Laffont and Tirole (1993).

Another strand focusses on transaction cost from ex post adaption (and renegotiation) due to initially incomplete catalogues of requirements. An example of this literature is Bajari and Tadelis (2006)[9] who study two widely used contracts, the fixed price and the cost plus contract. The former offers the supplier a price for completing the project as specified, and any changes are negotiated separately. The latter reimburses the supplier for his occurring cost plus an additional fee. Under the natural assumption that a contract's value and its complexity are positively correlated, Bajari and Tadelis (2006) derive a result that contradicts ours, and consistently, also the financial ceilings for direct awards in procurement policies: they suggest that the fixed price contract is favorable for simple projects and should be awarded by tendering procedures. Complex projects with high uncertainty, however, should be awarded by negotiating with an expert and offering her a cost plus contract. These insights rely on a different trade-off than between suitability and rent skimming: in simple projects (with rather complete catalogues of requirements), the gains from incentivizing the supplier to reduce production cost exceeds gains from avoiding renegotiation for adaption during implementation, while the opposite applies for complex contracts.

Closer to our the model is Ganuza and Pechlivanos (2000). They study a procurement from two potential suppliers who are horizontally differentiated on Hotelling's line. Depending on how far they are away from the buyer's

[9]Their chapter in Dimitri et al. (2006), which provides an intuitive overview of several issues of procurement, is based on the more formal version in Bajari and Tadelis (2001).

required specification they enjoy (or suffer from) comparative cost advantage (or disadvantage). Thus, by announcing his required specification the buyer may discriminate in favor of one supplier. Ganuza and Pechlivanos (2000) find that his optimal announcement promotes discrimination rather than homogenizing cost advantages to foster competition because gains from saving production cost exceed gains from intensified competition.

In a similar vein, Ganuza (2007) identifies a rationale for the empirical observation that public procurement authorities underinvest in the preparation of their catalogues of requirements. He models a two-stage game, in which the buyer first invests in developing his specific need on Salop's circle, and then awards the contract to a differentiated supplier. After the buyer's need realizes, the parties' (putative) renegotiation yields cost overruns. Ganuza (2007) finds that mitigated competition due to horizontal differentiation renders underinvestment and costly renegotiation optimal.

In chapter 2, pages 65f, of this thesis we review literature on differentiation with demand uncertainty, and thereby emphasize our contribution of introducing differing priors, i.e., of analyzing procurements with a self-proclaimed expert.

We provide this analysis in the following structure. In section 1.2 the buyer directly awards a contract to the expert. In section 1.3 the expert competes with a non-expert in a tendering procedure. In section 1.4 we compare these two procurement policies in terms of welfare and the buyer's cost. Section 1.5 concludes.

1.2 Direct award

1.2.1 Benchmark model

A buyer awards an indivisible project (e.g., the construction of a tunnel) to a firm. The implementation of the project requires satisfying several characteristics with impact on the buyer's utility (e.g., the design of the tunnel). We assume that all these non-monetary characteristics can be aggregated into a unidimensional variable S that we call *specification*. A project's specification differentiates horizontally, i.e., at equal prices the buyer's optimal choice depends on his particular need.

Ex ante the buyer's need s is unknown (e.g., because it depends on unexplored geological conditions). The buyer and the firm agree that the need is uniformly distributed. However, their priors on its support differ. While the buyer believes that his need lies within the unit interval, the firm is convinced that it is drawn from a smaller support. This captures the notion that the contracting authority ex ante has less information about a project's requirements than the contractor. We thus refer to the latter as (a female) *expert*. According to his prior, the buyer asks the expert for an offer that consists of a specification within $[0, 1]$ and a price. We define the expert's prior as $S \sim U[\underline{s}, \overline{s}]$ with $0 < \underline{s} < \overline{s} < 1$. The density of S thus measures the precision of the firm's information: the higher $f = 1/(\overline{s} - \underline{s})$, the more precise expert knowledge. Note that the interval $[\underline{s}, \overline{s}]$ may be located asymmetrically in $[0, 1]$. That is, the mean of $E[S] = (\underline{s} + \overline{s})/2$ may differ from $1/2$. Further note that–at this point–we do not specify whether the buyer's or the firm's prior is correct, nor whether they are common knowledge. Because the expert is firmly convinced of $S \sim U[\underline{s}, \overline{s}]$, she does not update her prior (or, therefore equivalently, her "belief") and the absence of an assumption on the need's true support has no influence on her equilibrium offer.[10]

We denote the firm's offer by (s_M, p_M) where M represents the monopolistic position of the expert in case of a directly awarded project. The expert initially decides on the specification, and subsequently on the price. This timing captures the notion of sticky specifications and flexible prices. At the last stage, the buyer receives offer (s_M, p_M), learns its horizontal need s, and decides whether to implement the project. Deviations from s to the specification s_M cause disutility. The disutility is quadratic in the difference and "costs" t per unit. The buyer thus compares total cost $p_M + t\,(s - s_M)^2$ with his willingness to pay, denoted w, in order to decide whether to accept or reject the offer. If he rejects, the expert's profit is zero because we normalize the cost of preparing the offer to zero. However, we consider the firm's cost for project implementation, $0 \leq c < w$, which is incurred if the buyer accepts the offer. In this case, the expert's expected profit is thus $\pi_M = (p_M - c)\,\rho_M$, where ρ_M denotes her expected winning probability.

We focus on subgame perfect equilibria in pure strategies. They are characterized by the profit maximizing specification s_M^* at the first stage, and price $p_M^*(s_M)$ for all $s_M \in [0, 1]$ at the second stage.

[10]We introduce an assumption on the true support of the buyer's need in section 1.4 in order to analyze procurement policies.

1.2.2 Offers for a directly awarded contract

We solve the game by backward induction. At the last stage, the buyer learns his need s, and either accepts or rejects the offer (s_M, p_M). This determines the firm's expected implementation probability, $\rho = E\Pr\left(w-p_M-t(s-s_M)^2 \geq 0\right)$. Given the expert's prior, ρ is equivalent to the cumulative probability that $s \in \left[s_M-\sqrt{(w-p_M)/t}, s_M+\sqrt{(w-p_M)/t}\right]$, and therefore, endogenous. Figure 1.2 depicts the two conjunctions of this interval with $[\underline{s}, \overline{s}]$ that, in equilibrium, determine the expert's expected implementation probability.[11]

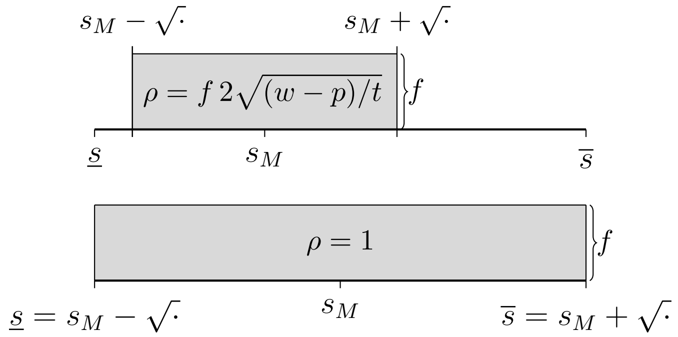

Figure 1.2: Expected implementation probabilities of direct award offers

At the second stage, the expert anticipates these probabilities, takes the specification $s_M \in [0,1]$ as given, and decides on her price $p_M(s_M)$. Note that focussing on strictly positive profits restricts equilibrium strategies to $w > p_M(s_M) > c$. We refer to the interval length of price candidates (the contract's net value) weighted by the buyer's sensitivity to unsuitability, $\omega \equiv (w-c)/t$, as

[11] Actually, there are four possible conjunctions of the two intervals (see figure 1.11 in the appendix). However, there are only two equilibrium candidates: suppose the expert bids a price p_M such that the "acceptance interval" $\left[s_M-\sqrt{(w-p_M)/t}, s_M+\sqrt{(w-p_M)/t}\right]$ is a strict subinterval of $[\underline{s}, \overline{s}]$, and thus, $\rho = f\,2\sqrt{(w-p)/t} < 1$. Given this (rather high) price, the expert's specification ensures that the acceptance interval never overlaps her prior on the need's support because this would reduce her winning probability, i.e., she abstains from bidding a specification in the peripheries of the unit interval. Now suppose that the expert bids a price p_M, which ensures that the buyer accepts her offer, i.e., that $\rho = 1$. This (rather low) price is such that the lengths of the two intervals coincide because any further price reduction directly reduces her expected profit, while her winning probability still equals 1. Given this (rather low) price, any deviation from $s_M = E[S]$ (implying a shift of the acceptance interval to the left or right) would reduce her winning probability. It follows that the expert's specification equals her expected value of the buyer's need. Proposition 1.1 shows that both equilibrium candidates depicted in figure 1.2 indeed constitute equilibrium offers.

the expert's *pricing power*. It is crucial for the trade-off between a high price and a high implementation probability, and thus, together with the precision of the expert's information, $f = 1/(\overline{s} - \underline{s})$, determines the equilibrium offer for a directly awarded project.

Proposition 1.1 (Direct award). *The offers for a directly awarded contract are*

(i) $s^r_M \in \left[\underline{s} + \sqrt{(w-c)/(3t)}, \overline{s} - \sqrt{(w-c)/(3t)}\right]$ *and* $p^r_M = (2w+c)/3$ *if and only if* $\omega < 3\left(\overline{s} - \underline{s}\right)^2/4$*, and,*

(ii) $s^*_M = (\underline{s} + \overline{s})/2$ *and* $p^*_M = w - t\left(\overline{s} - \underline{s}\right)^2/4$ *otherwise.*

The proof of proposition 1.1 is in the appendix. It shows that the positive effect of a high price on the expert's profit exceeds the negative effect of a low implementation probability if and only if

$$\omega < 3\, f^{-2}/4. \tag{1.1}$$

If condition (1.1) is satisfied, the pricing power is limited for a given precision of expert knowledge f. Equivalently, for a given pricing power ω, the expert's knowledge is imprecise, and the support of her belief about the buyer's need thus similar to the latter's catalogue of requirements $s_M \in [0, 1]$. In this case, the expert optimally bids $p^r_M = (2w + c)/3$, a linear combination of the buyer's willingness to pay and the cost of implementation, which is independent of specification s_M. Intuitively, the expert abstains from compensating the buyer for unsuitable project specifications if her pricing power is limited and her knowledge moderate. Consistently, the expected implementation probability at the second-stage is also independent of s_M and strictly smaller than 1. Formally, $\rho^r_M = f \cdot 2\sqrt{\omega/(3\,t)} < 1$, i.e., if the expert's information is imprecise she exploits her limited pricing power, and thereby risks that the buyer rejects her offer. Her specification in this case ensures that the interval in which the buyer accepts her offer is a strict subinterval of her prior on the support of his need.

In contrast, if condition (1.1) is violated, i.e., if either the expert's information is precise or her pricing power high, she ensures that the buyer implements her offer by granting a price discount that fully compensates him for unsuitable project specifications. In this case, the second-stage price $p^*_M(s_M)$ is a function of $s_M \in$

$[0,1]$ because it corresponds to the highest price that sets $\rho = 1$, i.e., that satisfies $s_M - \sqrt{(w-p)/t} = \underline{s}$ and $\overline{s} = s_M + \sqrt{(w-p)/t}$. Consequently, the expert at the first stage minimizes expected unsuitability cost by offering $s_M^* = E[S] = (\underline{s}+\overline{s})/2$. The proof of proposition 1.1 shows that the expert's profit is maximized if she exploits her precise information to fully compensate the buyer even in the worst case. More precisely, if either her pricing power is high or her information precise, the expert appropriates the buyer's willingness to pay up to a price discount equal to the maximal unsuitability cost. Formally, if and only if $\omega \geq 3\,tf^{-2}/4$ the expert offers $(s_M^*, p_M^*) = \left(E[S], w - t\,(\overline{s}-\underline{s})^2/4\right)$, which implies $\rho_M^* = 1$.

In section 1.4, we compare this latter equilibrium offer for a directly awarded contract with the offers in a tendering procedure, which we determine in the next section.

1.3 Tendering procedure

1.3.1 Add a non-expert competitor

Now we add a (male) competitor to the procurement process described in the benchmark model in section 1.2.1.[12] That is, the buyer chooses one out of two firms $i = 0, 1$ to carry out the project.

Ex ante, when the buyer calls for tenders, his need s is unknown. All players agree that the need is uniformly distributed. However, their priors on its support differ. Let the expert be firm $i = 0$, and accordingly, denote her prior by $S_0 \sim U[\underline{s}, \overline{s}]$ with $0 < \underline{s} < \overline{s} < 1$. Her competitor, firm 1, by contrast, believes in the buyer's announcement, i.e., that the need is drawn from the unit interval. We denote the buyer's and firm 1's prior by $S_1 \sim U[0,1]$.

At this point, we do not specify whether S_0 or S_1 is correct.[13] However, both priors are common knowledge. This captures the notion of a *self-proclaimed*

[12]That is, we introduce differing priors in Hotelling's line of spatial competition with demand uncertainty. In chapter 2 of this thesis we discuss the assumptions of this specification-then-price game, carefully study its equilibrium types, and determine conditions for expert knowledge to be profitable.

[13]The absence of an assumption on the need's true support has no impact on the results because firms do not update their priors before handing in the offers, and the buyer learns its need after having received them. However, the need's true support is crucial for the welfare analysis. We thus introduce an appropriate assumption in section 1.4.

expert: firm 0 is convinced that her ex ante information on the support of the buyer's need is superior to the other players' information. Firm 1, in contrast, assesses the expert's information as wrong, and complies with the more broadly formulated call for tenders.

In this call for tenders, the buyer asks both firms to hand in offers that contain a specification within the unit interval $s_i \in [0,1]$, and a price $p_i \geq 0$. We denote the offers by (s_0, p_0) and $(1-s_1, p_1)$, respectively. At the first stage, firms simultaneously choose their specifications. Let $s_0 < 1-s_1$, i.e., the expert offers a specification to the left of her competitor.[14] At the second stage, firms observe the specifications, and then simultaneously decide on their prices.

At the third and last stage, the buyer receives the offers, and learns his need s. He compares total cost $p_0 + t\,(s-s_0)^2$ with $p_1 + t\,(1-s_1-s)^2$, and awards the contract to the firm with the cheaper offer. We assume that $|p_0 - p_1| < t$, and the buyer's willingness to pay, w, high enough so that he certainly implements one of the two projects. Moreover, we normalize the cost for preparing an offer to zero, and assume firms to face identical constant cost for project implementation, $c \geq 0$.

We focus on subgame perfect equilibria in pure strategies. In equilibrium, the buyer minimizes cost, and both firms $i = 0, 1$ maximize their profits $\pi_i \equiv (p_i - c)\,\rho_i$, where ρ_i denotes firm i's expected winning probability. Equilibria are characterized by the profit maximizing specifications $(s_0^*, 1-s_1^*)$ and $(p_0^*(s_0), p_1^*(s_1))$ for all admissible s_0 and s_1. The corresponding equilibrium path is $(s_0^*, 1-s_1^*)$ and $(p_0^*(s_0^*), p_1^*(s_1^*))$.

1.3.2 Offers in a tendering procedure

In this section, we informally derive the equilibrium offers in the tendering procedure, which we then present in proposition 1.2.

We solve the game by backward induction. At the last stage, the buyer learns his need s, and chooses the cheaper of the two offers (s_0^*, p_0^*) and $(1-s_1^*, p_1^*)$. He

[14] In contrast to symmetric formulations of spatial competition, this assumption is crucial for our results: it excludes strategies that aim at maximizing differentiation by offering specifications in the competitor's special field. See the comment on proposition 1.2, page 22.

is thus indifferent at specification

$$\hat{s} \equiv s_0 + \frac{1 - s_0 - s_1}{2} + \frac{p_1 - p_0}{2\,t\,(1 - s_0 - s_1)}.$$

Firms anticipate $\hat{s}$, which determines their expected winning probabilities

$$\rho_0 \equiv \begin{cases} 0, & \text{if } \hat{s} \leq \underline{s} \\ f_0\,h_0, & \text{if } \underline{s} < \hat{s} < \overline{s}, \\ 1, & \text{if } \overline{s} \leq \hat{s}, \end{cases} \quad \text{and} \quad \rho_1 \equiv \begin{cases} 1, & \text{if } \hat{s} \leq 0, \\ f_1\,h_1 & \text{if } 0 < \hat{s} < 1, \\ 0, & \text{if } 1 \leq \hat{s}, \end{cases}$$

where $f_0 = 1/(\overline{s} - \underline{s})$, $f_1 = 1$, $h_0 = \hat{s} - \underline{s}$, and $h_1 = 1 - \hat{s}$.

Figure 1.3 depicts these probabilities, and illustrates that ρ_0 and ρ_1 need not sum up to 1. That is, the expert expects her competitor to win with probability $1 - \rho_0$, which may differ from ρ_1. Analogously, firm 1 expects the expert to win with probability $1 - \rho_1$, which may differ from ρ_0.

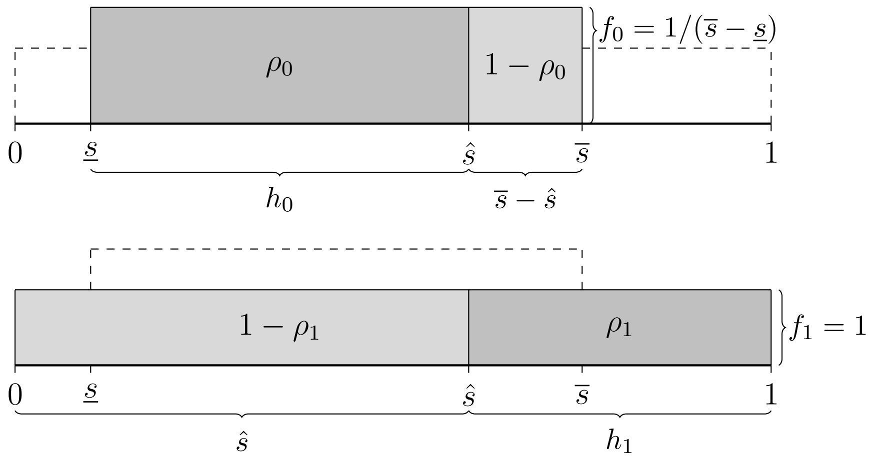

Figure 1.3: Expected winning probabilities in tendering procedures

Due to this discrepancy between the firms' expectations, focussing on strictly positive probabilities does not imply $\rho_i < 1$, $\forall i = 0, 1$. Consequently, the profit functions $\pi_i = (p_i - c)\,\rho_i$ exhibit a kink at the highest price p_i that sets $\rho_i = 1$. This is important in order to understand the pricing strategies at the second stage. While firm 1's profit $\pi_1 = (p_1 - c)\,(1 - \hat{s})$ is independent of the expert's prior, $S_0 \sim U[\underline{s}, \overline{s}]$ influences $\pi_0 = (p_0 - c)\,f_0\,(\hat{s} - \underline{s})$ in two ways: π_0 is strictly increasing in the precision of expert knowledge, $f_0 = 1/(\overline{s} - \underline{s})$, which in turn is increasing in

$\underline{s}$. The expert's hometurf $h_0 = \hat{s} - \underline{s}$, however, is strictly decreasing in $\underline{s}$. We call this negative effect of knowledge *unsuitable project specifications* in the expert's special field. Analogously, we interpret $1 - \overline{s}$ as unsuitable specifications that affect the expert's competitor. If $1 - \overline{s}$ is relatively low, that is, if the expert assesses her competitor's special field as suitable for the buyer's need, she bids a price that leaves her competitor a chance to win, i.e., in this case $1 - \rho_0 < 1$. Otherwise, if $1 - \overline{s}$ is relatively high, the expert's best price response for given specifications s_i stifles competition. That is, she bids the highest price reaction $p_0(p_1)$ that solves $\rho_0 = 1$.

It is simple to show that the latter "play it safe" strategy is never optimal for firm 1 who's price reaction is independent of $\underline{s} < \overline{s}$. The expert's prior S_0 thus solely determines the type of price equilibrium, which we outline in lemma 1.1. Its proof corresponds to the one of proposition 2.1 in chapter 2 of this thesis. We refer to the equilibrium with $\rho_0 < 1$ as *Duopoly*, and the equilibrium with $\rho_0 = 1$ as *Limit* equilibrium.

Lemma 1.1 (Price equilibria)**.** *Given the specifications* $0 \leq s_0 < 1 - s_1 \leq 1$, *there exist two types of pure strategy equilibria, which are unique in the respective parameter range. If and only if* $1-\overline{s} < 1/2+(-s_0+s_1-2\underline{s})/6$ *and* $\underline{s} < (3+s_0-s_1)/4$, *firms bid Duopoly prices*

$$p_0^D(s_0) \equiv c + t\,(1 - s_0 - s_1)\,(1 + (s_0 - s_1 - 4\underline{s})/3)\,, \text{ and,} \tag{1.2}$$

$$p_1^D(s_1) \equiv c + t\,(1 - s_0 - s_1)\,(1 + (-s_0 + s_1 - 2\underline{s})/3)\,. \tag{1.3}$$

If and only if $1 - \overline{s} \geq 1/2 + (-s_0 + s_1 - 2\underline{s})/6$, *firms bid Limit prices*

$$p_0^L(s_0) \equiv c + t\,(1 - s_0 - s_1)\,(s_0 - s_1 - 1 + 4(1 - \overline{s}))\,, \text{ and,} \tag{1.4}$$

$$p_1^L(s_1) \equiv c + 2\,t\,(1 - s_0 - s_1)\,(1 - \overline{s})\,. \tag{1.5}$$

The condition that determines the equilibrium type corresponds to a comparison of firm 1's expected winning probabilities. Let ρ_1^D denote his expected winning probability at prices (1.2) and (1.3), and consistently, ρ_1^L his expected probability at prices (1.4) and (1.5). If the expert assesses firm 1's expected winning probabilities as rather accurate, her Duopoly pricing leaves her competitor a chance to win. More precisely, this is the case for $\rho_1^L < \rho_1^D$, i.e., if the expert agrees on the ranking of her competitor's winning probabilities (since

$1 - \rho_0^L = 0 < 1 - \rho_0^D$). Otherwise, that is, if and only if $\rho_1^D \leq \rho_1^L$, firms bid their Limit prices. Intuitively, if the expert believes that her competitor is highly overconfident in case of Limit pricing, she indeed bids her Limit price reaction.

For the example of identical specifications $s_0 = s_1 \in [0, 1/2)$ figure 1.4 depicts the region of Duopoly pricing in grey, and the region of Limit pricing in white. In the hatched area, there exists no price equilibrium given the specifications in this example. Along each of the dashed lines one of the effects of knowledge on the expert's profit is constant. Consider $1 - \overline{s} = a - \underline{s}$ for $a < 1/2$, along which precision $f_0 = 1/(1-a)$ is low, and firms bid their Duopoly prices. Otherwise, when precision is high and unsuitability low enough[15], the expert makes sure to win the contract herself by Limit pricing. Similarly, consider $1 - \overline{s} = b\,\underline{s}$. For $b < 1/3$, the expert's special field is relatively unsuitable for the buyer's need. In this case, her pricing leaves her competitor a chance to win. If, instead, the expert assesses firm 1's range of specifications as relatively unsuitable (and precision f_0 is high enough), her price deters competition.

[15] Specifically, in the example of figure 1.4 (where $s_0 = s_1 \in [0, 1/2)$), the Limit price equilibrium exists if $a \geq 1/2$ and $\underline{s} \leq \min\{3(a - 1/2)/2, 3/4\}$.

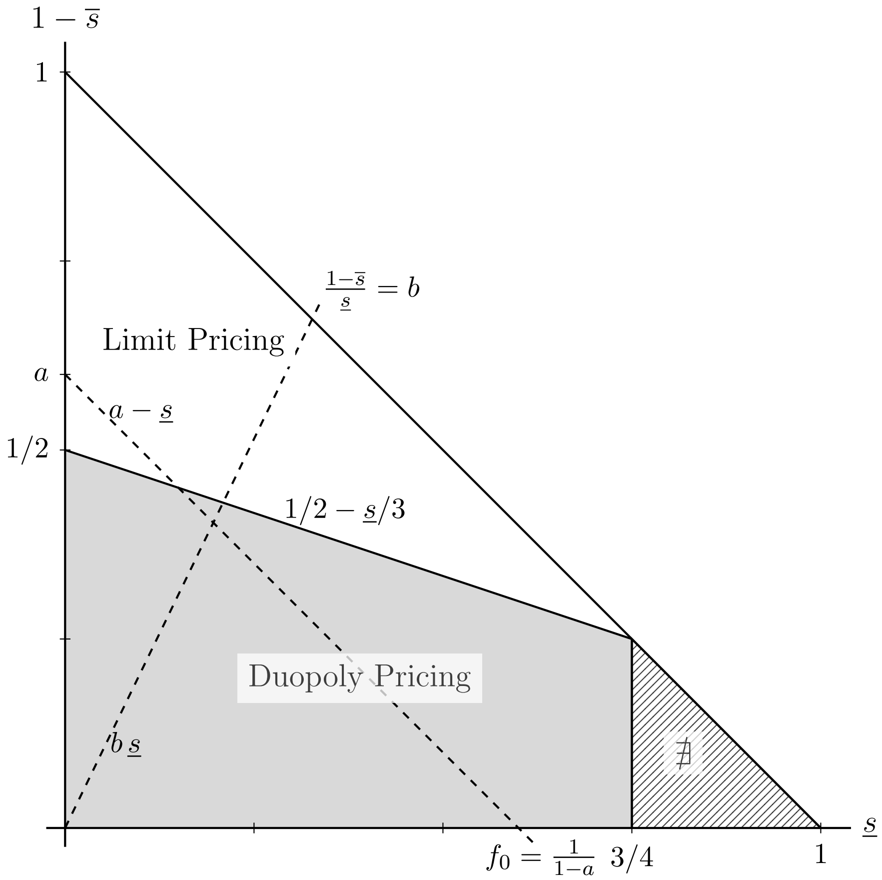

Figure 1.4: Types of second-stage price equilibria for $s_0 = s_1 \in [0, 1/2)$

At the first stage, competitors take Duopoly prices (1.2) and (1.3) as well as Limit prices (1.4) and (1.5) as given and decide on their specifications. In Duopoly price equilibrium, both firms thereby face the trade-off between mitigating price competition by offering a differentiated project (i.e., bidding a low specification s_i) and stealing the competitor's business by offering a customized project (i.e., bidding a high s_i). For the expert's competitor, who does not believe in unsuitable specifications, the positive effect of the maximally differentiated specification on its price markup unambiguously exceeds the positive effect of potential customization on its home turf. Firm 1 therefore bids $1 - s_1^* = 1$. The expert's awareness of unsuitable specifications, in contrast, is an additional incentive for customization. Given the Duopoly prices, she thus bids $s_0^{DC} > 0$ if she assesses her own special field as unsuitable, i.e., if $\underline{s}$ is high enough.

In Limit price equilibrium, the expert faces the same trade-off: differentiation increases the maximum price that deters competition, and customization increases her home turf. The first effect is captured by the total degree of differentiation $1 - s_0 - s_1$, and thus, is analogous to the Duopoly case. The second effect, however, is independent of $\underline{s}$ as its mechanism differs from the one in the Duopoly equilibrium. Intuitively, because the expert's second-stage Limit pricing implies $h_0^L = \overline{s} - \underline{s} \Leftrightarrow \rho_0^L = 1$, it relies on the difficulty to set the buyer indifferent at $\hat{s} = \overline{s}$: suppose that $\hat{s} < \overline{s}$. In order to steal the competitor's business (increase $\hat{s}$ up to $\overline{s}$) the expert grants a high price discount if $\overline{s}$ is high (or equivalently, if $1 - \underline{s}$ low), and, a low price discount otherwise. In the former case, customization is optimal because the indirect reduction of a high price discount through lower unsuitability cost exceeds the effect of a stronger threat of direct price competition. If $\overline{s}$ is low, in contrast, deterring direct price competition by means of maximal differentiation is more profitable than indirectly reducing the already low price discount.

To understand firm 1's specification choice in Limit equilibrium, recall the discrepancy due to differing priors, i.e., $\rho_0^L = 1$ does not imply $\rho_1^L = 0$. In fact, firm 1 is convinced that the expert's information is wrong. Consequently, his expected home turf corresponds to the degree of his competitor's "misjudgement" concerning his own special field. Formally, $\rho_1^L = h_1^L = 1 - \overline{s}$. Therefore, firm 1 in Limit equilibrium faces no trade-off, and the only effect of customization is fiercer price competition. Firm 1 thus bids his maximally differentiated specification $1 - s_1^* = 1$.

Substituting these profit maximizing specifications of the first stage in the second-stage prices (1.2) to (1.5) yields the four equilibrium types of the entire tendering game, which we present in proposition 1.2. Its proof corresponds to the one of proposition 2.2 in chapter 2 of this thesis. Besides existence, it shows that each equilibrium type is unique in the corresponding range of parameters. Note that proposition 1.2 crucially relies on the assumption $s_0 < 1 - s_1$, which prevents strategies that aim at maximizing differentiation by offering specifications in the competitor's special field, and thus, jeopardize existence of customized equilibrium types: given the expert offers close enough to $1 - s_1 = 1$, her competitor would benefit from deviating to $1 - s_1 = 0$.

Proposition 1.2 (Tendering procedure)**.** *If the buyer calls for tenders, firms bid their* *(i) Differentiated Duopoly offers*

$$\left(s_0^{DD},\, p_0^{DD}\right) = (0,\, c + 2\,t\,(1/2 - 2\underline{s}/3))\,,\ \textit{and}$$
$$\left(1 - s_1^{*},\, p_1^{DD}\right) = (1,\, c + 2\,t\,(1/2 - \underline{s}/3))\,,$$

if and only if $\underline{s} \leq 1/4$ *and* $1 - \overline{s} < 1/2 - \underline{s}/3$.
(ii) Customized Duopoly offers

$$\left(s_0^{DC},\, p_0^{DC}\right) = \left((4\underline{s} - 1)/3,\, c + 32\,t\,(1 - \underline{s})^2/27\right),\ \textit{and}$$
$$\left(1 - s_1^{*},\, p_1^{DC}\right) = \left(1,\, c + 40\,t\,(1 - \underline{s})^2/27\right),$$

if and only if $\underline{s} > 1/4$ *and* $1 - \overline{s} < 5(1 - \underline{s})/9$.
(iii) Differentiated Limit offers

$$\left(s_0^{LD},\, p_0^{LD}\right) = (0,\, c + t\,(4(1 - \overline{s}) - 1))\,,\ \textit{and}$$
$$\left(1 - s_1^{*},\, p_1^{LD}\right) = (1,\, c + 2\,t\,(1 - \overline{s}))\,,$$

if and only if $1/2 \leq 1 - \overline{s}$. *(iv) Customized Limit offers*

$$\left(s_0^{LC},\, p_0^{LC}\right) = \left(1 - 2(1 - \overline{s}),\, c + 4\,t\,(1 - \overline{s})^2\right),\ \textit{and}$$
$$\left(1 - s_1^{*},\, p_1^{LC}\right) = \left(1,\, c + 4\,t\,(1 - \overline{s})^2\right),$$

if and only if

$$1/2 > 1 - \overline{s} \geq \begin{cases} 1/2 - \underline{s}/3, & \textit{for } \underline{s} \leq 1/4,\ \textit{and,} \\ 5(1 - \underline{s})/9, & \textit{for } 1/4 < \underline{s}. \end{cases}$$

Above, we outline that the expert's second-stage pricing strategy determines whether there exists a Duopoly or Limit equilibrium. Note that her optimality condition $\rho_1^D < \rho_1^L$ (and $\rho_1^L \leq \rho_1^D$, respectively), remains unchanged when solving the entire game since $\rho_1^{DD} = 1/2 - \underline{s}/3$ and $\rho_1^{DC} = 5(1-\underline{s})/9$. Figure 1.5 depicts this condition that constitutes the limit between the region of Duopoly equilibria (grey shaded area), and the region of Limit equilibria (white area). It illustrates that the firms offer specifications such that there exists a equilibrium for all $0 < \underline{s} < \overline{s} < 1$.

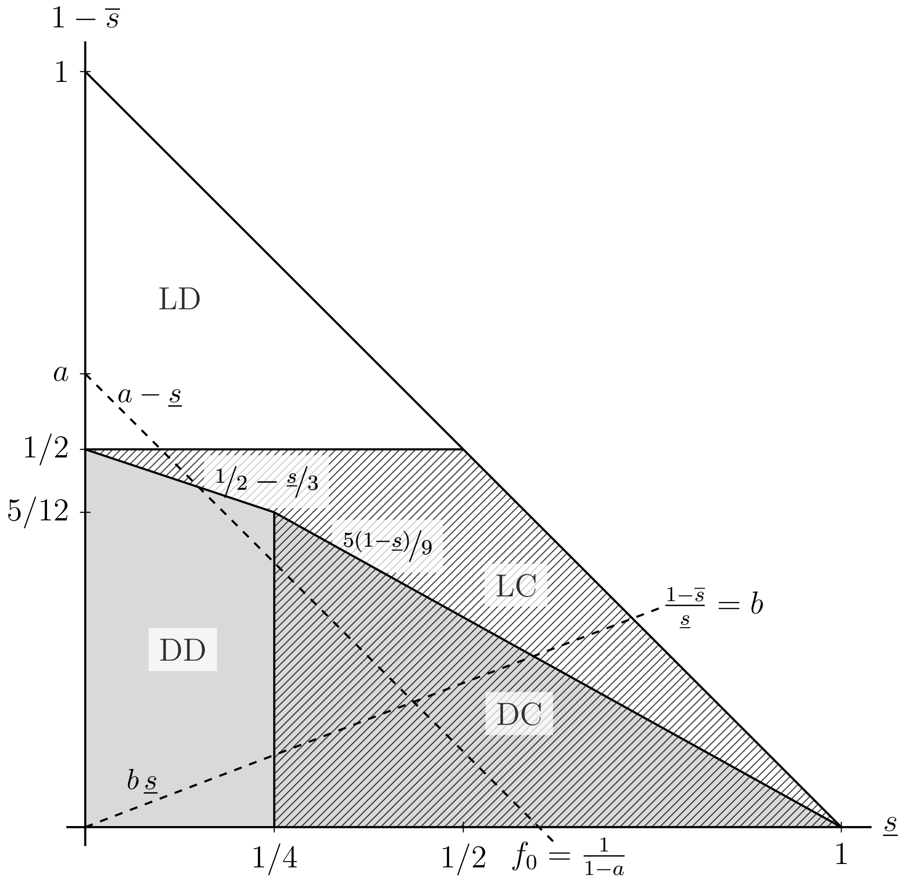

Figure 1.5: Types of equilibria in the entire tendering game

Moreover, figure 1.5 shows whether the expert customizes her project (in the hatched areas) or differentiates (elsewhere). Recall that her competitor always opts for the latter, i.e., firm 1 bids $1 - s_1^* = 1$. Therefore, in analogy to the pricing stage, the intuition behind existence of either a customized or differentiated equilibrium depends on the expert's knowledge S_0 only: for a given precision f_0 (along the dashed line $a - \underline{s}$), the expert rather customizes its project if the unsuitability of its own special field is high. Accordingly, for a given unsuitability ratio $(1 - \overline{s})/\underline{s}$ (along the dashed line $b\,\underline{s}$) the expert rather customizes if its ex ante information is precise enough, and vice versa.

We regard the tendering procedure described in this section and the direct award of a contract studied in the previous section 1.2 as alternatives for the procurement of a complex project if the buyer's catalogue of requirements is incomplete when he calls for tenders, and a potential contractor is a, at least self-proclaimed, expert. We analyze the outcomes of these alternatives in what follows.

1.4 Policy analysis

In this section, we compare the offer for a directly awarded contract, presented in proposition 1.1, with the tendering offers of proposition 1.2 in terms of welfare and the buyer's total cost. Both crucially rely on the buyer's cost for unsuitable project specifications: it is a natural measure for welfare, and the buyer's total cost equals the sum of price and unsuitability cost.

Unsuitability cost is quadratic in the distance between the buyer's need s and the specification of the winning offer, i.e., $s_M^* = E[S_0]$ in case of a direct award, and either s_0^k or $1 - s_1^* = 1$ in any tendering equilibrium $k \in \{DD, DC, LD, LC\}$. Note that we ignore the risky offer for a directly awarded contract, (s_M^r, p_M^r), in what follows.[16] Formally, we define unsuitability cost as

$$l_M(s) \equiv (s - E[S_0])^2, \tag{1.6}$$

[16] We ignore the offer (s_M^r, p_M^r) because our analysis compares the offers for a directly awarded contract and in a tendering procedure. These comparisons are meaningful only for existing equilibria. In the tendering game, existence crucially relies on the assumption of a high enough willingness to pay such that the buyer certainly implements one of the offers. This assumption implies $\omega \geq 3\,(\overline{s} - \underline{s})^2/4$, and therefore, that the expert offers (s_M^*, p_M^*) for a directly awarded contract.

and,

$$l^k(s) \equiv \begin{cases} (s - s_0^k)^2, & \text{if } s \leq \hat{s}, \\ (1-s)^2, & \text{if } \hat{s} < s, \end{cases} \tag{1.7}$$

where $k \in \{DD, DC, LD, LC\}$. For the example of $\underline{s} = 1/2$, $\overline{s} = 4/5$ and $t = 3$ figure 1.6 depicts the unsuitability cost $t\,l_M$ in case of a direct award (dashed), and $t\,l^{DC}$ in case of tendering (solid) as functions of the buyer's need $s \in [0,1]$.

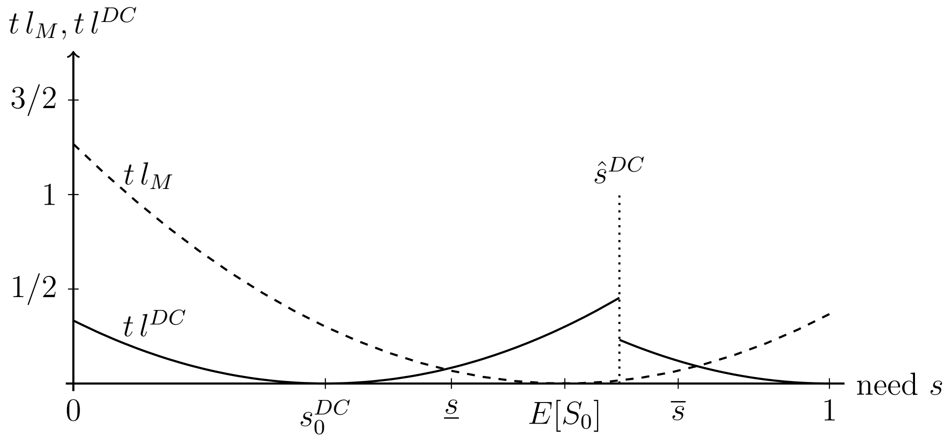

Figure 1.6: Unsuitability cost for $\underline{s} = 1/2$, $\overline{s} = 4/5$ and $t = 3$

Figure 1.6 illustrates that $l^k(s)$ consists of two parabolas with vertices equal to s_0^k and 1 that have the identical curvature as $l_M(s)$. Lemma 1.2 is a corollary thereof together with the observation that the expert's specification in case of direct award, $E[S_0]$, lies strictly between the tender specifications s_0^k and 1, for all equilibrium types k. Intuitively, if the buyer awards the contract directly, the expert maximally customizes her offer, i.e., minimizes her expectation about the former's unsuitability cost. This allows her to charge a higher price without risking a rejection of the offer. In a tendering procedure, in contrast, customization intensifies price competition. This negative effect exceeds the positive effect of stealing the competitor's business. Therefore, firm 1 offers his maximally differentiated specification. The expert, however, is aware of unsuitable project specifications that jeopardize her home turf in case of differentiation, and thus, has an additional incentive to customize. The (price-driven) incentive for customization in case of a direct award is clearly stronger than the sum of the two customization incentives (concerning the winning probability) in a tendering procedure. Together with the functional forms of $l_M(s)$ and $l^k(s)$, this implies that there are realizations of the buyer's need for which unsuitability cost in a tendering exceed unsuitability cost in case of a directly awarded contract. Lemma 1.2 formally describes this, as well as additional observations on the unsuitability cost. Its proof is in the appendix.

Lemma 1.2 (Unsuitability cost). *(i) For all* $0 < \underline{s} < \overline{s} < 1$, $s \in [0,1]$, *and* $\forall k \in \{DD, DC, LD, LC\}$ *there exists an interval of needs* $s \in [s_L^k, s_H]$, *with* $s_L^k = (s_0^k + E[S_0])/2$ *and* $s_H = (1 + E[S_0])/2$, *that imply* $l^k(s) \geq l_M(s)$.

(ii) Moreover, $l^k(s)$ *is maximal at need* $\hat{s}^k \in (s_L^k, s_H]$ *in both differentiated equilibria* $k = DD, LD$, *as well as in* $k = DC$ *iff* $\underline{s} \leq 10/19$, *and in* $k = LC$ *iff* $1/3 \leq 1 - \underline{s}$. *Otherwise, in customized equilibria* $l^k(s)$ *is maximal at need* $s = 0$, *however, still lower than cost of a direct award, i.e.* $l_M(0) > l^k(0)$. *Cost* l_M *in case of direct award is minimal at* $E[S_0]$, *where* $E[S_0] \in (s_L^k, s_H)$, *and maximal at* $s = 0$ *or* $s = 1$.

(iii) The bounds of $[s_L^k, s_H]$ *lie closer to the bounds of* $[\underline{s}, \overline{s}]$ *than to the ones of* $[0, 1]$.

Not surprisingly, the specification of a directly awarded project is less costly compared to the two specifications of a tendering procedure if and only if the buyer's need realizes close to the former. To deepen insight we consider the ex ante *expected* cost for unsuitability in what follows. For this purpose, we introduce an assumption on the true support of the buyer's need.

1.4.1 Welfare

So far, firm 0 was a self-proclaimed expert only, i.e., we have not specified whether her own prior, $S_0 \sim U[\underline{s}, \overline{s}]$, or her competitor's prior, $S_1 \sim U[0,1]$, is correct because there has been no reason to do so: a judgement on the firms' convictions does not affect the equilibria established in propositions 1.1 and 1.2. Welfare considerations, however, require an assumption on the need's true support.

We introduce the exogenous parameter $\gamma \in (0,1)$, which denotes the probability that s is drawn from $[\underline{s}, \overline{s}]$. In other words, γ is the probability that the expert's prior is correct. Therefore, we call γ the *reliability* of the expert's ex ante information.

To gain intuition for the main results presented in propositions 1.3 and 1.5, recall that the firms' priors $S_0 \sim U[\underline{s}, \overline{s}]$ and $S_1 \sim U[0,1]$ are two random variables. Their realization is subject to uncertainty and mutually exclusive: either S_0 realizes with probability γ or S_1 realizes with complementary probability $1-\gamma$. For all $\gamma \in (0,1)$, the expected joint density of S_0 and S_1 thus is

$$f_{S_0,S_1} \equiv f_X = \begin{cases} 1-\gamma, & \text{if } 0 \leq x \leq \underline{s}, \\ 1-\gamma+\gamma\, f_0, & \text{if } \underline{s} < x < \overline{s}, \\ 1-\gamma, & \text{if } \overline{s} \leq x \leq 1, \end{cases}$$

where $x \in [0,1]$ denotes a draw from the joint density f_X. Figure 1.7 shows f_X, i.e., the need's true distribution for the example $\underline{s} = 1/2$, $\overline{s} = 4/5$ and $\gamma = 1/2$.

In the small interval $[\underline{s}, \overline{s}]$, the density of the buyer's need is $1-\gamma+\gamma\, f_0$. It corresponds to the sum of the baseline density $1-\gamma$, which relies on firm 1's prior $f_1 = 1$, and $\gamma\, f_0$, the product of the reliability and precision of the expert's information. We define $\nu \equiv \gamma\, f_0$ and call ν the *validity* of expert knowledge. The validity of expert knowledge (as opposed to the base line density $1-\gamma$) is crucial for the following welfare comparisons.

Since $\underline{s} < \hat{s}^k \leq \overline{s}$ is implied by $\rho_0^k \in (0,1]$ in all types of tendering equilibria k, the expectations over X of the unsuitability cost (1.6) and (1.7) are

$$\begin{aligned} E_X[l_M] = & \quad \left[\nu \int_{\underline{s}}^{\overline{s}} (x - s_M^*)^2 \, dx + (1-\gamma) \int_0^1 (x - s_M^*)^2 \, dx\right] \\ = & \frac{1}{12}\left[\nu\, (\overline{s} - \underline{s})^3 + (1-\gamma)\left((2 - \underline{s} - \overline{s})^3 + (\underline{s} + \overline{s})^3\right)/2\right] \end{aligned} \tag{1.8}$$

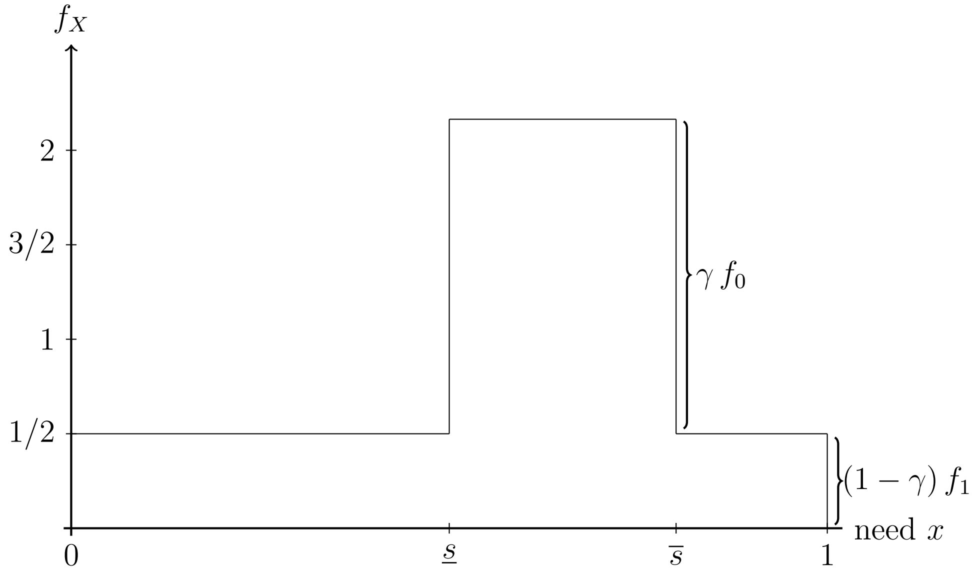

Figure 1.7: The need's true distribution for $\underline{s} = 1/2$, $\overline{s} = 4/5$, $\gamma = 1/2$

$$\begin{aligned} E_X[l^k] = &\left[\nu \left(\int_{\underline{s}}^{\hat{s}^k} \left(x - s_0^k\right)^2 \, dx + \int_{\hat{s}^k}^{\overline{s}} (1-x)^2 \, dx\right)\right. \\ &\left. + (1-\gamma) \left(\int_0^{\hat{s}^k} \left(x - s_0^k\right)^2 \, dx + \int_{\hat{s}^k}^{1} (1-x)^2 \, dx\right)\right] \\ = &\left[F_X(\hat{s}^k) \left(x - s_0^k\right)^2 + \left(1 - F_X(\hat{s}^k)\right) (1-x)^2\right], \end{aligned} \tag{1.9}$$

where $F_X(\hat{s}^k) = \nu\,(\hat{s}^k - \underline{s}) + (1-\gamma)\,\hat{s}^k$.

The results of lemma 1.3 and proposition 1.3 are both based on comparisons of $E_X[l_M]$ and $E_X[l^k]$ for each $k \in \{DD, DC, LD, LC\}$. Its proofs are in the appendix.

Lemma 1.3 (Validity and unsuitability cost)**.** *The difference in expected unsuitability cost of direct awards and tendering procedures is strictly decreasing in the validity of expert knowledge, i.e., $\partial E_X[l_M - l^k]/(\partial\nu) < 0$, for all $k \in \{DD, DC, LD, LC\}$.*

In other words, the more valid expert knowledge, the lower expected unsuitability cost of a direct award compared to a tendering. This insight relies on the observations of lemma 1.2, which are illustrated in figure 1.8 for the example of $\underline{s} = 1/2$, $\overline{s} = 4/5$, $t = 3$ and $\gamma = 1/2$.

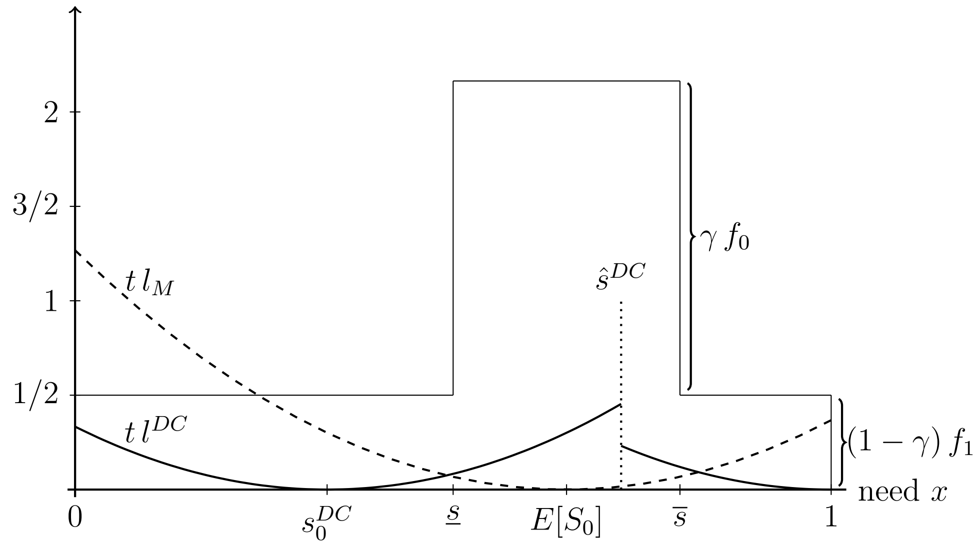

Figure 1.8: Unsuitability cost for $\underline{s} = 1/2$, $\overline{s} = 4/5$, $t = 3$ and $\gamma = 1/2$

High validity $\nu = \gamma\, f_0$ corresponds to a high relative density in the smaller interval $[\underline{s}, \overline{s}]$, which contains $E[S_0]$, the need's realization that minimizes $l_M(s)$, as well as $\hat{s}^k$, where cost in tendering is maximal in both differentiated equilibria, as well as in DC if $\underline{s} \leq 10/19$, and in LC if $1/3 \leq 1 - \underline{s}$. Otherwise, in customized equilibria $l^k(s)$ is maximal at need $s = 0$, however, is still lower than cost of a direct award, i.e., $l_M(0) > l^k(0)$. Note that $[\underline{s}, \overline{s}]$ either contains $[s_L^k, s_H]$ as in the example of figure 1.8 or at least its bounds lie closer to the bounds of $[\underline{s}, \overline{s}]$ than to the ones of $[0, 1]$. Therefore, the higher the validity the more weight on those realizations of the buyer's need for which $l_M(s) \leq l^k(s)$, and the lower the relative density towards the peripheries of the unit interval where $l_M(s) > l^k(s)$. Consequently, the higher the validity of expert knowledge the lower the unsuitability cost if the buyer awards the contract directly to the expert rather than calling for tenders.

Similar considerations drive the results of proposition 1.3, which determines whether expected cost under direct awards or tendering procedures is higher in absolute terms.

Proposition 1.3 (Welfare). *For each type of tendering equilibrium $k \in \{DD, DC, LD, LC\}$ there exists τ^k such that $\gamma/(1-\gamma) \geq \tau^k \Leftrightarrow E_X[l_M - l^k] \leq 0$. That is, direct award is welfare superior to tendering if and only if the expert's relative reliability exceeds τ^k.*

(i) For both Duopoly equilibria $k \in \{DD, DC\}$ threshold τ^k may be non-binding. More precisely, $\tau^{DD} \leq 0 \Leftrightarrow \underline{s}/3 \leq 1 - \overline{s} \leq 5\underline{s}/3$ and $\tau^{DC} \leq 0 \Leftrightarrow \alpha_L \leq \overline{s} \leq \alpha_H$ and $\underline{s} < .2698$, with α_L, α_H defined in (1.28) of the proof.

(ii) For both Limit equilibria $k \in \{LD, LC\}$ threshold τ^k is binding, i.e., $\tau^k > 0$ for all admissible $\underline{s}$ and $\overline{s}$.

(iii) There exists τ^k_γ such that $\gamma < \tau^k_\gamma \Leftrightarrow E_X[l_M - l^k] > 0$. Threshold τ^k_γ is binding, i.e., $\tau^k_\gamma < 1, \forall k \in \{DD, DC, LD, LC\}$.

In the appendix we prove existence of threshold τ^k on the relative reliability of the expert's information[17] such that direct awards to the expert are welfare superior to tender procedures if and only if $\gamma/(1-\gamma) \geq \tau^k$. Analysis of threshold τ^k yields the three main insights of proposition 1.3:

(i) in both Duopoly equilibria $k \in \{DD, DC\}$ there is a set of parameters for which a direct award to the expert is more efficient than a tendering procedure no matter how reliable expertise is. That is, even if the expert's information is wrong (formally, if $\gamma \to 0$) the unsuitability cost of a tendering procedure exceeds cost for a directly awarded contract. Note that this also holds if the expert's information is imprecise, i.e., if $f_0 \to 1$. In other words, there is a subset of the Duopoly parameter range where direct awards are welfare superior to tendering procedures, even though there is no expert (i.e., if expert knowledge is wrong, and/or negligible).

(ii) in both Limit equilibria the threshold on the reliability ratio is binding, i.e., $0 < \tau^k$ for $k \in \{LD, LC\}$. That is, in Limit equilibrium a direct award is only welfare superior to a tendering if the reliability of expert knowledge is high enough.

To understand this note that solving $E_X[l_M - l^k] = [\nu\, a^k + (1-\gamma)\, b^k]/4 \leq 0$ for the reliability ratio is equivalent to $\gamma/(1-\gamma) \geq \tau^k = -b^k/(a^k\, f_0)$. From lemma

[17] Since $\nu = \gamma\, f_0$, precision f_0 is a substitute for the reliability of the expert's information γ, and thus, has similar effects on expected unsuitability cost. However, f_0 is a function of $\underline{s}$ and $\overline{s}$, and therefore, its effects are harder to formally disentangle.

1.3, we know that $a^k < 0$ for all $k = \{DD, DC, LD, LC\}$. It follows that τ^k is negative if and only if $b^k < 0$. In Limit equilibria this is never the case, i.e., the marginal effect of a higher baseline density $1-\gamma$ on the cost difference $E_X[l_M - l^k]$ is positive. An increase in the reliability γ therefore unambiguously lowers cost of a direct award compared to a tendering. Moreover, the expert's Limit pricing strategy entails high unsuitability cost in the small interval because it sets the buyer indifferent at $\hat{s}^L = \overline{s}$. That is, the expert's Limit price ensures that the buyer implements the expert's offer if $s \in [\underline{s}, \overline{s}]$, which entails higher unsuitability cost since there is no competing offer in this interval. Figure 1.9 depicts this for the example of $\underline{s} = 1/10$, $\overline{s} = 9/20$, $t = 3$ and $\gamma = 1/2$.

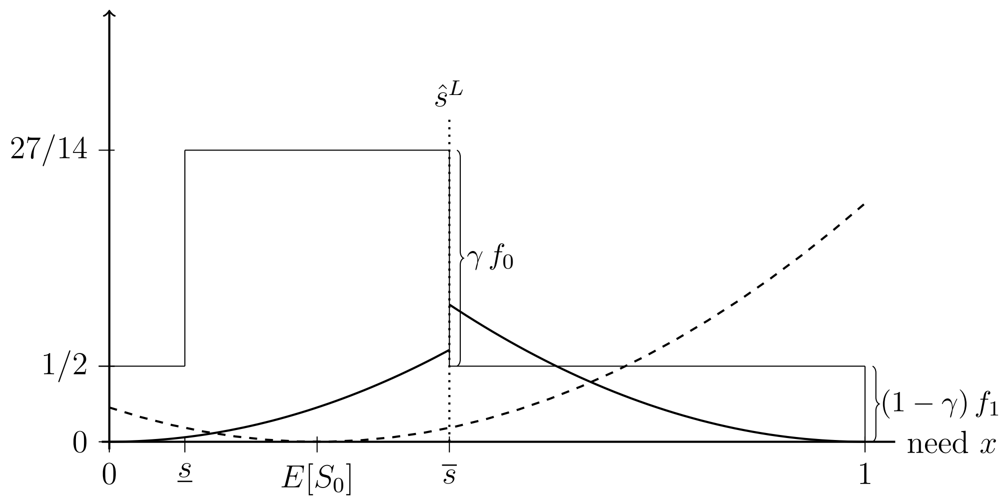

Figure 1.9: Unsuitability cost for $\underline{s} = 1/10$, $\overline{s} = 9/20$, $t = 3$ and $\gamma = 1/2$

However, figure 1.9 also illustrates that unsuitability cost of the Limit offers in the peripheral intervals $[0, \underline{s}]$ and $[\overline{s}, 1]$ is low. In fact, it is low enough to prevent the directly awarded contract from being welfare superior to a tendering $\forall \gamma \in (0, 1)$. Consequently, for $k = LD, LC$ directly awarding a contract is welfare superior to calling for tenders if and only if the expert's reliability is high enough, i.e., if $\gamma \geq \tau^k$.

In Duopoly equilibria, in contrast, b_k might be negative. In this case, reliability γ has two opposite effects on the cost difference $E_X[l_M - l^k]$ since it is decreasing in the baseline density $1-\gamma$, or equivalently, partly (through b_k) increasing in γ. That is, tendering procedures become less costly compared to direct awards as the buyer's need is likelier to realize in the small rather than in the unit interval. However, a_k is negative too. Consequently, in the small interval $[\underline{s}, \overline{s}]$ the relative cost of tendering procedures are increasing in the reliability γ. This second effect is stronger than the first, i.e., the increase of the cost benefit of direct awards in the smaller interval is higher than the increase of the cost benefits of tendering procedures in the peripheries. Interestingly, in absolute terms, the cost of direct awards are unambiguously, i.e. for all $\gamma \in (0,1)$, lower than the cost of tendering procedures if and only if $b_k \leq 0$, i.e., if the marginal effects of γ are counteracting.

(iii) tendering is never unambiguously welfare superior to directly awarding a contract. Formally, $E_X[l_M - l^k] \geq 0 \Leftrightarrow \gamma \leq \tau_\gamma^k$, where $\tau_\gamma^k < 1, \forall k \in \{DD, DC, LD, LC\}$. It follows that for any $\underline{s}$ and $\overline{s}$ there exists a $\gamma \in (\tau_\gamma^k, 1)$ such that unsuitability cost of a tendering exceeds cost for a directly awarded contract.

These insights of proposition 1.3 rely on welfare comparisons that ignore the project's *implementation probability*: while the buyer certainly implements one of the two tendering offers by assumption of a high enough willingness to pay w, he may reject the offer for a direct award. To see the latter note that

$$w - p_M^* - t\, E_X\left[(x - s_M^*)^2\right] \geq 0 \tag{1.10}$$

is neither implied by the implementation probability of 1 in case of tendering nor by $\omega \geq 3\,(\overline{s} - \underline{s})^2/4$, which ensures implementation under direct award from the expert's point of view (see proposition 1.1). The reason is that the expert's price for a directly awarded contract, p_M^*, appropriates the buyer's willingness to pay w, and moreover, is independent of implementation cost c.[18] Formally, condition

[18]Nevertheless, we know that condition (1.10) is stronger than $\omega \geq 3\,(\overline{s} - \underline{s})^2/4 \Leftrightarrow w - p_M^* - t\, E_{S_0}\left[(s - s_M^*)^2\right] \geq 0 \Leftrightarrow \rho_M^* = 1$ because the expert has no doubts about her prior $S_0 \sim U[\underline{s}, \overline{s}]$, and thus, overrates her winning probability ρ_M^*. In terms of the need's true density f_X, this is equivalent to $\gamma = 1$. For any $\gamma < 1$ it thus follows that $E_X\left[(x - s_M^*)^2\right] > E_{S_0}\left[(s - s_M^*)^2\right]$.

(1.10) is equivalent to

$$\gamma \geq \frac{2 - 3(\underline{s} + \overline{s}) + 6\underline{s}\overline{s}}{2 - 3(\underline{s} + \overline{s}) + 2\underline{s}\overline{s} + (\underline{s} + \overline{s})^2} \equiv \tau_\gamma^M. \tag{1.11}$$

Not all results of proposition 1.3 hold if we focus on the set of parameters satisfying (1.11), i.e., on the case where expert knowledge is reliable enough to ensure implementation of a directly awarded project. Proposition 1.4 outlines the insights of comparing unsuitability cost in this case of certain implementation. Its proof is in the appendix.

Proposition 1.4 (Welfare under certain implementation)**.** *Given that the buyer certainly implements* (s_M^*, p_M^*)*, or equivalently, given* $\gamma \in [\tau_\gamma^M, 1)$

(i) *threshold* τ^k*, defined by* $\gamma/(1-\gamma) \geq \tau^k \Leftrightarrow E_X[l_M - l^k] \leq 0$*, may be non-binding in both Duopoly equilibria* $k \in \{DD, DC\}$*. That is, there is a set of parameters such that a direct award is welfare superior to tendering* $\forall \gamma \in (0,1)$*.*

(ii) *a direct award is welfare superior to tendering in both Limit equilibria* $k \in \{LD, LC\}$. *Formally,* $0 < \tau_\gamma^k \leq \tau_\gamma^M$, *where* $\gamma \geq \tau_\gamma^k \Leftrightarrow E_X[l_M - l^k] \leq 0$.

(iii) *tendering is welfare superior to a direct award if and only if* $\gamma < \tau_\gamma^k$. *This set of parameters exists in both Duopoly equilibria* $k \in \{DD, DC\}$, *and the threshold is binding, i.e.,* $\tau_\gamma^k < 1$.

Note that results (i) and (iii) of proposition 1.4 are similar to proposition 1.3: in the Duopoly range, there exists a subset of parameters that satisfy $\tau_\gamma^k < \tau_\gamma^M$. In this case a direct award is less costly than a tendering. Moreover, there exists another subset that satisfies $\tau_\gamma^M < \tau_\gamma^k$. In this case, a direct award is less costly if and only if the reliability of expert knowledge is high enough, i.e., if $\gamma \geq \tau_\gamma^k$. Because both thresholds, τ_γ^M and τ_γ^k, might be negative there exists a subset of parameters for which direct awards are welfare superior to tenderings even if expert knowledge is not valid (i.e., if $\nu \to 0$). However, since $\tau_\gamma^M < 1$ and $\tau_\gamma^k < 1$, the opposite is not true. That is, tendering is welfare superior to a direct award only if the reliability of expert knowledge is low enough.

In the Limit range of parameters, in contrast, welfare considerations drastically change if we focus on certain implementation. See result (ii) in proposition 1.4: direct awarding a contract is unambiguously welfare superior to tendering because $\tau_\gamma^k \leq \tau_\gamma^M$ for both $k = LD, LC$. Consequently, the welfare benefits of tendering procedures in proposition 1.3, which prevail if expert knowledge is unreliable, stem from the assumption that the buyer certainly implements one of the two offers in a tendering but may reject the offer for a direct award. As soon as we ensure implementation of the latter, direct award is unambiguously welfare superior. Intuitively, certain implementation of the expert's offer for a direct award is equivalent to a high reliability of the expert's information. Given such high reliability, it is more efficient to directly award the contract to the expert rather than calling for tenders.

All findings in terms of welfare are consistent with the exemption clause in public procurement policies that allows direct awards of contracts to experts. However, a more prominent criterion for the lawfulness of a direct award is the project's value: public procurement policies usually determine expenditure ceilings for the admissibility of direct awards. Additionally to unsuitability cost, the buyer's total cost considers this monetary factor in form of prices. In the next section, we compare the buyer's total cost if he directly awards the project with his cost in case of tendering.

1.4.2 The buyer's total cost

The buyer's total cost, denoted $E_X[g_M]$ in case of a direct award and $E_X[g^k]$ in any type of tendering equilibrium k, is the sum of unsuitability cost and price. The former are given in (1.8) and (1.9), respectively. For a directly awarded project, the expert bids price $p_M^* \equiv w - t\,m_M$, where $m_M = (\overline{s} - \underline{s})^2/4$ denotes the *unsuitability discount* she grants. Accordingly, we rewrite the tendering prices $p_i^k \equiv c + t\,m_i^k$, and refer to m_i^k as the *differentiation premium* of firm $i = 0, 1$ in equilibrium $k \in \{DD, DC, LD, LC\}$. Using this notation the buyer's total cost are

$$\begin{aligned} E_X[g_M] &= p_M + t\,E_X[l_M] \\ &= w - t\,m_M + t\,E_X[l_M], \end{aligned}$$

and,

$$\begin{aligned} E_X[g^k] = \nu &\left(\int_{\underline{s}}^{\hat{s}^k} p_0^k + t\left(x - s_0^k\right)^2 dx + \int_{\hat{s}^k}^{\overline{s}} p_1^k + t\,(1-x)^2\, dx \right) \\ &+ (1-\gamma)\left(\int_0^{\hat{s}^k} p_0^k + t\left(x - s_0^k\right)^2 dx + \int_{\hat{s}^k}^{1} p_1^k + t\,(1-x)^2\, dx \right) \\ = c &+ t\,E_X[m^k] + t\,E_X[l^k], \end{aligned}$$

where $E_X[m^k] = F_X(\hat{s}^k)\,m_0^k + \left(1 - F_X(\hat{s}^k)\right) m_1^k$.

Costs $E_X[l_M]$ and $E_X[l^k]$ as well as the unsuitability discount m_M and the differentiation premium $E_X[m^k]$ are functions of $\underline{s}$, $\overline{s}$, and γ. Due to prices, however, the buyer's total cost additionally depends either on his willingness to pay w because the expert's in case of a direct award appropriates the buyer's rent, or on the implementation cost c because of competition in case of tendering. Proposition 1.5 is a corollary of this observation. Its proof is in the appendix.

Proposition 1.5 (The buyer's total cost)**.** *The buyer's total cost from offer* (s_M^*, p_M^*) *is lower than in any tendering equilibrium* $k \in \{DD, DC, LD, LC\}$ *if and only if* $\omega \leq m_M + E_X[m^k] - E_X[l_M - l^k]$ *and the former is implemented with certainty.*

That is, the buyer's cost in a tendering exceeds total cost of a direct award if and only if the contract's value (or alternatively, the expert's pricing power) is low and expert knowledge is valid enough to ensure implementation of the directly awarded project. Or conversely, if expert knowledge is not reliable enough to ensure implementation of the direct award offer, better call for tenders (independent of the contract's value and the expert's pricing power).

Figure 1.10 depicts the example of $\underline{s} = 1/2$, $\overline{s} = 4/5$ and $t = 3$, $w = 3/2$, $c = 0$ and $\gamma = 1/2$, in which the buyer's cost of a directly awarded contract clearly exceeds his cost of the tendering procedure because his willingness to pay is rather high. The result of proposition 1.5 is in line with the financial

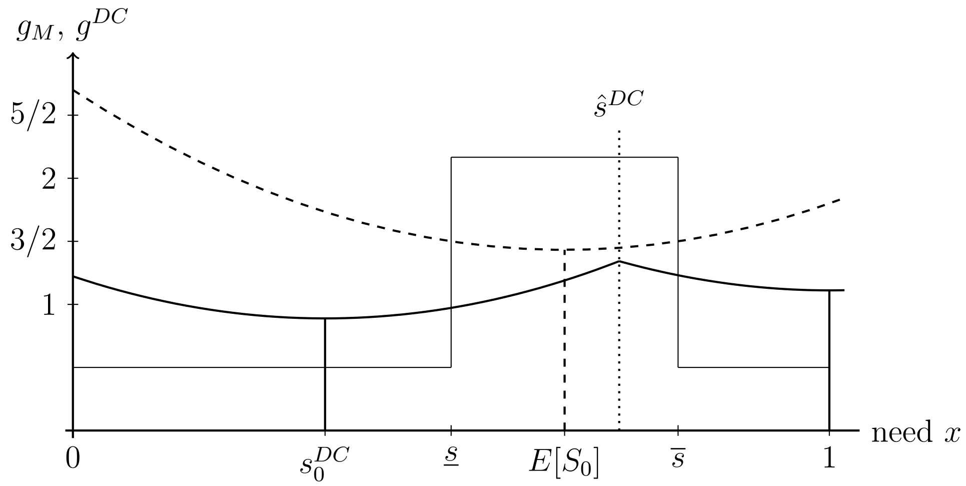

Figure 1.10: The buyer's total cost for $\underline{s} = 1/2$, $\overline{s} = 4/5$ and $t = 3$, $w = 3/2$, $c = 0$ and $\gamma = 1/2$

ceilings for direct awards that are anchored in most policies for public procurement, however, emphasizes another trade-off beyond the net cost and benefit of tendering procedures (described in the introduction): in case of a direct award, the suitability gains due to valid expert knowledge exceed the expert's ability to appropriate the buyer's rent if the contract's net value is low.

1.5 Conclusion

This chapter introduces the notion of a self-proclaimed expert in procurement situations, in which the buyer ex ante is uncertain about his horizontal need, and thus, launches an incomplete catalogue of requirements. We find that the expert's price in case of a directly awarded contract appropriates the buyer's rent and grants a discount that–in the expert's view–fully compensates the buyer for unsuitable project specifications. Consistently, the expert minimizes unsuitability by customizing her specification to the buyer's need. In a tendering procedure where the expert competes with a non-expert supplier, however, prices equal implementation cost plus a premium that competitors charge due to their differentiated specifications.

If the expert's information is valid, i.e., reliable and precise, and her pricing power is low, the buyer's cost of a directly awarded project is lower than his cost of the tendering offers. This constitutes a rationale for financial ceilings on the lawfulness of direct awards stipulated in most public procurement policies. It supports these policies if validity ensures that the expert's direct award offer is implemented with certainty, and because her pricing power is low if either the project's net value is low or the buyer's sensitivity for unsuitable specifications is high.

In terms of unsuitability cost, direct awards to the expert are welfare superior than tendering procedures if her information is precise enough to allow her a pricing strategy that stifles competition in case of tendering, and moreover, reliable enough to ensure implementation of her direct award offer. However, if the expert's information is rather imprecise implying a pricing strategy that leaves her competitor a chance to win in tendering, it might be welfare superior to directly awarding the contract to the expert even if her information is not reliable. This is the case if her (wrong) belief on the buyer's need induces her to customize her specification to his (true) need and thereby crucially reduces his unsuitability cost.

Intuitively, while a generalization of the need's uniform distribution to mean-preserving log-concave distributions provides stronger incentives for both competitors to customize their specifications in a tendering, it has no impact on the expert's specification if the contract is directly awarded. Therefore, we conjecture that such generalizations enhance welfare of tendering procedures compared to direct awards. We expect the same effect from adding more competitors (whether non-experts or experts) to the tendering. This latter driver of a tendering's welfare superiority is mitigated if we—somewhat realistically—assume strictly positive fixed cost for the offer preparation. This, together with an endogenous entry decision, creates a trade-off between lower unsuitability cost due to a higher number of competitors and sunk cost. A more urgent question for practitioners, however, concerns the buyer's catalogue of requirements, which in this chapter, is given. Future research thus might take a mechanism design perspective and derive the buyer's efficient announcement when calling for tenders.

1.A Appendices

A.1 Proof of proposition 1.1 (Direct award)

Proof. The firm first decides on the specification and subsequently on the price. Given its specification of the first stage, $s_M \in [0,1]$, its second stage problem writes

$$\max_{p_M} \; (p_M - c) \cdot \rho$$
$$\text{s.t.} \quad \rho = \Pr\Big(w - p_M - t\,(s - s_M)^2 \geq 0\Big)\,, \tag{1.12}$$

where the constraint (1.12) pins down firm 0's expected probability that the offered project is implemented. It is equivalent to the cumulative probability that the realization of the buyer's need s realizes within $\left[s_M - \sqrt{(w-p_M)/t}, s_M + \sqrt{(w-p_M)/t}\right]$. Figure 1.11 depicts the four possible conjunctions of this interval with the expert's prior on the need's support, $[\underline{s}, \overline{s}]$. We first, in (a), argue that only two of these cases are equilibrium candidates. Then, in (b), we formally determine the expert's case-specific offers. In (c), we compare the her expected profits, and thereby derive the existence condition.

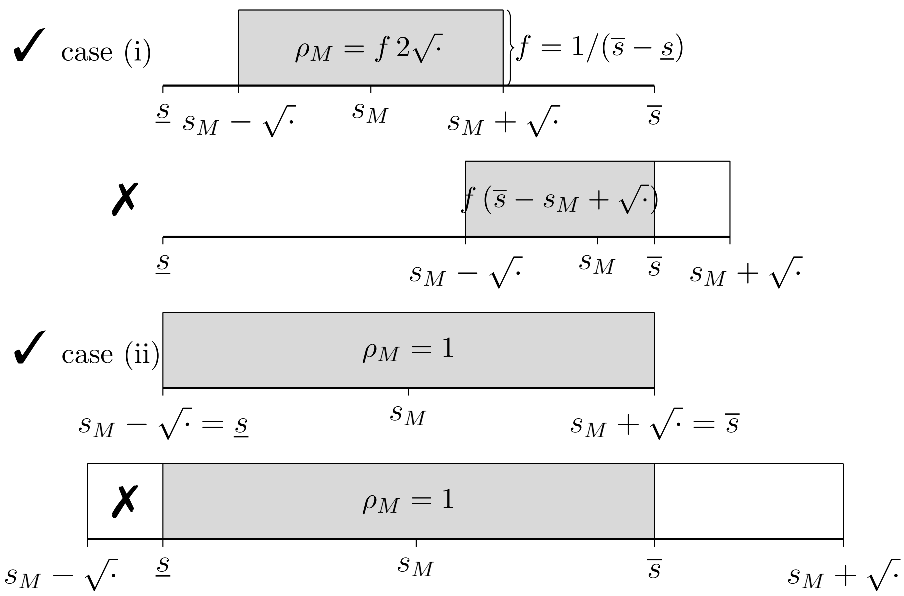

Figure 1.11: Cases (i) to (iv) of implementation probability ρ

(a) Equilibrium candidates

An equilibrium offer (s_M, p_M) implies that the "acceptance interval" $\left[s_M - \sqrt{(w-p_M)/t}, s_M + \sqrt{(w-p_M)/t}\right]$ either is a strict subinterval of $[\underline{s}, \overline{s}]$ (as in case (i) of figure 1.11) or that the two intervals coincide (as in case (ii) of figure 1.11):

Case (i). Suppose the expert bids a price p_M such that $\rho = f\,2\sqrt{(w-p_M)/t} < 1$. Given this (rather high) price, the expert's specification ensures that the acceptance interval never overlaps her prior on the need's support because this would reduce her winning probability, i.e., she abstains from bidding a specification in the peripheries of the unit interval.

Case (ii). Suppose that the expert bids a price p_M, which ensures that the buyer accepts her offer, i.e., that $\rho = 1$. This (rather low) price is such that the lengths of the two intervals coincide because any further price reduction directly reduces her expected profit, while her winning probability still equals 1. Given this (rather low) price, any deviation from $s_M = E[S]$ (implying a shift of the acceptance interval to the left or right) would reduce her winning probability. It follows that the expert's specification equals her expected value of the buyer's need.

Moreover, note that any putative equilibrium price satisfies $c \leq p_M \leq w$ to avoid a loss, and any putative equilibrium specification satisfies $\underline{s} < s_M < \overline{s}$ for the reasons described above. In what follows, we show that both equilibrium candidates (i) and (ii) indeed constitute equilibrium offers.

(b) Optimal offer per case

Case (i) is defined by $\underline{s} < s_M - \sqrt{(w-p_M)/t} \leq s_M + \sqrt{(w-p_M)/t} < \overline{s}$ for $s_M \in [0,1]$, $\forall w \geq p_M \geq 0$, $\forall t > 0$. Equivalently, in case (i)

$$p_M > w - t\,(s_M - \underline{s})^2 \text{ and } p_M > w - t\,(\overline{s} - s_M)^2 \tag{1.13}$$

have to hold simultaneously. This implies that the expert ex ante expects the buyer to accept her offer with probability $\rho = f \cdot h = (\overline{s} - \underline{s})^{-1} \cdot 2\sqrt{(w-p_M)/t} \in (0,1)$. Her expected profit in the second stage, $\pi = (p-c)\,\rho$ is well-behaved[19] and maximizing yields the unique solution

$$p_M = (2w + c)/3. \tag{1.14}$$

[19] It is simple to show that $\partial\pi/(\partial p) = f \cdot (2w - 3p + c)/\sqrt{t\,(w-p)} \geq 0 \Leftrightarrow p \leq (2w+c)/3$ and $\partial^2\pi/(\partial p)^2 = -f/2 \cdot (4w - 3p - c) \cdot (w-p)^{-2/3} t^{-1/2} < 0 \Leftrightarrow 4w > 3p + c$, which holds in equilibrium due to condition $c \leq p_M \leq w$.

The winning probability at this price is $\rho = f\,2\sqrt{(w-c)/(3\,t)} = f\,2\sqrt{\omega/3}$ and substituting (1.14) in (1.13) is equivalent to

$$w - c < 3\,t\,(s_M - \underline{s})^2 \text{ and } w - c < 3\,t\,(\overline{s} - s_M)^2\,. \tag{1.15}$$

The firm's first stage problem in case (i) is thus

$$\max_{s_M} \; f \cdot \frac{4(w-c)^{3/2}}{3^{3/2}\sqrt{t}} \tag{1.16}$$
$$\text{s.t. } (1.15).$$

The objective function is independent of the choice variable s_M. The constraint (1.15) is most relaxed at $s_M = E[S] = (\underline{s} + \overline{s})/2$ because $s_M < E[S]$ implies $3\,t\,(s_M - \underline{s})^2 < 3\,t\,(E[S] - \underline{s})^2 = 3\,t\,(\overline{s} - E[S])^2 < 3\,t\,(\overline{s} - s_M)^2$, i.e., the first constraint of (1.15) is stronger than the second, and stronger than the constraint at $s_M = E[S]$. Analogue, $s_M > E[S] \Rightarrow 3\,t\,(\overline{s} - s_M)^2 < 3\,t\,(E[S] - \underline{s})^2 = 3\,t\,(\overline{s} - E[S])^2 < 3\,t\,(s_M - \underline{s})^2$. It thus follows that the range of given $w - c$ that satisfy the expert's first stage problem is largest if she bids $s_M = E[S]$ because this minimizes the buyer's cost for unsuitable project specifications. The offer $((\underline{s} + \overline{s})/2, (2w + c)/3)$ is therefore a possible equilibrium strategy for all $w - c < 3\,t\,(\overline{s} - \underline{s})^2/4$. However, this intuitively reasonable offer is not the unique solution to the expert's problem: all specifications $s_M \in \left(\underline{s} + \sqrt{\omega/3}, \overline{s} - \sqrt{\omega/3}\right)$ simultaneously satisfy the constraints (1.15) and yield expected profit (1.16). Note that existence of equilibrium offer (s_M^r, p_M^r) in case (i) thus requires $\underline{s} + \sqrt{\omega/3} < \overline{s} - \sqrt{\omega/3} \Leftrightarrow \omega < 3\,(\overline{s} - \underline{s})^2/4$.

Case (ii) is defined by $s_M - \sqrt{(w-p)/t} \le \underline{s} < \overline{s} \le s_M + \sqrt{(w-p)/t}$ for all $s_M \in [0,1]$, $\forall w \ge p_0 > 0$, $\forall t > 0$. Equivalently, in case (ii)

$$p_M \le w - t\,(s_M - \underline{s})^2 \text{ and } p_M \le w - t\,(\overline{s} - s_M)^2 \tag{1.17}$$

have to hold simultaneously. This implies that the firm ex ante expects the buyer to accept its offer with certainty, i.e., $\rho = 1$. Thus, its profit $\pi = (p - c)$ is linearly increasing in p. Therefore, the firm in the second stage sets a price equal to the binding constraint in (1.17). That is

$$p_M(s_M) = \begin{cases} w - t\,(s_M - \underline{s})^2\,, & \text{if } s_M \ge (\underline{s} + \overline{s})/2 \\ w - t\,(\overline{s} - s_M)^2\,, & \text{if } s_M \le (\underline{s} + \overline{s})/2. \end{cases}$$

Because, in the respective range of s_M, the former is strictly decreasing while the latter is strictly increasing in s_M, the price $p_M(s_M)$ and the first stage profit, are both maximized at $s_M = (\underline{s} + \overline{s})/2$ where both constraints bind. The corresponding price is $p_M = w - t\,(\overline{s} - \underline{s})^2/4$. By construction, the case (ii) offer is a possible equilibrium strategy for all parameter values. To see this substitute

$(s^*_M, p^*_M) = \left((\underline{s} + \overline{s})/2, w - t\,(\overline{s} - \underline{s})^2/4\right)$ in restrictions (1.17).

Knowing the potential offers (i) and (ii) we compare the case-specific profits in order to determine the expert's equilibrium strategy in the next step.

(c) Comparison of profits

The expert optimally bids its case (i) offer if $\omega < 3f^{-2}/4$, and its case (ii) offer otherwise since:

$$\pi_{(i)} - \pi_{(ii)} \geq 0 \quad \Leftrightarrow \quad \frac{4\,f}{\sqrt{t}}\left(\frac{(w-c)}{3}\right)^{3/2} - w + t\,(2f)^{-2} + c \geq 0.$$

First note that, for $\omega < 3f^{-2}/4$, this difference is strictly decreasing in w since

$$\begin{aligned}\frac{\partial(\pi_{(i)} - \pi_{(ii)})}{\partial w} = \frac{2\,f}{\sqrt{t}}\left(\frac{w-c}{3}\right)^{1/2} - 1 < 0 \\ \Leftrightarrow \quad w - c < \frac{3\,t}{4\,f^2}.\end{aligned}$$

Then, note that $\pi_{(i)} - \pi_{(ii)}$ is positive if and only if $\omega < 3f^{-2}/4$ as it is zero at the supremum of w:

$$\left.\frac{4\,f}{\sqrt{t}}\left(\frac{w-c}{3}\right)^{3/2} - w + (2\,f)^{-2} + c\right|_{w=3\,t\,f^{-2}/4+c} = \frac{4\,f}{\sqrt{t}}\left(\frac{t}{4f^2}\right)^{3/2} - \frac{2\,t}{4\,f^2} = 0.$$

Both offers, (s^r_M, p^r_M) and (s^*_M, p^*_M), satisfy the conditions $c \leq p_M \leq w$ and $\underline{s} < s_M < \overline{s}$, and therefore indeed constitute the expert's equilibrium offers for a directly awarded contract. $\square$

A.2 Proof of lemma 1.1 (Price equilibria in tendering)

We thoroughly study the tendering game in the next chapter 2 of this thesis. Lemma 1.1 (Price equilibria in tendering procedures) corresponds to proposition 2.1. Its proof is in the corresponding appendix A.2.

A.3 Proof of proposition 1.2 (Tendering procedure)

Proposition 1.2 (Tendering procedure) corresponds to proposition 2.2 in chapter 2 of this thesis. Its proof in in appendix A.4.

A.4 Proof of lemma 1.2 (Unsuitability cost)

Proof. The proof is structured as follows. In (i) we show that $s_M^* = E[S_0]$ lies strictly between the specifications s_0^k and $1 - s_1^* = 1$ for all equilibrium types $k = \{DD, DC, LD, LC\}$. We describe the cost curves' identical slope and curvature as well as their intercepts s_L^k and s_H. Then, in (ii), we mainly show that unsuitability cost in tendering is maximized at $s = \hat{s}^k$ or $s = 0$, and determine the conditions equivalent to $s_L^k < \hat{s}^k \leq s_H$. Finally, in (iii), we compare the bounds of the three intervals.

(i) Clearly, $s_0^{DD} = s_0^{LD} = 0 < s_M^* = E[S] < 1 - s_1^* = 1$ holds for all $0 < \underline{s} < \overline{s} < 1$. For the customized equilibrium types, recall from proposition 1.2 that $s_0^{DC} = (4\underline{s} - 1)/3$ and $s_0^{LC} = 1 - 2(1 - \overline{s})$. Note that $s_0^{DC} < s_M^* \Leftrightarrow (4\underline{s} - 1)/3 < (\underline{s} + \overline{s})/2 \Leftrightarrow \underline{s} < (3\overline{s} + 2)/5$ is satisfied for all admissible parameters because the lowest value of its RHS (evaluated at $\overline{s} = (4 + 5\underline{s})/9$), converges to $4/15 + 1/3\underline{s} + 2/3$, an expression that exceeds $\underline{s}$ for all $\underline{s} \leq 1$. Moreover, $s_0^{LC} < s_M^* \Leftrightarrow 2\overline{s} - 1 < (\underline{s} + \overline{s})/2 \Leftrightarrow \overline{s} < (2 + \underline{s})/3$ is implied by the LC existence conditions $\overline{s} \leq 1/2 + \underline{s}/3$ if $\underline{s} \leq 1/4$, and $\overline{s} \leq (4 + 5\underline{s})/9$ otherwise.

The cost function in case of tendering is $l_k = (s - s_0^k)^2$ if $s \leq \hat{s}$, and $l_k = (1 - s)^2$ otherwise. In case of a direct award, unsuitability cost is $l_M = (s - E[S_0])^2$. These three U-shaped parabolas have identical curvature and vertexes s_0^k, 1 and $E[S_0]$, respectively. From $s_0^k < E[S_0] < 1$ it thus follows that, l_k and l_M intercept at $s_L^k \equiv (s_0^k + E[S_0])/2$ and at $s_H \equiv (E[S_0] + 1)/2$, and that $l_k \geq l_M$ for all $s \in [s_L^k, s_H]$. Substituting s_0^k the intercepts are

$$s_L^{DD} = s_L^{LD} = (\underline{s} + \overline{s})/4, \tag{1.18}$$

$$s_L^{DC} = (3\overline{s} + 11\underline{s})/12 - 1/6, \tag{1.19}$$

$$s_L^{LC} = (5\overline{s} + \underline{s})/4 - 1/2, \tag{1.20}$$

and,

$$s_H = 1/2 + (\underline{s} + \overline{s})/4. \tag{1.21}$$

(ii) We now argue that unsuitability cost l^k in case of tendering is maximal either at need $\hat{s}^k$ or 0, and then, for all k separately, proof $s_L^k < \hat{s}^k < s_H$. First, recall that

$$\hat{s}^{DD} = 1/2 + \underline{s}/3 \tag{1.22}$$

$$\hat{s}^{DC} = (4 + 5\underline{s})/9 \tag{1.23}$$

$$\hat{s}^{LD} = \hat{s}^{LC} = \overline{s}. \tag{1.24}$$

If $\hat{s}^k < s$, then $l_k = 0$ holds at $s = 1$. Thus, this parabola reaches its supremum if $s \to \hat{s}^k$. If $s < \hat{s}^k$, $l_k = 0$ holds at $s = s_0^k$. And, $0 \leq s_0^k < \hat{s}^k/2$ implies that l_k reaches its maximum at $s = \hat{s}^k$.[20] In both differentiated equilibrium types $k = DD, LD$, unsuitability cost l^k is thus clearly maximal at $s = \hat{s}^k$. In what follows we determine the conditions for this to hold in the customized equilibrium types:

$$\begin{aligned} s_0^{DC} = (4\underline{s} - 1)/3 < \hat{s}^{DC}/2 = (4 + 5\underline{s})/18 \\ \Leftrightarrow 24\underline{s} - 6 < 4 + 5\underline{s}, \end{aligned}$$

which is satisfied even if its LHS equals the infinum $\overline{s} = (4 + 5\underline{s})/9$ iff

$$8\underline{s} - 2 < (4 + 5\underline{s})/3 \Leftrightarrow 19\underline{s} < 10.$$

And, in case of Customized Limit equilibrium

$$\begin{aligned} s_L^{LC} = (\underline{s} + \overline{s})/4 < \overline{s} = \hat{s}^{LC} \\ \Leftrightarrow \underline{s}/3 < \overline{s} \end{aligned}$$

is satisfied by definition $\underline{s} < \overline{s}$.

Substituting (1.18) to (1.21) as well as (1.22) to (1.24) and rearranging proves that $\hat{s}^k \in (s_L^k, s_H]$ and $l_M(0) > l_k(0)$ in all equilibrium types k. Further note that $E[S_0] \in (s_L^k, s_H)$ follows from part (i), and $\hat{s}^k \in (\underline{s}, \overline{s}]$ from $\rho_0^k \in (0, 1]$.

(iii) Finally, substitute (1.18) to (1.21) in

$$\begin{aligned} (s_L^k - \underline{s})^2 < (s_L^k - 0)^2 \\ (\overline{s} - s_H)^2 < (1 - s_H)^2 \end{aligned}$$

to see that the bounds of $[s_L^k, s_H]$ lie closer to the bounds $[\underline{s}, \overline{s}]$ than to the ones of $[0, 1]$. □

A.5 Proof of lemma 1.3 (Validity and unsuitability cost)

Proof. We explicitly prove this lemma for each k separately. Note that we thereby implicitly show that $E_X[l_M - l^k]$ is decreasing in reliability γ because in the peripheries of the unit interval, where the need's density is $f_X = 1 - \gamma$, the cost difference $E[l_M - l^k]$ is positive: if γ increases, the density in the smaller interval $[\underline{s}, \overline{s}]$, where the cost difference is negative, increases while the density in the peripheries decreases.

Differentiated Duopoly. The difference in unsuitability cost between direct

[20] Note that this does not imply continuity of l_k, which indeed exhibits a jump at $\hat{s}^k$ for $k = DD, DC, LD$.

award and the DD tendering offers is

$$E_X[l_M - l^{DD}] = \frac{1}{4}\Big(\nu\left(1 + \underline{s}^3 + 4\underline{s}^2/9 - \overline{s}^3 + 4\overline{s}^2 - 4\overline{s} - \underline{s}\,\overline{s}\left(\overline{s} - \underline{s}\right)\right) + (1-\gamma)\left(5\underline{s} + 3\overline{s} - 3\right)\left(\underline{s} + 3\overline{s} - 3\right)/9\Big).$$

This difference is decreasing in the validity ν since

$$\frac{\partial E_X[l_M - l^{DD}]}{\partial \nu} = \frac{1}{4}\left(1 + \underline{s}^3 + 4\underline{s}^2/9 - \overline{s}^3 + 4\overline{s}^2 - 4\overline{s} - \underline{s}\,\overline{s}\left(\overline{s} - \underline{s}\right)\right) \leq 0 \tag{1.25}$$

is satisfied as shown in what follows. Solving

$$\frac{\partial LHS(1.25)}{\partial \underline{s}} = 3\underline{s}^2 + 2\underline{s}\left(\overline{s} - 4/9\right) - \overline{s}^2$$

for $\underline{s}$ yields 2 roots. Exclude $\underline{s}' = \left(4/3 - \overline{s} - 2\sqrt{81\overline{s}^2 - 18\overline{s} + 4}/3\right)/9 \leq 0$, $\forall \overline{s} \in [0,1]$, and note that $\underline{s} = \left(4/3 - \overline{s} + 2\sqrt{81\overline{s}^2 - 18\overline{s} + 4}/3\right)/9$ is the global minimum since

$$\frac{\partial^2 LHS(1.25)}{(\partial \underline{s})^2} = 6\underline{s} + 2\left(\overline{s} - 4/9\right) > 0 \Leftrightarrow 54\underline{s} + 18\overline{s} > 8$$

is clearly satisfied at the infinum of $\overline{s}$, i.e., at $\overline{s} = 1/2 + \underline{s}/3$. Therefore, if (1.25) is negative at both corner values of $\underline{s}$, then $\partial E[\cdot]/(\partial \nu) \leq 0$. This indeed is the case since

$$\left.\frac{\partial E_X[l_M - l^{DD}]}{\partial \nu}\right|_{\underline{s}=0} = \frac{1}{4}\left(1 - \overline{s}^3 + 4\overline{s}^2 - 4\overline{s}\right) < 0,\ \forall \overline{s} \in [1/2, 1),$$

and,

$$\left.\frac{\partial E_X[l_M - l^{DD}]}{\partial \nu}\right|_{\underline{s}=1/4} = \frac{1}{64}\left(569/36 - 16\overline{s}^3 + 60\overline{s}^2 - 63\overline{s}\right) < 0,\ \forall \overline{s} \in [1/2, 1).$$

Customized Duopoly. The difference in unsuitability cost between direct award and the DC tendering offers is

$$\begin{aligned}E_X[l_M - l^{DC}] = \frac{1}{4}\Big(&\nu\big(-\overline{s}^3 + \overline{s}^2(4-\underline{s}) - \overline{s}(4-\underline{s}^2)\\ &+ (115\underline{s}^3/3 - 196\underline{s}^2 + 196\underline{s} + 128/3)/81\big)\\ &+ (1-\gamma)\big(\overline{s}^2 + 2\underline{s}\,\overline{s} - 2\overline{s} + (-560\underline{s}^3/3 + 65\underline{s}^2 - 2\underline{s} + 128/3)/81\big)\Big).\end{aligned}$$

This difference is decreasing in the validity ν since

$$\begin{aligned}\frac{\partial E_X[l_M - l^{DC}]}{\partial \nu} = \frac{1}{4}\big(&-\overline{s}^3 + \overline{s}^2(4-\underline{s}) - \overline{s}(4-\underline{s}^2)\\ &+ (115\underline{s}^3/3 - 196\underline{s}^2 + 196\underline{s} + 128/3)/81\big) \le 0,\end{aligned} \tag{1.26}$$

which is satisfied as shown in what follows. Solving

$$\frac{\partial LHS(1.26)}{\partial \overline{s}} = \underline{s}^2 - 3\overline{s}^2 + 2\overline{s}\,(4-\underline{s}) - 4$$

for $\overline{s}$ has 2 solutions. Exclude $\overline{s}' = 2 - \underline{s} > 1, \forall \underline{s} \in [1/4, 1)$, and note that $\overline{s} = (2+\underline{s})/3$ is the global minimum since

$$\frac{\partial^2 LHS(1.26)}{(\partial \underline{s})^2} = 8 - 6\overline{s} - 2\underline{s} > 0$$

is clearly satisfied. Therefore, if (1.26) is negative at both corner values of $\overline{s}$, then $\partial E[\cdot]/(\partial \nu) \le 0$. This indeed is the case since, $\forall \underline{s} < 1$,

$$\left.\frac{\partial E_X[l_M - l^{DC}]}{\partial \nu}\right|_{\overline{s}=(4+5\underline{s})/9} = \frac{100}{729}\,(\underline{s}-1)^3 < 0,$$

and,

$$\left.\frac{\partial E_X[l_M - l^{DC}]}{\partial \nu}\right|_{\overline{s}=1} = \frac{115}{4\cdot 243}\,(\underline{s}-1)^3 < 0.$$

Differentiated Limit. The difference in unsuitability cost between direct award and the LD tendering offers is

$$E_X[l_M - l^{LD}] = \frac{1}{4}\,(\overline{s} - \underline{s})\,\big(-\nu(\underline{s} + \overline{s})^2 + (1-\gamma)(2 - 3\overline{s} - \underline{s})\big)\,.$$

This difference is decreasing in the validity ν since

$$\frac{\partial E_X[l_M - l^{LD}]}{\partial \nu} = -\frac{1}{4}\,(\overline{s} - \underline{s})\,(\underline{s} + \overline{s})^2 < 0.$$

Customized Limit. The difference in unsuitability cost between direct award and the LC tendering offers is

$$\begin{aligned} E_X[l_M - l^{LC}] = \frac{1}{4}\Big(& -\nu\,(\overline{s} - \underline{s})\,(2 - 3\overline{s} + \underline{s})^2 \\ & + (1-\gamma)\big(-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}\big)\Big). \end{aligned}$$

This cost difference is decreasing in the validity ν since

$$\frac{\partial E_X[l_M - l^{LC}]}{\partial \nu} = -\frac{1}{4}\,(\overline{s} - \underline{s})\,(2 - 3\overline{s} + \underline{s})^2 < 0.$$

It follows from above calculations that the more valid the expert's prior, the more efficient direct awards compared to tendering procedures in terms of welfare. □

A.6 Proof of proposition 1.3 (Welfare)

Proof. This proof is structured as follows. First, in (i), we determine the thresholds τ^k for both Duopoly equilibrium types $k = DD, DC$ separately and thereby show, that it might be negative. Then, in (ii), we determine the thresholds τ^k for the Limit equilibrium types $k = LD, LC$ and show that it is strictly positive. Finally, in (iii), we derive threshold τ^k_γ and show that it is strictly lower than 1 in all equilibrium types k.

(i) Differentiated Duopoly. In the range of DD equilibrium, unsuitability cost of a tendering procedure exceed cost of a directly awarded contract if and only if

$$\begin{aligned} & E_X[l_M - l^{DD}] \leq 0 \\ \Leftrightarrow & f_0\,\frac{\gamma}{1-\gamma} \geq -\frac{1}{9}\,\frac{(5\underline{s} + 3\overline{s} - 3)\,(\underline{s} + 3\overline{s} - 3)}{\big(1 + \underline{s}^3 + 4\underline{s}^2/9 - \overline{s}^3 + 4\overline{s}^2 - 4\overline{s} - \underline{s}\,\overline{s}\,(\overline{s} - \underline{s})\big)} \equiv -\frac{1}{9}\,\frac{B\,C}{A}. \end{aligned} \tag{1.27}$$

First, generally note that the reliability ratio $\gamma/(1-\gamma)$ and precision f_0 are substitutes. More precisely, direct awards are more efficient than tendering procedures if either the reliability or the precision or the reliability of expert knowledge is high enough. Further note that the denominator A in (1.27) is negative as shown in the proof of lemma 1.3 from inequality (1.25) on. However, in what follows we study the threshold

$$\tau^{DD} \equiv \frac{(\overline{s} - \underline{s})}{9} \frac{B\,C}{-A}$$

on the reliability ratio $\gamma/(1-\gamma)$: if $\tau^{DD} \leq 0$, then a directly awarded contract to the expert is more efficient than a tendering procedure for all ratios of reliability, i.e., even if the probability γ that the expert's prior is correct converges to 0. This requires the numerator $B\,C$ to be negative, or formally,

$$\tau^{DD} \leq 0 \Leftrightarrow (5\underline{s} + 3\overline{s} - 3)\,(\underline{s} + 3\overline{s} - 3) \leq 0,$$

which is the case if and only if

$$\underline{s} + 3\overline{s} - 3 \leq 0 \Leftrightarrow 1 - \overline{s} \geq \underline{s}/3$$

and

$$5\underline{s} + 3\overline{s} - 3 \geq 0 \Leftrightarrow 1 - \overline{s} \leq 5\underline{s}/3.$$

Note that the range $\underline{s}/3 \leq 1 - \overline{s} \leq 5\underline{s}/3$ exists $\forall \underline{s} \in [0, 1/4]$ as shown in figure 1.12.

(i) Customized Duopoly. In the range of DC equilibrium, unsuitability cost of a tendering procedure exceed cost of a directly awarded contract if and only if

$$\begin{aligned}
&E_X[l_M - l^{DC}] \leq 0 \\
&\Leftrightarrow f_0 \frac{\gamma}{1-\gamma} \geq \\
&- \frac{\overline{s}^2 + 2\underline{s}\,\overline{s} - 2\overline{s} + (-560\underline{s}^3/3 + 65\underline{s}^2 - 2\underline{s} + 128/3)/81}{\big(- \overline{s}^3 + \overline{s}^2(4 - \underline{s}) - \overline{s}(4 - \underline{s}^2) + (115\underline{s}^3/3 - 196\underline{s}^2 + 196\underline{s} + 128/3)/81\big)} \\
&\qquad \equiv -\frac{H}{G},
\end{aligned}$$

where $G < 0$ is shown in the proof of lemma 1.3 from inequality (1.26) on. In what follows we show that there exists a set of parameters (in the DC range) that satisfies

$$\begin{aligned}
&\tau^{DC} \equiv (\overline{s} - \underline{s}) \frac{H}{-G} \leq 0 \quad \Leftrightarrow \quad H \leq 0 \\
&\Leftrightarrow \overline{s}^2 + 2\underline{s}\,\overline{s} - 2\overline{s} + (-560\underline{s}^3/3 + 65\underline{s}^2 - 2\underline{s} + 128/3)/81 \leq 0 \\
&\Leftrightarrow \alpha_L \equiv 1 - \underline{s} - \sqrt{1680\underline{s}^3 + 144\underline{s}^2 - 1440\underline{s} + 345}/27 \\
&\qquad\qquad \overline{s} \leq 1 - \underline{s} + \sqrt{\cdot}/27 \equiv \alpha_H.
\end{aligned} \tag{1.28}$$

Note that these values of $\overline{s}$ implying $\tau^{DC} \leq 0$ exist for $\underline{s} \lesssim .2698$ roughly, i.e., in DC range the set of parameters for which direct award is less costly than tendering for all reliability levels $\gamma \in (0,1)$ is small, since such an $\overline{s}$ only exists for $\underline{s} \in (.25, .2698)$. In figure 1.12 (roughly) this area is shaded in grey.

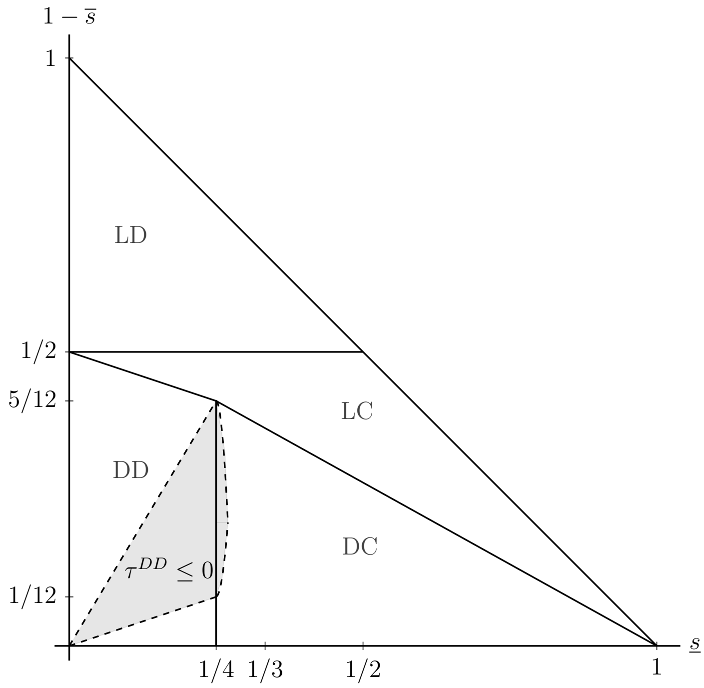

Figure 1.12: Unbinding thresholds on the reliability (direct awards are welfare superior in the shaded area)

(ii) Differentiated Limit. In the range of LD equilibrium, unsuitability cost of a tendering procedure exceed cost of a directly awarded contract if and only if

$$\begin{aligned} &E_X[l_M - l^{LD}] \leq 0 \\ \Leftrightarrow\ & f_0 \frac{\gamma}{1-\gamma} \geq \frac{(2-3\overline{s}-\underline{s})}{(\underline{s}+\overline{s})^2}, \end{aligned}$$

which is always positive since $2-3\overline{s}-\underline{s} < 0$, $\forall \underline{s} \in (0, \overline{s})$ and $\overline{s} \in [\underline{s}, 1/2]$.

(ii) Customized Limit. In the range of LC equilibrium, unsuitability cost of a tendering procedure exceed cost of a directly awarded contract if and only if

$$\begin{aligned} &E_X[l_M - l^{LC}] \leq 0 \\ \Leftrightarrow\ & f_0 \frac{\gamma}{1-\gamma} \geq \frac{\big(-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}\big)}{(\overline{s}-\underline{s})\,(2-3\overline{s}+\underline{s})^2}, \end{aligned}$$

which is always positive since

$$-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s} \geq 0, \tag{1.29}$$

as we show in the following, first for $\underline{s} \in (0, 1/4]$, then for $\underline{s} \in (1/4, 1)$. For all $\underline{s} \in (0, 1/4]$ we show that (1.29) is satisfied as its LHS is increasing in $\overline{s}$, and still positive at the lowest value of $\overline{s}$.

$$\frac{\partial LHS(1.29)}{\partial \overline{s}} = 2(-12\overline{s}^2 + 9\overline{s} + \underline{s} - 1) \geq 0, \tag{1.30}$$

because, in LC, the LHS of (1.30) is decreasing in $\overline{s}$, and still positive at the highest value of $\overline{s}$:

$$\frac{\partial LHS(1.30)}{\partial \overline{s}} = 18 - 48\overline{s} < 0, \ \forall \overline{s} \in (1/2, 1/2 + \underline{s}/3], \text{ and,}$$
$$LHS(1.30)\Big|_{\overline{s}=1/2+\underline{s}/3} = 1 - 8\underline{s}^2/3 > 0, \forall \underline{s} \in (0, 1/4].$$

Therefore, the LHS of (1.29) is increasing in $\overline{s}$, and it is clearly positive at the lowest value of $\overline{s}$:

$$-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}\Big|_{\overline{s}=1/2} = (1/2 - \underline{s})^2 \geq 0.$$

It follows that

$$\frac{\gamma}{1-\gamma} \geq \frac{-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}}{(2 - 3\overline{s} + \underline{s})^2} \equiv \tau^{LD} > 0.$$

In a nutshell, in contrast to the Duopoly case, in both Limit equilibria there exists no set of parameters such that direct award is less costly than tendering for all levels of reliability $\gamma \in (0, 1)$.

(iii) Differentiated Duopoly. In the remainder of this proof we show that there exists no set of parameters for which tendering is more efficient than a directly awarded project $\forall \gamma \in (0, 1)$. To show that, we solve $E_X[l_M - l^{DD}] \leq 0$ for the reliability γ. Then, we separately show that the resulting threshold $\tau_\gamma^k < 1$ for each equilibrium types k. In the range of DD equilibrium,

$$E_X[l_M - l^{DD}] \leq 0 \Leftrightarrow \gamma \geq \tau_\gamma^{DD}, \text{ where } \tau_\gamma^{DD} \equiv -\frac{B\,C\,(\overline{s} - \underline{s})}{9A - B\,C\,(\overline{s} - \underline{s})}.$$

$$\tau_\gamma^{DD} < 1 \Leftrightarrow 1 + \frac{B\,C\,(\overline{s} - \underline{s})}{9A - B\,C\,(\overline{s} - \underline{s})} > 0 \tag{1.31}$$

From the proof of lemma 1.3 from inequality (1.25) on we know that $A \leq 0$, and from part (i) of this proof we know that $B\,C \leq 0 \Leftrightarrow \underline{s}/3 \leq 1 - \overline{s} \leq 5\underline{s}/3$. Otherwise, $B\,C > 0$, in which case inequality (1.31) is obviously satisfied as it is equivalent to $9A - B\,C\,(\overline{s} - \underline{s}) + B\,C\,(\overline{s} - \underline{s}) < 0$. This holds if the denominator is negative. We thus show in the following, that $9A - B\,C\,(\overline{s} - \underline{s}) < 0$ also in the less obvious case where $B\,C \leq 0 \Leftrightarrow \underline{s}/3 \leq 1 - \overline{s} \leq 5\underline{s}/3$. First, note that

$$\begin{aligned} &9A - B\,C\,(\overline{s} - \underline{s}) = \\ &\qquad 14\underline{s}^3 + 22\underline{s}^2\overline{s} - 18\underline{s}\overline{s}^2 - 18\overline{s}^3 - 22\underline{s}^2 + 54\overline{s}^2 + 9\underline{s} - 45\overline{s} + 9 < 0 \end{aligned} \tag{1.32}$$

because, for $\overline{s} \in (1/2 + \underline{s}/3, 1)$ it is a strictly convex function with an interior minimum, and, evaluating the LHS of (1.32) at the corner values of $\overline{s}$ shows that it is negative. Solving

$$\frac{\partial LHS(1.32)}{\partial \overline{s}} = 22\overline{s} - 36\underline{s}\overline{s} - 54\overline{s}^2 + 108\overline{s} - 45 = 0$$

for $\overline{s}$ yields two roots. While $\overline{s}' = 1 - \underline{s}/3 + \sqrt{168\underline{s}^2 - 216\underline{s} + 54}$ exceeds 1, and thus, is irrelevant, the other root

$$\overline{s} = 1 - \underline{s}/3 - \sqrt{168\underline{s}^2 - 216\underline{s} + 54}$$

is a minimum since

$$\left.\frac{\partial^2 LHS(1.32)}{(\partial \overline{s})^2}\right|_{\overline{s} = 1 - \underline{s}/3 - \sqrt{168\underline{s}^2 - 216\underline{s} + 54}} = 6\sqrt{168\underline{s}^2 - 216\underline{s} + 54} > 0.$$

Because the minimum is interior, i.e. $1/2 < 1 - \underline{s}/3 - \sqrt{168\underline{s}^2 - 216\underline{s} + 54} < 1, \forall \underline{s} \in (0, 1/4]$, inequality (1.32) holds if it is negative at the corner values of $\overline{s}$, which indeed is the case since

$$\left. LHS(1.32)\right|_{\overline{s} = 1 - 5\underline{s}/3} = 32\underline{s}^3/3 + 60\underline{s}^2 - 24\underline{s} \geq 0,\ \underline{s} \in (0, 1/4],$$

and,

$$\left. LHS(1.32)\right|_{\overline{s} = 1 - \underline{s}/3} = 16\underline{s}^3/3 + 12\underline{s}^2 - 12\underline{s} \geq 0,\ \underline{s} \in (0, 1/4].$$

(iii) Customized Duopoly. In the range of DC equilibrium,

$$E_X[l_M - l^{DC}] \leq 0 \Leftrightarrow \gamma \geq \tau_\gamma^{DC}, \text{ where } \tau_\gamma^{DC} \equiv -\frac{H\,(\overline{s}-\underline{s})}{G - H\,(\overline{s}-\underline{s})}.$$

$$\tau_\gamma^{DC} < 1 \Leftrightarrow 1 + \frac{H\,(\overline{s}-\underline{s})}{G - H\,(\overline{s}-\underline{s})} > 0 \tag{1.33}$$

From the proof of lemma 1.3 from inequality (1.26) on, we know that $G \leq 0$ for all admissible $\underline{s}, \overline{s}$, and from (i) of this proof we know that $H \leq 0 \Leftrightarrow \underline{1} - \underline{s} - \sqrt{\cdot}/27 \leq \overline{s} \leq 1 - \underline{s} + \sqrt{\cdot}/27$ and $\underline{s} \in (1/4, .2698)$. Otherwise $H > 0$, in which case inequality (1.33) obviously is satisfied as it is equivalent to $G - H\,(\overline{s}-\underline{s}) + H\,(\overline{s}-\underline{s}) < 0$. This equivalence applies whenever the denominator is negative. We thus show in the following that $G - H\,(\overline{s}-\underline{s}) < 0$ also in the less obvious case where $H \leq 0$. First, note that

$$\begin{aligned} &G - H\,(\overline{s}-\underline{s}) < \\ &G - H = 25\underline{s}^3/9 - 29\underline{s}^2/9 + \underline{s}^2\overline{s} - \underline{s}\overline{s}^2 - 2\underline{s}\overline{s} - \overline{s}^3 + 3\overline{s}^2 + 22\underline{s}/9 - 2\overline{s} < 0 \end{aligned} \tag{1.34}$$

because, for the admissible set of parameters (1.34) there exists an interior minimum in $\underline{s} \in (1/4, 1)$, and, evaluating the LHS of (1.34) at the corner values of $\underline{s} \in (1/4, .2698)$, a necessary condition for $H \leq 0$, shows that it is negative. We omit the explicit calculations as the procedure is exactly as in above DD case, however, due to simplicity focusses on $\underline{s}$ (not $\overline{s}$ as in DD). Since $G - H\,(\overline{s}-\underline{s}) < 0$ is negative, $\tau_\gamma^{DC} < 1$.

(iii) Differentiated Limit. In the range of LD equilibrium,

$$\begin{aligned} &E_X[l_M - l^{LD}] \leq 0 \\ \Leftrightarrow\ &\gamma \geq \tau_\gamma^{LD} \equiv \frac{(2-3\overline{s}-\underline{s})(\overline{s}-\underline{s})}{(\underline{s}+\overline{s})^2 + (2-3\overline{s}-\underline{s})(\overline{s}-\underline{s})}. \end{aligned} \tag{1.35}$$

Since all terms in (1.35) are strictly positive $\tau_\gamma^{LD} < 1$ is equivalent to

$$(2-3\overline{s}-\underline{s})(\overline{s}-\underline{s}) < (\underline{s}+\overline{s})^2 + (2-3\overline{s}-\underline{s})(\overline{s}-\underline{s}),$$

and obviously satisfied.

(iii) Customized Limit. In the range of LC equilibrium,

$$\begin{aligned} &E_X[l_M - l^{LC}] \leq 0 \\ \Leftrightarrow\ &\gamma \geq \tau_\gamma^{LC} \equiv \frac{-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}}{(2-3\overline{s}+\underline{s})^2 - 8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s}} \end{aligned} \tag{1.36}$$

Since all terms in (1.36) are strictly positive $\tau_\gamma^{LC} < 1$ is equivalent to

$$\begin{aligned}&-8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s} < \\ &\qquad (2 - 3\overline{s} + \underline{s})^2 - 8\overline{s}^3 + 9\overline{s}^2 - 2\overline{s} + 2\underline{s}\overline{s} + \underline{s}^2 - 2\underline{s},\end{aligned}$$

and obviously satisfied. □

A.7 Proof of proposition 1.4 (Certain implementation)

Proof. This proof is structured as follows. First, in (i), we show that in both Duopoly equilibria $k = DD, DC$ there exists a set of parameters for which $\tau_\gamma^M \leq \tau_\gamma^k \leq 0$. Then, in (ii), we explicitly show that $\tau_\gamma^k \leq \tau_\gamma^M$ for the example of the Differentiated Limit equilibrium. The logic for $k = LC$ as well as for showing (iii), i.e., that for the Duopoly equilibria $k = DD, DC$ there exists a subset of parameters for which $\tau_\gamma^M \leq \tau_\gamma^k < 1$ is analogous.

(i) Differentiated Duopoly. As shown in the proof of proposition 1.3.i), $\tau_\gamma^{DD} \leq 0 \Leftrightarrow \underline{s}/3 \leq 1 - \overline{s} \leq 5\underline{s}/3$. The buyer accepts the direct award offer iff $\tau_\gamma^M \leq \gamma$, and $\tau_\gamma^M \leq 0 \Leftrightarrow \underline{s} \leq \frac{3\overline{s}-2}{3(2\overline{s}-1)}$. Both is satisfied iff

$$\underline{s} \leq \begin{cases} \frac{3\overline{s}-2}{3(2\overline{s}-1)}, & \text{if } \overline{s} \leq 5/6 \\ 1/4, & \text{if } 5/6 < \overline{s} \leq 11/12 \\ 3(1-\overline{s}) & \text{if } 11/12 < \overline{s} < 1. \end{cases}$$

In this case, direct award is better than tendering $\forall \gamma \in (0,1)$.

(i) Customized Duopoly. As shown in the proof of proposition 1.3.i), $\tau_\gamma^{DC} \leq 0 \Leftrightarrow \alpha_L \leq \overline{s} \leq \alpha_H$. The buyer accepts the direct award offer iff $\tau_\gamma^M \leq \gamma$, and $\tau_\gamma^M \leq 0 \Leftrightarrow \overline{s} \geq \frac{3\underline{s}-2}{3(2\underline{s}-1)}$. Since $\alpha_L < \frac{3\underline{s}-2}{3(2\underline{s}-1)} < \alpha_H$ for all $\underline{s} \in (1/4, .2698)$ both thresholds are negative iff

$$\frac{3\underline{s}-2}{3(2\underline{s}-1)} \leq \overline{s} < \alpha_H.$$

In this case, direct award is better than tendering $\forall \gamma \in (0,1)$.

(ii) Differentiated Limit. The difference $\tau_\gamma^M - \tau_\gamma^{LD}$ is strictly increasing in

$\underline{s}$ and 0 at its infinum $\underline{s} = 0$

$$\begin{aligned} &\tau_\gamma^M - \tau_\gamma^{LD} \geq 0 \\ \Leftrightarrow &\frac{2 - 3(\underline{s} + \overline{s}) + 6\underline{s}\overline{s}}{2 - 3(\underline{s} + \overline{s}) + 2\underline{s}\overline{s} + (\underline{s} + \overline{s})^2} - \frac{(2 - 3\overline{s} - \underline{s})(\overline{s} - \underline{s})}{(\underline{s} + \overline{s})^2 + (2 - 3\overline{s} - \underline{s})(\overline{s} - \underline{s})} \geq 0 \\ &- \underline{s}^4 + 6\underline{s}^3\overline{s} + 18\underline{s}^2\overline{s}^2 - 2\underline{s}\overline{s}^3 + 3\overline{s}^4 - \underline{s}^3 \\ &\quad - 15\underline{s}^2\overline{s} - 3\underline{s}\overline{s}^2 - 5\overline{s}^3 + 2\underline{s}^2 + 4\underline{s}\overline{s} + 2\overline{s}^2 \geq 0, \end{aligned} \tag{1.37}$$

which is satisfied as its derivative with respect to $\underline{s}$ is positive, and, (1.37) is positive evaluated at the infinum $\underline{s} = 0$:

$$(1.37)\Big|_{\underline{s}=0} = 3\overline{s}^4 - 5\overline{s}^3 + 2\overline{s}^2 \geq 0 \tag{1.38}$$

which is increasing iff

$$\begin{aligned} \frac{\partial(1.38)}{\partial \overline{s}} &= 12\overline{s}^3 - 15\overline{s}^2 + 4\overline{s} \geq 0 \\ &\Leftrightarrow 0 \leq \overline{s} \leq 5/8 - \sqrt{33}/24 \text{ or } 5/8 + \sqrt{33}/24 \leq \overline{s}, \end{aligned}$$

i.e. (1.38) has a local (in the LD range) maximum at $\overline{s} = 5/8 - \sqrt{33}/24$, which is interior. Evaluating (1.38) at the corner values of $\overline{s}$ shows that it is indeed positive: at the infinum 0 it's 0, and moreover, $(1.38)\Big|_{\overline{s}=1/2} = 1/16$. It follows that $\tau_\gamma^M \geq \tau_\gamma^{LD}$.

(ii) LC, (iii) DD, and (iii) DC. Analogous steps prove that $\tau_\gamma^M \geq \tau_\gamma^{LC}$, and, that there exists a subset of parameters in the Duopoly range for which $\tau_\gamma^M \leq \tau_\gamma^k < 1$. We omit the corresponding (brute force) calculations. □

A.8 Proof of proposition 1.5 (The buyer's total cost)

Proof. Total cost is lower under direct award than in tendering equilibrium k if and only if

$$\begin{aligned} E[g_M - g^k] = w - c - t\,m_M - t\,E[m^k] + t\,E[l_M] - t\,E[l^k] \leq 0 \\ \Leftrightarrow \omega \leq m_M + E[m^k] - E[l_M - l^k]. \end{aligned}$$

Recall that certain implementation is a necessary condition for existence of tendering equilibrium types k in proposition 1.2. Therefore, the result of proposition 1.5 applies if

$$\begin{aligned} w - c - t\,E[m^k] - t\,E[l^k] &\geq 0 \\ \Leftrightarrow \omega &\geq E[m^k] + E[l^k] \end{aligned} \tag{1.39}$$

is satisfied.[21] It follows that the set of ω, which is equivalent to $E[g_M - g^k] \leq 0$, is non-empty if and only if

$$\begin{aligned} E[m^k] + E[l^k] &\leq m_M + E[m^k] - E[l_M - l^k] \\ \Leftrightarrow E[l_M] &\leq m_M, \end{aligned}$$

which in turn is equivalent to a certain implementation of the direct award offer (s^*_M, p^*_M), i.e., to condition (1.10). $\square$

[21] Actually, this is true for all results except of the direct award offers in proposition 1.1. Because (1.39) implies $\omega \geq 3\,(\overline{s} - \underline{s})^2/4$ we ignore the expert's risky offer and focus on (s^*_M, p^*_M) in the comparisons of propositions 1.3 and 1.4. Besides that, however, (1.39) has no direct effect on the results of these propositions, which are independent of w and c.

Chapter 2

Profitability of a Self-Proclaimed Expert's Knowledge

2.1 Introduction

"In the beginner's mind
there are many possibilities,
but in the expert's there are few."
—Shunryu Suzuki (1904)

Satisfying specific needs is a major challenge of project implementation and procurement contracts. The construction of a tunnel, e.g., entails consideration of building standards, local soil properties, size and utilization requirements, as well as softer criteria like the contracting authority's design preferences.

Because the success of project implementation crucially depends on satisfying such needs, and moreover, procurement is an important expenditure item[1], practitioners and scholars of different fields study the selection of contractors and suppliers for a long time.[2]

In economics, theory on multidimensional and scoring auctions analyzes awards of contracts based on multiple criteria besides the price. Examples include Che (1993), Asker and Cantillon (2010), and Rezende (2009). The latter cites Gene Richter, former Chief Purchasing Officer of IBM: "There is nothing that a company buys that I can think of where only the price is important." We share this view. However, in contrast to the literature on scoring auctions, we use horizontal project specifications rather than vertical evaluation schemes as second dimension to analyze competition for a contract.[3]

High complexity of large projects demands the executing firm not only to fulfill certain technological and capacity standards, but renders expertise necessary. Expertise is especially important if the buyer is uncertain about his specific need, and thus, launches an incomplete call for offers. A prominent example for a

[1]Procurement accounts for a notable part of total spending, in the public as well as in the private sector: in 2018, EU public authorities spend 14% of GDP (around €2 trillions) on the purchase of services, works and supplies (see European Commission, 2019b). IBM in 2000 spent over 50% (roughly $45 billions) of revenue on outside suppliers (see Lester, 2000; IBM, 2001).

[2]Stević (2017) provides a literature review in supply chain management, and an overview of criteria for the selection of suppliers.

[3]For a discussion on similarities between scoring auctions and horizontally differentiated markets see Colucci et al. (2011).

contracting authority's failure to specify an accurate catalog of requirements is the Big Dig in Boston. According to the Boston Globe (Lewis and Murphy, 2003) "Construction on virtually all of the Big Dig's major contracts began with incomplete and error-filled designs, which led to nearly $750 million in other construction cost overruns."

To avoid such cost overruns, firms eligible for implementing complex projects ought to be highly specialized. Naturally, specialized firms might differ in their levels as well as fields of expertise. Amongst others, Athey and Levin (2001) and Bajari et al. (2004, 2014) provide empirical evidence for asymmetric information amongst bidders as well as amongst bidders and the contracting authority in U.S. Forest Service timber auctions and highway procurement, respectively. Both explicitly account for these asymmetries in structural estimations of auction models, and find that bids are strategically manipulated.

This chapter analyzes horizontal competition for a contract between duopolists of whom one proclaims herself an expert. It thereby challenges the positive connotation of expert knowledge. On the one hand, the expert uses her deeper expertise to mitigate competition, and increase her probability of being awarded the contract. On the other hand, the expert is aware of project specifications that are unsuitable for the buyer's need. If unsuitability affects her own special field she compensates the buyer by means of price discounts. Therefore, it is not evident whether expert knowledge is profitable or not.

We study this ambiguity by introducing differing, but commonly known, priors in Hotelling's line of spatial competition. More precisely, we extend Hotelling's location-then-price game with uniform consumer distribution and quadratic transportation cost by allowing duopolists to hold differing priors about the consumers' location, i.e., about their distribution's support. In our interpretation, there is a single buyer who's preference for horizontal project specifications ex ante is unknown.[4] While the non-expert firm believes that the buyer's need is drawn from the unit interval, his competitor believes that it realizes

[4]As in this chapter, Colucci et al. (2011) assume differentiated suppliers to be uninformed about the buyer's taste. However, their firms compete in price only, and the buyer's valuation of the products' second dimension is drawn from a commonly known distribution. Celik (2014) uses a related idea to examine whether a monopolist should reveal information about its differentiated product to an uncertain buyer. In his article, not only the latter's taste, but also the monopolist's location are random variables with privately known realizations.

in a strict subinterval thereof. We refer to the latter firm as the *self-proclaimed expert* because we abstain from specifying the true support of the buyer's need, i.e., the expert's prior might be wrong.[5] Actually, the non-expert firm is convinced that the expert is wrong and vice versa, i.e., firms agree to disagree. Knowing this, both competitors decide on their offers that consist of a horizontal specification and a price.

We prove existence of pure strategy equilibria, and their uniqueness in mutually exclusive parameter ranges. The firms' expected equilibrium profits serve as benchmarks for alternative belief structures in order to answer three main questions: (i) Is expert knowledge profitable? (ii) Would the expert be better off disclosing evidence for her knowledge to her competitor or the buyer? (iii) Do firms assess expert knowledge as market power?

In content, this chapter thus contributes to the literature on the effect of asymmetric information among firms on profits. An example that studies question (i) is Chokler et al. (2006), who moreover provide earlier sources. In line with our result, they find that more precise information might be disadvantageous. However, while Chokler et al. (2006) compare equilibrium profits of an expert and an uninformed firm in the same game, we extend the main game described in section 2.2 by an ex ante stage, in which the expert decides whether to rely on her knowledge and prepare the offer herself (as in the main game) or to delegate offer preparation to a third party who shares the prior of her non-expert competitor. Thereby, we identify situations in which the expert is better of if she delegates, i.e., forgoes applying her knowledge.

Note that this result does not contradict our answer to question (ii), which is, the expert never persuades another player to adopt her knowledge. In other words, she is always better off by holding a differing, more precise, prior than by

[5]Note that the absence of specifying the true support of the buyer's need has no impact on the firms' offers. However, knowing the true support is necessary for formally analyzing welfare. In chapter 1 of this thesis, we introduce an appropriate assumption in order to compare welfare impacts of a tendering procedure, as described in the present chapter, with a directly awarded contract.

Further note that expert knowledge as defined here cannot be interpreted as a head start according to auction or contest theory that studies equilibrium behavior and information rents in presence of head starts, as e.g., Siegel (2014), Kirkegaard (2012) or Seel (2014). While head starts unambiguously favor some of the contestants, our expert is self-proclaimed, i.e., her prior might be wrong and misleading. Moreover, and more interestingly, knowledge in this chapter not only entails expertise, but also awareness of unsuitable project specifications, which constraints the expert in her decision making.

persuading her competitor or the buyer to adopt her knowledge. The rationale for the strict advantage of differing priors in terms of the expert's profit is the higher degree of horizontal differentiation, and the entailed alleviation of price competition. Therefore, the expert in our model never discloses (potentially existing) evidence for her ex ante information. For a general approach to examine incentives for information disclosure, as well as a literature survey, see Raith (1996).[6] Both mentioned papers, Chokler et al. (2006) and Raith (1996), mainly study asymmetric information if firms compete in quantities. Information in this chapter, in contrast, is symmetric since priors are commonly known, and moreover, firms compete in horizontal specifications, not quantities.

This is important to understand question (iii), because we interpret expected winning probabilities as market power. On Hotelling's line, a firm's expected winning probability equals the product of its home turf, i.e., the market segment it serves, and the consumers' density in this market segment. The latter cancels out when comparing a firm's expectations about both competitors' winning probabilities since our firms are convinced of their priors on the need's density. Thus, we separately compare each firm's own expected home turf with her or his expectation about the competitor's home turf to study whether firms actually assess expert knowledge as market power. The answer crucially depends on the equilibrium type under consideration. Therefore, we postpone a more detailed outline of this result. First, we emphasize our main contribution, and describe the model as well as its equilibria.

Technically, our model contributes to the literature on spatial competition with demand uncertainty by introducing differing priors. To the best of our knowledge, Jovanovic (1981) is the only article so far that extends Hotelling's line with differing priors. He studies entry in a constrained market under the assumption that firms form their beliefs about the consumers' location based on private observations. In contrast to our model, the uncertainty in Jovanovic (1981) exclusively concerns the mean of the consumer distribution since his firms share an identical prior on the density. Moreover, Jovanovic (1981) exogenously fixes a uniform price, and firms that enter the market compete in locations only. Consequently, two stage location-then-price games with demand uncertainty and asymmetric information among competing firms are closer to our model—albeit, in the only paper we know

[6] For a survey on information sharing among firms and its effects on competition and welfare, rather see Kühn and Vives (1995).

that applies all three assumptions, information asymmetry among firms is based on a common prior.[7] Therefore, the deviation from the common prior assumption is a novelty in the literature on spatial competition, and also a contribution to the still emerging field of theories with differing priors.

In his seminal paper, Aumann (1976) shows that agents who share the same prior cannot agree to disagree, i.e., even if agents with a common prior have different information, their posteriors must coincide. This result triggered a debate on the common prior assumption. For instance, Gul (1998) points out that "the prior view is an inherently dynamic story" and "it amounts to asserting that at some moment in time everyone must have identical beliefs". In the long run, when agents learn or communicate this seems reasonable, see e.g., Geanakoplos and Polemarchakis (1982). However, in a noncooperative one-shot game, as analyzed in this chapter, it is conceivable that agents enter the game with differing beliefs. This by no means jeopardizes the concept of Nash equilibrium.[8] Aumann and Brandenburger (1995) specify equilibrium conditions under which common knowledge plays a subordinate role. Their agents share a common prior if they "had the same [prior] information and probability assessments, and then got different information." Similar to Jovanovic (1981), we may thus interpret differing priors as differing posteriors of an ex ante stage to our one-shot game: at the very beginning, when the buyer asks firms for offers, the expert already has experience in implementing similar projects, while her competitor has not.

We model this assumption in Hotelling's line of length 1 with quadratic transportation cost and uniformly distributed demand. In this market, two firms with zero marginal cost compete for a contract by first choosing a horizontal specification, and then a price. Since d'Aspremont et al. (1979) used this framework without uncertainty as an example, it became well-known that there exists a unique equilibrium in pure strategies in which firms maximally differentiate

[7] We review some of the literature on spatial competition with demand uncertainty at the end of this introduction. There, we describe in more detail the only examples with asymmetric information among firms that we are aware of–although in our model, only priors are asymmetric but information is not.

[8] For formal characterizations of the common prior assumption and deviations thereof see, amongst others, Rubinstein and Wolinsky (1990) and Feinberg (2000). For a more philosophical treatise consider Morris (1995) who asks why utilities and preferences are accepted as personal, but probabilities not. Van den Steen (2001)'s dissertation consists of a methodological essay on differing priors and two early applications in principal agent frameworks. There are vast recent models that deviate from the common prior assumption, especially in the literature on financial markets and overconfidence.

to mitigate price competition.[9] Note that there is also a counteracting incentive for firms to customize their projects, i.e., locate towards the market center: since customization reduces transportation cost, it increases the firms' winning probabilities. Lambertini (1993) shows that the two driving forces of differentiation balance outside the market if horizontal specifications are unconstrained.

These findings remain valid, however, are amended in our extension with differing priors. We model differing priors by assuming that the buyer's need ex ante is unknown. While the non-expert firm's prior is according to the standard Hotelling line of length 1, his competitor proclaims herself an expert. That is, the latter believes to have more precise information about the buyer's need due to more experience with similar projects. Formally, the expert's prior on the support of the buyer's need is a strict subinterval of $[0, 1]$. The difference to other Hotelling frameworks with demand uncertainty is that priors are commonly known, but not common: both firms know both priors, and are convinced that their own is true, whereas their competitor's is wrong. Moreover, we analyze a static game without any new information additional to the ex ante priors. Consequently, information is not asymmetric, and firms do not update their beliefs. In other words, throughout the entire game, firms agree to disagree.

There exist four types of pure strategy equilibria that uniquely exist in mutually exclusive parameter ranges. In all of these equilibria, the non-expert firm (holding a prior according to Hotelling's line) opts for maximal differentiation and adjusts his price to the expert's. The latter's prior, or more precisely, its deviation from the non-expert's prior, thus determines the type of equilibrium.

The information the expert believes in, or her *expert knowledge*, consists of two divergent components. On the one hand, knowledge stems from deeper *expertise* that increases her expected winning probability. On the other hand, knowledge includes awareness of *unsuitable project specifications*. If unsuitability affects the

[9] We normalize the length of Hotelling's line to 1. This normalization and the introduction of demand uncertainty with a self-proclaimed expert are the only differences to the example in d'Aspremont et al. (1979). In a more general framework, Anderson (1988) proves existence of pure strategy equilibrium for sufficiently convex transportation cost. With linear transportation cost, Osborne and Pitchik (1987) characterize the unique symmetric subgame perfect equilibrium of the location stage based on the mixed strategy equilibrium in the pricing stage of Dasgupta and Maskin (1986). For equilibrium existence under asymmetric consumer densities see Anderson et al. (1997), and for reviews on spatial competition, e.g., Eiselt et al. (1993), Gabszewicz and Thisse (1992) or any textbook on industrial organization.

expert's own special field she compensates the buyer by means of price discounts. Otherwise, i.e., if the expert assesses her competitor's specification as unsuitable, she charges a price premium. The relative degree of both components, expertise and unsuitability, determines the firms' equilibrium offers.

If expert knowledge is moderate, i.e. if priors are rather homogenous, our model is close to Hotelling's line. In this case, there exists a unique equilibrium in pure strategies with maximally differentiated specifications. Compared to Hotelling, prices are asymmetric and slightly lower because the expert grants a discount that launches price competition.

If the expert's perceived unsuitability of her maximally differentiated project for the buyer's need is high, but her expertise rather moderate, the expert customizes her project. That is, she offers an interior specification that reduces differentiation and intensifies price competition. Consequently, equilibrium prices are lower than under maximal differentiation. However, horizontal customization reduces unsuitability, and allows the expert to lower her discount. Therefore, prices are more harmonized.

In both mentioned equilibria, expertise relative to the expert's perceived unsuitability is moderate, and she expects both firms to win with strictly positive probability. We refer to this situation as the *Duopoly case*.

In contrast, if expertise relative to the expert's unsuitability is deep enough, she offers a structurally different price that stifles competition, i.e., that ensures winning the contract. Following vast literature we call this strategy *Limit pricing.*[10] Whether the expert maximally differentiates or customizes her project in *Limit equilibria* depends on the perceived unsuitability of her competitor's project: if it is low, she customizes her project. If her competitor's unsuitability is high, the expert maximally differentiates and offers a higher price than under customization.

These equilibria serve as benchmarks for alternative belief structures. Comparative statics of the expert's expected profits allow us to answer two of the three main questions.

(i) In Duopoly equilibria the expert is better off delegating the offer's preparation to a third party who shares the non-expert's prior (and accordingly bids the well-known Hotelling offer) if her maximally differentiated project rather

[10] Among others, Bain (1956) and Modigliani (1958) started the literature on pricing to deter entry in the 1950s. Afterwards, the term *limit price* has been widely used, for example by Dixit (1979) or Milgrom and Roberts (1982).

suits the buyer's need. In contrast, if she faces a high enough unsuitability, the expert optimally applies her knowledge by customizing her specification, and granting the entailed price discount. The same is true in Limit equilibria: if the expert's maximally differentiated project is relatively unsuitable, she better prepares the offer by herself and adjusts her price. In case that her competitor faces high unsuitability too, she charges a price premium. Otherwise, she grants a price discount. However, if the expert's relative unsuitability is low, she better delegates, and thereby, forgoes applying her knowledge.

Note that this by no means implies our answer to question (ii): The expert never benefits from disclosing evidence for her knowledge, neither to her competitor nor the buyer. The reason is that sharing a common prior reduces horizontal differentiation, and consequently, intensifies price competition. Therefore, the expert's expected profit is higher with differing priors.

To answer question (iii), we consider the outcomes of both duopolists by comparing each firm's own expected home turf with her or his expectation about the competitor's home turf. In the Duopoly case, the non-expert firm's own expected home turf is smaller than his expectation about the expert's home turf, in our words, that is, he assesses knowledge as market power. This is true for the expert if and only if her competitor's unsuitability is high enough. In the Limit case, the expert conjectures herself a monopolist. Her competitor, however, believes that knowledge is power if and only if the expert customizes her specification.

We discuss these contentwise results in more detail in section 2.5, and focus on technically related literature in what follows.

Initial theory on demand uncertainty in location-then-price games typically analyzes random utility models as, for example, De Palma et al. (1985) or described in Anderson et al. (1992, chapter 9). This literature assumes uncertainty concerning horizontal tastes of individual consumers, i.e., the latters' utilities from a product's specification are random. In aggregate, individual demand uncertainty disappears, and symmetric firms locate towards the center of Hotelling's line if individual tastes are heterogeneous enough. In this case, the consumers' density at the ends of Hotellings line is low, i.e., demand realizations are unlikely at extreme locations. Therefore, firms move towards the populated market center. In our model, there is no individual taste heterogeneity. The intuition behind the expert's enhanced incentive to offer an interior specification, however, is similar: if she assesses her maximally differentiated specification as unsuitable for the buyer's need, she customizes her project.

The customization force in the literature on aggregate demand uncertainty in Hotelling markets has another rationale than random individual utility. Early work includes Balvers and Szerb (1996) who introduce a common quality shock to the demand function. They fix a uniform price, and characterize the firms' location choices that might deviate from the "principle of maximum differentiation". This result is in line with more recent work by Christou and Vettas (2005) who additionally model price competition after the aggregate quality shock is revealed to firms. They show that the duopolists' incentives to locate close to each other are increasing in the ratio between the maximum possible quality difference and the consumers' transportation cost. If this ratio is sufficiently high, firms offer identical specifications. The quality difference crowds out any incentives for horizontal differentiation because, for consumers, positive quality shocks and price reductions are perfect substitutes. Therefore, quality shocks directly increase their willingness to pay. This strengthens the firms' incentives to customize their specifications as it allows to jointly charge mark ups *and* steal the competitor's business by offering homogeneous projects. In our model, firms are uncertain about the buyer's horizontal need, not the projects' quality levels. Our expert customizes her project in order to reduce the buyer's cost for unsuitable specifications, not because both firms profit from a higher willingness to pay due to uncertain quality.

In a similar vein, Gerlach et al. (2005) model quality in an innovation contest with uncertainty. Uncertainty in their article, however, concerns the success probability of a project, which depends on the firms' horizontally differentiated R&D approaches. They find a price equilibrium in which, for any locations and a high enough quality difference between contestants, one of them wins with probability 1. A related price equilibrium may arise in Letina and Schmutzler (2019). They study contest design using a definition of quality that accounts for the buyer's unsuitability cost, which is random and contingent on the firms' differentiated approaches. They find that the efficient horizontal variety can be implemented with an appropriate bonus tournament that sets the firms' expected profits equal to zero. In both mentioned equilibria with a sure winner, the latter is revealed before firms decide on prices, and moreover, the sure winner is commonly accepted. Both is not true in our model's Limit equilibria: we have no (or only an ex post) revelation stage, and our duopolists are convinced of their differing, but commonly known, priors. Therefore, potential ex post realizations of the buyer's unsuitability cost do not affect the firm's equilibrium offers. Another consequence of differing priors in Limit equilibria is that the expert perceives herself a monopolist, while the non-expert firm's expected winning probability as well as his price are strictly positive.

In contrast to the mentioned papers on innovation contests, uncertainty in our model has no vertical dimension. It is about the support of the buyer's horizontal need, and thus, about its distribution's mean and density. The expert's expected value of the buyer's taste incorporates her perception of the maximally differentiated projects' unsuitability levels, while the expert's expected density represents her expertise. The latter is rarely modelled in the literature on demand location uncertainty as, except for Meagher and Zauner (2008, 2011), this literature focuses on shifting the mean of a unit mass of consumers, while refraining from uncertainty about the consumers' density.

An early example is Jovanovic (1981), which is the only article we know that equips firms with differing beliefs stemming from private signals about the mean of the consumers' unit mass (see above). Based on their differing beliefs, his firms decide whether and where to enter the market for a given uniform price. In equilibrium, firms that enter the market locate at their private signal because the absence of price competition suppresses their incentive to differentiate.

As this incentive is important in the present chapter, it is closer to Harter (1997) who models uncertainty in the vein of Jovanovic (1981). He considers the case where the shift parameter of the consumers' location is drawn from a commonly known distribution, and provides simulation results for sequential location choices of firms followed by price competition. In his Duopoly case, the first firm that enters the market locates very close to the center, and has a positive probability of earning monopoly profits. In contrast to our Limit equilibria, however, this stems from the possibility that the entire mass of consumers belong to the market segment served by the first firm. This strategic incentive of the first mover to customize its project is absent in our model because competitors simultaneously choose their horizontal specifications. Moreover, the expert's expected winning probability of 1 in Limit equilibria entirely relies on her pricing decision, and is independent of the specifications. Nevertheless, our expert has a related rationale as Harter (1997)'s first mover: if her competitor's unsuitability is low compared to her own, the expert customizes her project in order to minimize the price discount and comply with the entire support of the buyer's need at the same time.

Bonein and Turolla (2009) find a similar incentive for customization in a sequential entry game with asymmetric information. They assume that one firm exactly knows the consumers' location, while its competitor is uniformed. To the best of our knowledge, this is the only extension of Hotelling's line in which a commonly accepted expert competes with an uninformed rival. In our model, in contrast, priors are asymmetric, but information is not. Nevertheless, in line with our results, Bonein and Turolla (2009) show that the informed firm's incentive to offer a customized specification is stronger than the uniformed firm's. Actually, their expert customizes her project, while her competitor differentiates. Which effect prevails in terms of total differentiation depends on the timing of entry: the first mover's reaction is stronger than the follower's, i.e., if the first mover is informed, uncertainty is a customization force. If, instead, the follower is informed, the degree of horizontal differentiation increases.

The different incentives for customization that we described so far in the models with random utility, with aggregate quality shocks and on entry decisions, stand in contrast to findings in Casado-Izaga (2000) and Meagher and Zauner (2004). The latter study simultaneous location-then-price choices of firms that face the same uncertainty as in Jovanovic (1981) and Harter (1997). They identify uncertainty as a differentiation force: the randomness of the consumers' mean shifts the support of the consumer's distribution as the latter is a unit interval. In the vein of Bonein and Turolla (2009), this lowers the uninformed duopolists' incentives to steal their respective competitor's business by offering customized specifications. Following a similar intuition, our expert in Limit equilibria maximally differentiates if she assesses her competitor's specification as unsuitable. Facing an unsuitable competing offer makes business stealing redundant, and at the same time, allows the expert to charge a price premium.

So far, we have cited models with demand uncertainty about the consumers' location that consider a random shift in the mean of a unit mass of consumers, and ignore uncertainty about the density. Meagher and Zauner (2008, 2011) fill this gap, characterize the subgame perfect equilibria, and determine conditions for existence and uniqueness. For the case of symmetric densities, as in our model, they find a closed form solution that depends on the distribution at the mean. In this equilibrium, uncertainty is a differentiation force. More precisely, the firms' projects are more differentiated under a mean preserving spread of a symmetric consumer distribution than under the initial distribution. This rationale also exists in our model: the more homogeneous the firms' priors on the support of the buyer's need, i.e. if the expert's prior is close to the unit interval, she opts for maximal differentiation, and firms charge higher prices than in any other equilibrium.

In contrast to this chapter, firms in all mentioned articles, except for the location game with a given price in Jovanovic (1981), share a common prior. Moreover, except for the sequential entry game in Bonein and Turolla (2009), information is symmetric, i.e., there is no expert. Actually, the latter also applies in our model. However, by assuming differing, but commonly known, priors we introduce the notion of a *self-proclaimed* expert in Hotelling's location-then-price game.

In the next section we outline the model. We solve it backwards, and describe the ex ante considerations in section 2.3.1, the price subgame in section 2.3.2, and the specification subgame in section 2.3.3. Section 2.4 presents the equilibrium types of the entire game. In section 2.5 we derive the answers to our research questions (i) to (iii), and section 2.6 concludes.

2.2 Model

In this section we introduce differing priors in Hotelling's location-then-price game. We first describe the assumptions, then the timing, and finally the equilibrium concept.

A buyer chooses one of two firms $i = 0, 1$ to carry out a single indivisible project (e.g., the construction of a tunnel). The project requires the choice of several characteristics with impact on the buyer's utility (e.g., the design of the tunnel). We shall assume that all these nonmonetary characteristics can be aggregated into a unidimensional variable s_i that we refer to as *specification* of project $i = 0, 1$.

Specifications differentiate projects horizontally, i.e., at equal prices the buyer's optimal choice depends on his particular need. Ex ante the buyer's need s is unknown (e.g., because it depends on unexplored geological conditions). All players agree that the need is uniformly distributed. However, their priors on its support differ. While the buyer and (the male) firm 1 both believe that the former's need is drawn from the unit interval, (the female) firm 0 believes in a smaller support. We denote the buyer's and firm 1's prior by $S_1 \sim U[0, 1]$, and take it as a benchmark for the extent of ex ante knowledge. We define firm 0's prior by $S_0 \sim U[\underline{s}, \overline{s}]$ with $0 < \underline{s} < \overline{s} < 1$. The length of the support of S_0 thus measures the degree of heterogeneity: the smaller the support of S_0, the larger the priors' heterogeneity. Note that the interval $[\underline{s}, \overline{s}]$ may be located asymmetrically in $[0, 1]$. That is, in contrast to $E[S_1] = 1/2$, the mean of S_0 may differ from $1/2$.

We do not specify whether S_0 or S_1 is correct. Moreover, both priors are common knowledge.[11] This captures the notion of a *self-proclaimed expert*: firm 0 believes that her ex ante information, or her *knowledge*, on the support of the buyer's need is superior than the information of the other players. Firm 1, in

[11]The absence of an assumption on the need's true support has no impact on our results because players do not update their priors before firms hand in the offers and the buyer awards the contract. Therefore, we use *prior* and *belief* as synonyms.

Common knowledge of the priors, however, is crucial for the determination of closed-form solutions. While the price equilibria with given specifications are robust against randomization of the firm 0's prior, there is no explicit solution to the specification decision. However, numerical approximations suggest that the profit maximizing specifications converge to constants. Daring interpretations of the common knowledge assumption may rely on industrial espionage, or the notion that highly specialized duopolists, which probably do not compete for the first time, know their only competitor's experience and view.

contrast, assesses the expert's information as wrong, and complies with the more broadly formulated call for offers.

The buyer asks both firms $i = 0, 1$ for offers that contain a specification within the unit interval, $s_i \in [0, 1]$, and a price $p_i \geq 0$. The offers are denoted (s_0, p_0) and $(1 - s_1, p_1)$. The firms have identical marginal cost normalized to 0. They initially decide on their specifications. Let $s_0 < 1 - s_1$, i.e., firm 0 offers a specification to the left of firm 1.[12] After observing the specifications, firms decide on their prices.[13] The buyer receives the offers, and learns his need s. Deviations from s to the specifications s_0 and $1 - s_1$ cause disutility. The disutility is quadratic in the difference and "costs" t per unit. The buyer compares total cost $p_0 + t\,(s - s_0)^2$ with $p_1 + t\,(1 - s_1 - s)^2$, and tasks the firm with the cheaper offer. We assume that $|p_0 - p_1| < t$, and the buyer's willingness to pay high enough so that he certainly implements one of the two projects.[14]

Ex ante, the buyer asks firms $i = 0, 1$ to hand in their offers with $s_i \in [0, 1]$ and $p_i \geq 0$. The buyer's need s is unknown. The firms' priors are $S_0 \sim U[\underline{s}, \overline{s}]$ and $S_1 \sim U[0, 1]$, and commonly known. This game has three stages. In the first stage, firms simultaneously choose their specifications. In the second stage, they observe the specifications and decide on their prices. In the third stage, the buyer receives the offers (s_0, p_0) and $(1 - s_1, p_1)$, learns the realization of his need $s \in [0, 1]$, and picks the better of the two offers.

We focus on subgame perfect equilibria in pure strategies. In equilibrium, the buyer minimizes cost, and both firms $i = 0, 1$ maximize their profits $\pi_i \equiv p_i\,\rho_i$,

[12]In contrast to symmetric formulations of spatial competition, this assumption is crucial for our results: it excludes strategies that aim at maximizing differentiation by offering specifications on the competitor's market side. See the comment on proposition 2.2, page 91.

[13]This timing is in line with the literature on spatial competition, and necessary for existence of pure strategy equilibria. Moreover, it captures the idea of cumbersome decision processes concerning the specifications of a project (or, their "stickiness"), and higher flexibility in price setting. The observability of specifications during process of offer preparation, however, may be questioned. Nevertheless, there are cases, especially in public procurement procedures, in which the suppliers present the specifications of their projects, before handing in the offers including prices. A local example is the ongoing debate on the procurement of combat aircrafts in Switzerland (for a recent summary see, e.g., Rhyn, 2019). In this process, potential suppliers present their prototypes in an public air show before they hand in their offers.

[14]In chapter 1 of this thesis, we abstain from normalizing marginal cost, and moreover, we explicitly model the buyer's willingness to pay. The results of this chapter carry over to the richer setup.

where ρ_i denotes firm i's expected winning probability. Equilibria are characterized by the profit maximizing specifications $(s_0^*, 1 - s_1^*)$ and $(p_0^*(s_0), p_1^*(s_1))$ for all admissible s_0 and s_1. The corresponding equilibrium path is $(s_0^*, 1 - s_1^*)$ and $(p_0^*(s_0^*), p_1^*(s_1^*))$.

2.3 Subgames

2.3.1 Ex ante expected winning probabilities

We solve the game by backwards induction. Accordingly, we first study the buyer's decision at the last stage. This decision is anticipated by the firms, and determines their expected winning probabilities.

Ex ante both firms know that the buyer implements the cheaper project, and thus, is indifferent at the specification $\hat{s}$ that solves

$$p_0 + t\,(\hat{s} - s_0)^2 = p_1 + t\,(1 - s_1 - \hat{s})^2$$
$$\Leftrightarrow \hat{s} \equiv s_0 + \frac{1 - s_0 - s_1}{2} + \frac{p_1 - p_0}{2\,t\,(1 - s_0 - s_1)}. \tag{2.1}$$

The firms agree that, given $s_0 < 1 - s_1$, the interval of specifications to the left of $\hat{s}$ is covered by firm 0's projects and the interval to the right of $\hat{s}$ is covered by firm 1's project. However, their differing priors, $S_0 \sim U[\underline{s}, \overline{s}]$ and $S_1 \sim U[0, 1]$, give rise to two main sources of disagreement: firms disagree on the length and location of the interval as well as on the density of the buyer's need. Note that priors are common knowledge, and thus, firms agree on their disagreement.

We refer to the interval that belongs to firm i as *home turf*[15] of $i = 0, 1$ and denote it by h_i. Firms disagree on their home turfs as follows. Firm 0 expects her own home turf to equal $h_0 \equiv \hat{s} - \underline{s}$, and firm 1 to serve the interval $[\hat{s}, \overline{s}]$ of length $\overline{s} - \hat{s}$, while firm 1 expects his own home turf to be $h_1 \equiv 1 - \hat{s}$, and firm 0 to serve the interval $[0, \hat{s}]$.

Firms also disagree on the density of the buyer's need. Let $f_0 \equiv 1/(\overline{s} - \underline{s})$ denote the density of S_0, and $f_1 \equiv 1$ the density of S_1.

Figure 2.1 depicts the two sources of disagreement, the firms' ex ante expectations about their own home turfs that account for the lengths and locations

[15] We borrow this expression from Tirole (1988).

of the need's support, as well as the expected densities.

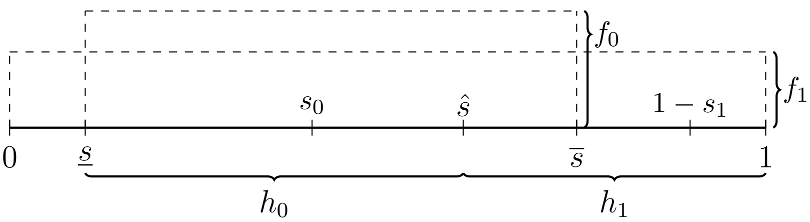

Figure 2.1: Disagreement on expected home turfs and density

However, firms agree on the need's uniform distribution. Therefore, both expect to win with a probability equal to the product of their own home turf's length and the need's density. Recall that ρ_i denotes the expectation of firm $i = 0, 1$ about her or his own winning probability. These expectations are

$$\rho_0 \equiv \begin{cases} 0, & \text{if } \hat{s} \leq \underline{s} \\ \frac{\hat{s}-\underline{s}}{\overline{s}-\underline{s}}, & \text{if } \underline{s} < \hat{s} < \overline{s}, \\ 1, & \text{if } \overline{s} \leq \hat{s}, \end{cases} \quad \text{and} \quad \rho_1 \equiv \begin{cases} 1, & \text{if } \hat{s} \leq 0, \\ 1-\hat{s}, & \text{if } 0 < \hat{s} < 1, \\ 0, & \text{if } 1 \leq \hat{s}. \end{cases}$$

The firms' disagreement implies that ρ_0 and ρ_1 need not sum up to 1. That is, firm 0 expects firm 1 to win with probability $1 - \rho_0$, which may differ from ρ_1. Analogically, firm 1 expects firm 0 to win with probability $1-\rho_1$, which is identical to ρ_0 if and only if $\hat{s} = \underline{s}/(1-\overline{s}+\underline{s})$. This potential discrepancy is shown in figure 2.2. Because it mainly drives our results it deserves a careful analysis.

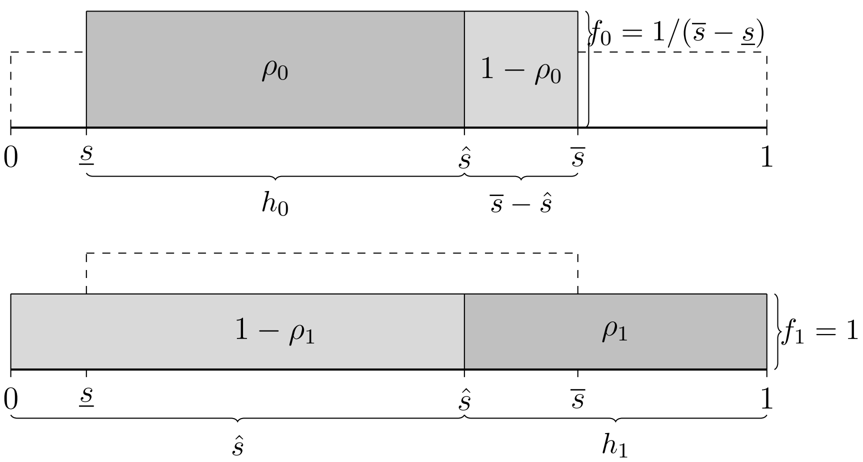

Figure 2.2: Expected probabilities of winning

Given $\underline{s} < \hat{s} < \overline{s}$, firm 0's ex ante expected chance of winning is

$$\rho_0 = \frac{1}{\overline{s} - \underline{s}} \left[(s_0 - \underline{s}) + \frac{1 - s_0 - s_1}{2} + \frac{p_1 - p_0}{2\,t\,(1 - s_0 - s_1)} \right]. \tag{2.2}$$

The expression in square brackets of equation (2.2) is firm 0's expected home turf $\hat{s} - \underline{s}$, i.e., her belief about the interval's length that corresponds to her own winning probability. The first term, $s_0 - \underline{s}$, captures firm 0's conviction that the lowest possible realization of the buyer's need is $\underline{s}$. Because s_0 lies to the left of $1 - s_1$ this affects firm 0's home turf. The expression is negative if $s_0 \leq \underline{s}$, i.e., if firm 0 offers a specification that–in her view–cannot meet the buyer's need because it is lower than the support of S_0.

The second and third term are as in equation (2.1) that determines the specification at which the buyer is indifferent between the offers 0 and 1. The second, $(1 - s_0 - s_1)/2$, captures the positive effect a higher degree of horizontal differentiation has on ρ_0: half the distance between the specifications s_0 and $1 - s_1$ is part of firm 0's home turf. The third term, $(p_1 - p_0)/(2\,t\,(1 - s_0 - s_1))$, captures the buyer's price sensitivity. Due to competition it is half the difference in prices that matters for the ex ante winning probability. The probability is decreasing in the own price and increasing in the competitor's one. The price difference is divided by the product of the unit cost for unsuitable project specifications, t, and the degree of horizontal differentiation. The buyer's price sensitivity is thus weaker the higher his disutility due to deviations from his need, and, the more differentiated the projects.

The effects of horizontal differentiation and price competition on firm 0's home turf are both independent of S_0. However, firm 0's prior directly shortens her home turf, the square bracket of (2.2), by $\underline{s}$.

Firm 0 weights her expected home turf, $\hat{s} - \underline{s}$, by her prior on the density of the buyer's need, $f_0 = 1/(\overline{s} - \underline{s})$. The smaller the support of S_0 the higher f_0, and the higher ρ_0. Think of f_0 as measure for the precision of firm 0's ex ante information, and recall that this information might be wrong. Weighting the home turf by f_0 then captures that firm 0 expects to win with higher probability because she believes to be better informed.

Let us now compare ρ_0 with firm 1's ex ante expected winning probability ρ_1.

Given $0 < \hat{s} < 1$, the latter is

$$\rho_1 = s_1 + \frac{1 - s_0 - s_1}{2} + \frac{p_0 - p_1}{2\,t\,(1 - s_0 - s_1)}. \tag{2.3}$$

The second and third term in (2.3) capture the effects that the degree of horizontal differentiation and price competition have on firm 1's expected home turf. They are symmetric to the corresponding terms in firm 0's winning probability (2.2). The discrepancy between ρ_0 and ρ_1 has two other causes.

First, note that the first term in equation (2.3) is $s_1 \Leftrightarrow 1 - (1 - s_1)$. This captures firm 1's conviction that his specification, $1 - s_1$, is in the support of the buyer's need. Assumption $s_0 < 1 - s_1$ implies that firm 1 is concerned about the upper bound of the support of S_1, which equals 1. Assumption $s_i \in [0, 1]$, $\forall i = 0, 1$ implies $s_1 \in [0, 1]$. It follows that the first term in (2.3) is $1 - (1 - s_1) = s_1$, weakly positive, and independent of $\underline{s}$ and $\overline{s}$. Thus, firm 1's expected home turf, $1 - \hat{s}$, as well as his expectation about firm 0's home turf, $\hat{s}$, are both independent of S_0. Recall that firm 0's expected home turf equals $\hat{s} - \underline{s}$, and thus depends on her prior S_0. The firms' disagreement on expected home turfs is the first source of discrepancy between ρ_0 and ρ_1. Its effect is increasing in $\underline{s}$, and thus, in the priors' heterogeneity.[16]

Second, note that firm 1 also weights his home turf (2.3) by his subjective belief about the density of the buyer's need $f_1 = 1$. In the limit where priors converge, f_1 equals the infimum of $f_0 = 1/(\overline{s} - \underline{s})$. That is, if $\underline{s} \to 0$ and $\overline{s} \to 1$ then $f_0 \to f_1 = 1$. The more heterogeneous the priors, i.e., the smaller the support of S_0, the higher is f_0. Therefore, given identical home turfs, firm 1 expects to win with probability $\rho_1 = 1/2$ while firm 0's expected winning probability is higher, i.e., $\rho_0 > 1/2$. The firms' disagreement on the density of the buyer's need is the second source of discrepancy between ρ_0 and ρ_1. Its effect is increasing in f_0/f_1, and thus, in the priors' heterogeneity.

Regardless of the source, the firms' disagreement unambiguously increases in $\underline{s}$ and f_0. However, contingent on the source, $\underline{s}$ has opposite effects on firm 0's expected winning probability ρ_0. Firm 0's home turf decreases in $\underline{s}$, and so does ρ_0. This negative effect reflects that, given $\hat{s}$, expert knowledge reduces the mass

[16]For the sake of completeness note that firm 0's expectation about firm 1's home turf is $\overline{s} - \hat{s}$. The discrepancy to firm 1's own expectation, $1 - \hat{s}$, is thus decreasing in $\overline{s}$. It follows that the effect of the firms disagreement on home turfs unambiguously decreases in the support of S_0. Equivalently, it increases in the priors' degree of heterogeneity.

of specifications within the support of the buyer's need due to the awareness of unsuitable specifications. In contrast, f_0 increases in $\underline{s}$ (and decreases in $\overline{s}$), and so does ρ_0. This positive effect reflects that expert knowledge allows to assess the buyer's need more precisely, and thus, enhances the merit of her home turf. Which effect of $\underline{s}$ dominates depends on f_0, and thus, on $\overline{s}$. We come back to this as we move along.

However, we now highlight that expert knowledge is not only a boon, but also a bane. Actually, expert knowledge reduces firm 0's expected winning probability as it shortens her home turf, the interval of specifications that might meet the buyer's need. We refer to this negative effect as firm 0's *perceived unsuitability.* It equals $\underline{s}$, and is thus increasing in this parameter.[17] We call the positive effect of expert knowledge on ρ_0 *perceived expertise.* The density f_0 captures firm 0's perceived expertise in her expected winning probability. It is increasing in the priors' heterogeneity, i.e., the smaller the support of S_0, the higher firm 0's perceived expertise, and thus, her expected winning probability.

Throughout the chapter we focus on strictly positive winning probabilities.[18] The firms' disagreement implies that ρ_0 and ρ_1 need not sum up to 1. Therefore, focusing on strictly positive probabilities does not exclude winning probabilities of 1, and thus, firms ex ante expect to win with

$$\rho_0 = \min\left\{\frac{1}{2(\overline{s}-\underline{s})}\left(\frac{p_1-p_0}{t\,(1-s_0-s_1)}+1+s_0-s_1-2\underline{s}\right),1\right\}, \tag{2.4}$$

[17] An increase in $\underline{s}$ allows no conclusion on the priors' heterogeneity. However, holding $\overline{s}$ constant, the heterogeneity is increasing in $\underline{s}$.

Further note that the concept of perceived unsuitability not only affects firm 0's home turf, but also firm 1's: due to the upper bound of S_0, firm 0 believes that firm 1's interval of unsuitable project specifications has length $1-\overline{s}$. We discuss this *perceived unsuitability of firm 1's project* and its implication in the subsequent section.

[18] Ruling out zero probabilities is not restrictive; neither in equilibrium as shown in the proof of proposition 2.2, nor ex ante as we argue in the following. The ex ante winning probabilities are strictly decreasing in the respective firm's price. Specifically, $\rho_0 = 0 \Leftrightarrow p_0 \geq p_1 + t\,(1-s_0-s_1)(1+s_0-s_1-2\underline{s})$ and $\rho_1 = 0 \Leftrightarrow p_1 \geq p_0 + t\,(1-s_0-s_1)(1-s_0+s_1)$ state the equivalence of a high price and an ex ante winning probability of 0. Such a high price implies zero profit and is no equilibrium candidate: the respective firm 0 could increase her expected profit by reducing her price–maximally down to marginal cost. This holds for both firms at the same level as they face identical marginal cost of zero. Thus, focussing on strictly positive winning probabilities is without loss of generality.

and

$$\rho_1 = \min\left\{\frac{1}{2}\left(\frac{p_0 - p_1}{t\,(1 - s_0 - s_1)} + 1 - s_0 + s_1\right), 1\right\}. \tag{2.5}$$

The possibility of probabilities equal to 1 is crucial for understanding price competition, which we analyze in the next section.

2.3.2 Duopoly and Limit prices

Now we determine the firms' pricing strategies of the second stage, $(p_0^*(s_0), p_1^*(s_1))$ for all admissible s_0 and s_1. Throughout the entire section, we thus take the specifications as given.

Both firms $i = 0, 1$ anticipate the buyer's project choice and expect the own profit to be $\pi_i \equiv p_i\,\rho_i$. Profit π_i is not differentiable everywhere with respect to the relevant price because the ex ante winning probabilities, ρ_0 given in (2.4) and ρ_1 given in (2.5), may both take their corner value 1. To determine the firms' best response functions we thus distinguish two cases for each firm $i = 0, 1$. First, we assume the ex ante probability to be strictly smaller than 1. If $\rho_i < 1$, firm i believes that her or his competitor has a chance to win the contract. We call this the *Duopoly* case and denote it by the superscript D. Second, we assume the probability to be 1. If $\rho_i = 1$, firm i believes that she or he wins the contract for sure. We call this the *Limit* case and denote it by the superscript L.

In what follows we derive and analyze the firms' best response functions for each case separately, and summarize the results in lemma 2.1.

Duopoly price reactions. Suppose firm i's ex ante expected winning probability is strictly lower than 1. If additionally $\rho_i > 0$, then firm i's ex ante profit is well-behaved and its unique maximum is

$$p_0^D(p_1) = \frac{1}{2}\big(p_1 + t\,(1 - s_0 - s_1)\,(1 + s_0 - s_1 - 2\underline{s})\big), \text{ for } i = 0, \tag{2.6}$$

and

$$p_1^D(p_0) = \frac{1}{2}\big(p_0 + t\,(1 - s_0 - s_1)\,(1 - s_0 + s_1)\big), \text{ for } i = 1. \tag{2.7}$$

Firms compete in prices, so their *Duopoly price reactions* (2.6) and (2.7) are both increasing in the competitor's price. The degree of the buyer's unit cost for unsuitable specifications, t, as well as the degree of horizontal differentiation, $1 - s_0 - s_1$, mitigate price competition if $\underline{s} \leq (1 + s_0 - s_1)/2$. That is, the more costly unsuitable specifications and the more differentiated the projects, the less price sensitive the buyer, and the higher the firms' markups.

Because firm 1 disagrees on firm 0's perceived unsuitability his price reaction $p_1^D(p_0)$ is independent of $\underline{s}$. Thus, it is increasing in both, t and $1 - s_0 - s_1$, for all $\underline{s} \in (0, \overline{s})$.

Firm 0's price, however, is increasing in both variables if and only if $\underline{s} < (1 + s_0 - s_1)/2$. In this case, the expert's perceived unsuitability is low, and so is her perceived expertise f_0. This implies that the positive direct effect of a price increase on her profit exceeds the negative indirect effect of a price increase through a reduced winning probability. Recall that the probability ρ_i is the product firm i's home turf and her or his prior density f_i. If $\underline{s}$ is low, firm 0 expects a rather large home turf and weights it by a rather low density. Then, the positive effect of a price premium on profit exceeds the marginal reduction in the expected winning probability. In this case, firm 0's Duopoly price reaction is the sum of half her competitor's price and the differentiation-specific price premium $t\,(1 - s_0 - s_1)\,(1 + s_0 - s_1 - 2\underline{s})\,/2 > 0$.

Conversely, if $\underline{s} \geq (1 + s_0 - s_1)/2$, the price premium is negative. That is, her high perceived unsuitability $\underline{s}$ induces firm 0 to compensate the buyer. Because, in this case, firm 0 expects a short home turf, $\hat{s} - \underline{s}$, and additionally weights it by a high density, the negative indirect effect of a price premium through a reduced winning probability exceeds its positive direct effect on profit. In this case, the expert offers a discount on half her competitor's price. The marginal compensation for unsuitability $\underline{s}$ is $t\,(1 - s_0 - s_1) > 0$, and thus, increasing in the buyer's cost and the degree of differentiation.

Another consequence of firm 0's perceived unsuitability concerns her project specification s_0. Given a certain degree of horizontal differentiation, both Duopoly price reactions are decreasing in the competitor's and increasing in the own specification. We call project the project of firm $i = 0, 1$ a *maximally differentiated project* if $s_i = 0$, and a *customized project* if $s_i > 0$. Given $1 - s_0 - s_1$, firm i may charge a higher price if her or his competitor differentiates, and, if she or he offers a customized project.

However, in case of firm 1, it is well-known that the negative effect of horizontal differentiation on his profit exceeds the positive effect of a customized project:

firm 1's Duopoly price reaction (2.7) is a best response function in the framework of Hotelling's line with length 1. Firm 1 considers the support's upper bound according to his prior S_1, and is convinced to face no unsuitability. Then, the effect of mitigated price competition due to horizontal differentiation is dominant, and firm 1 has no incentive to customize his project.[19] His corner specification $1 - s_1 = 1$ therefore maximizes his Duopoly price reaction. In case of firm 0, however, the positive effect of a customized project on her best response exceeds the negative effect of horizontal differentiation: firm 0's Duopoly price reaction has a unique maximum at $s_0 = \underline{s}$. This specification fills the gap between firm 0's project specification s_0 and the lower bound of the support of the buyer's need S_0. The expert's perceived unsuitability $\underline{s}$ is thus an incentive to offer a customized project.

Given price reactions (2.6) and (2.7), the prerequisites for the Duopoly case, $\rho_i < 1, \forall i = 0, 1$, are functions of both prices p_i. We postpone the analysis of conditions $\rho_i < 1$ until after knowing the equilibrium prices with given specifications. The conditions for the Limit case, $\rho_i = 1, \forall i = 0, 1$, are satisfied by construction as shown in the subsequent paragraph.

Limit price reactions. Suppose firm i's ex ante winning probability is 1. Then, her or his expected profit is $\pi_i = p_i$, and her or his problem thus

$$\max_{p_i} p_i \quad \text{s.t.} \quad \rho_i = 1.$$

The constraint $\rho_i = 1$ corresponds to the prerequisite for the Limit case, and, it determines the solution $p_i^L(p_j)$: the objective function is strictly increasing in the choice variable p_i. However, if $\rho_i < 1$, then the winning probability is strictly decreasing in p_i. Thus, solving $\rho_i = 1$ for p_i yields the highest price that ensures firm i–at least in her or his own view–to win the contract.[20] These *Limit price reactions* are

$$p_0^L(p_1) = p_1 + t\,(1 - s_0 - s_1)\,(-1 + s_0 - s_1 + 2(1 - \overline{s})) \tag{2.8}$$

[19]This is true for the constraint market we analyze here. For the case that horizontal specifications are unconstrained Lambertini (1993) shows that the two driving forces of differentiation balance outside the market.

[20]Note that the constraint on firm 0's probability, $\rho_0 = 1$, is equivalent to $p_0 \leq p_1 + t\,(1 - s_0 - s_1)\,(-1 + s_0 - s_1 + 2(1 - \overline{s}))$. Analogically, firm 1's constraint $\rho_1 = 1 \Leftrightarrow p_1 \leq p_0 + t\,(1 - s_0 - s_1)\,(-1 - s_0 + s_1)$.

and

$$p_1^L(p_0) = p_0 + t\,(1 - s_0 - s_1)\,(-1 - s_0 + s_1)\,. \tag{2.9}$$

Compared to the Duopoly case, price competition in the Limit case is twice as strong. Moreover, firm 1's Limit price reaction is decreasing in the cost for unsuitable specifications, t, as well as in the degree of horizontal differentiation, $1 - s_0 - s_1$. That is, in firm 1's view, the buyer is more price sensitive, the more costly unsuitable specifications and the more differentiated projects. In firm 0's view, however, this is only the case if $1 - \overline{s} < (1 - s_0 + s_1)/2$, i.e., for low enough $1 - \overline{s}$.

To interpret $1 - \overline{s}$ note that firms not only disagree on firm 0's perceived unsuitability $\underline{s}$, but also on the unsuitability affecting firm 1's home turf: the expert's prior on the upper bound of the need's support, $\overline{s}$, implies her belief that firm 1 faces an interval of unsuitable specifications with length $1 - \overline{s}$. Firm 1, however, believes that all projects with $s_i \in [0, 1]$ might meet the buyer's need. The important difference between firm 0's own perceived unsuitability, $\underline{s}$, and her belief about firm 1's unsuitability, $1 - \overline{s}$, is that the latter stems from firm 0's perceived expertise $f_0 = 1/(\overline{s} - \underline{s})$. The effect of $1 - \overline{s}$ on firm 0's expected winning probability is thus, in contrast to the effect of $\underline{s}$, unambiguously positive.

Consequently, firm 0's Limit price reaction is increasing in t and in $1 - s_0 - s_1$ if her perceived expertise f_0 is high enough, or equivalently, if $1 - \overline{s} \geq (1 - s_0 + s_1)/2$. Note that, holding $\underline{s}$ constant, high perceived expertise corresponds to firm 0's belief that firm 1's unsuitability is high, i.e., that her competitor faces a short home turf, $\overline{s} - \hat{s}$. This allows the expert to charge a markup, which is higher, the higher the buyer's cost for unsuitable specifications and the more differentiated the projects.

In the Duopoly case, firm 0 compensates the buyer for her perceived unsuitability $\underline{s}$. In the Limit case, however, she capitalizes on her perceived expertise, and charges a markup. More formally, because the ex ante probability $\rho_0 < 1$ increases in $f_0 = 1/(\overline{s} - \underline{s})$, it increases in $1 - \overline{s}$. Therefore, the higher firm 0's perception of firm 1's unsuitability, the higher her Limit price that guarantees $\rho_0 = 1$. Because the expert is sure to win the contract, she totally ignores her own unsuitability $\underline{s}$ when deciding on her Limit price reaction $p_0^L(p_1)$.

There is another consequence of firm 0's perceived expertise in the Limit case that is absent in the Duopoly case: the Limit price reaction (2.8) has a unique maximum at $s_0 = \overline{s}$. In firm 0's view, this specification prevents that her competitor's specification $1 - s_1$ is closer to the support of S_0 than her own project s_0. Analogically, firm 1 considers the support's lower bound according to his prior S_1, and is convinced that firm 0 faces no unsuitability. The supremum of firm 1's Limit price reaction (2.9) is thus at $1 - s_1 \to 0$. Here, both firms have incentives to offer specifications in their competitor's home turf.

Lemma 1 summarizes these results. Its proof formally completes above considerations, and is in the appendix.

Lemma 2.1 (Price reactions)**.** *If* $\rho_i \in (0,1), \forall i = 0,1$*, firm i's best response function is the Duopoly price reaction given by* (2.6) *for* $i = 0$ *and by* (2.7) *for* $i = 1$*. If* $\rho_i = 1$*, firm i's best response function is the Limit price reaction given by* (2.8) *for* $i = 0$ *and by* (2.9) *for* $i = 1$*.*

In lemma 2.1, the winning probabilities are given. However, they crucially depend on prices. The equilibrium prices thus not only require to be mutually best responses. Additionally, they have to satisfy the restrictions that lemma 1 imposes on the winning probabilities.

Four equilibrium candidates $(p_0(s_0), p_1(s_1))$ arise from pairwise equating the price reactions of lemma 2.1. In what follows we first list the two candidates that satisfy the equilibrium requirements of mutually best responses and positive prices. Then, we rearrange the conditions on the probabilities to determine the parameter spaces in which the two candidates actually constitute price equilibria. Finally, proposition 2.1 summarizes the results.

Equilibrium candidates. Pairwise equating the competitors' best response functions yields four candidates for price equilibria with given specifications. The proof of proposition 2.1 shows that two of these candidates fail to constitute an equilibrium. Here, we focus on the other two:

(i) Suppose both firms offer their Duopoly price reactions (2.6) and (2.7). The possible price equilibrium in this case is

$$p_0^D(s_0) \equiv t\,(1 - s_0 - s_1)\,(1 + (s_0 - s_1 - 4\underline{s})/3)\,, \tag{2.10}$$

$$p_1^D(s_1) \equiv t\,(1 - s_0 - s_1)\,(1 + (-s_0 + s_1 - 2\underline{s})/3)\,. \tag{2.11}$$

Both price candidates exceed marginal cost, i.e., are positive if

$$\underline{s} \leq (3 + s_0 - s_1)/4. \tag{2.12}$$

(ii) Suppose firm 0 offers her Limit price reaction (2.8), and firm 1 his Duopoly price reaction (2.7). The corresponding equilibrium candidate is

$$p_0^L(s_0) \equiv t\,(1 - s_0 - s_1)\,(s_0 - s_1 - 1 + 4(1 - \overline{s}))\,, \tag{2.13}$$

$$p_1^L(s_1) \equiv 2\,t\,(1 - s_0 - s_1)\,(1 - \overline{s})\,. \tag{2.14}$$

They are both positive if

$$1 - \overline{s} \geq (1 - s_0 + s_1)/4. \tag{2.15}$$

In both putative price equilibria (i) and (ii), firm 1 plays his Duopoly price reaction (2.7). The expert's best response thus solely determines the candidates' types: firm 0 plays her Duopoly price reaction (2.6) in the putative *Duopoly price equilibrium* (i), and her Limit price reaction (2.8) in the putative *Limit price equilibrium* (ii). Recall that lemma 1 establishes the best response functions taking the expected winning probabilities as given. Therefore, the putative equilibria (i) and (ii) exist if prices are positive, and if the respective conditions concerning the probabilities are satisfied. For the Duopoly price equilibrium (i) these conditions are $\rho_0 < 1$ and $\rho_1 < 1$, while the existence of the Limit price equilibrium (ii) requires $\rho_0 = 1$ and $\rho_1 < 1$. Additionally, in both price equilibria, $\rho_i > 0$ must be satisfied $\forall i = 0, 1$.

Given the putative *Duopoly equilibrium prices* $p_0^D(s_0)$ and $p_1^D(s_1)$ according to (2.10) and (2.11), firms expect to win with probabilities

$$\begin{aligned}\rho_0^D &\equiv \frac{1}{\overline{s}-\underline{s}}\,(1/2+(s_0-s_1-4\underline{s})/6)\,,\ \text{and} \\ \rho_1^D &\equiv 1/2+(-s_0+s_1-2\underline{s})/6. \end{aligned} \tag{2.16}$$

The proof of proposition 2.1 shows that $\rho_1^D \in (0,1)$ is satisfied for all admissible s_0, s_1 and $\underline{s}$. The conditions on firm 0's winning probability, however, restrict the parameter space in which Duopoly price equilibria exist:

$$\rho_0^D > 0 \quad \Leftrightarrow \quad \underline{s} < (3+s_0-s_1)/4 \tag{2.17}$$

$$\rho_0^D < 1 \quad \Leftrightarrow \quad 1-\overline{s} < 1/2+(-s_0+s_1-2\underline{s})/6. \tag{2.18}$$

Equilibrium condition (2.17) implies (2.12), the condition that the Duopoly prices according to (2.10) and (2.11) are positive. Given $p_1^D(s_1)$, it thus follows from the proof of lemma 2.1 that the Duopoly price $p_0^D(s_0)$ unambiguously maximizes firm 0's profit if (2.17) and (2.18) are satisfied.

In the opposite case, where (2.18) is violated, i.e., if $\rho_0^D = 1$, firm 0 expects to win with certainty when offering $p_0^D(s_0)$. This cannot be profit maximizing because firm 0 may offer a higher price without risking its–as certain perceived–victory. The highest price fulfilling this is firm 0's Limit price $p_0^L(s_0)$. Formally, condition (2.18) is equivalent to $p_0^D(s_0) > p_0^L(s_0)$, and consequently,

$$\rho_0^D = 1 \quad \Leftrightarrow \quad p_0^D(s_0) \leq p_0^L(s_0) \tag{2.19}$$

$$\Leftrightarrow \quad 1-\overline{s} \quad \geq 1/2+(-s_0+s_1-2\underline{s})/6. \tag{2.20}$$

Therefore, firm 0 offers her Limit equilibrium price if condition (2.19) is satisfied, and additionally, if $p_0^L(s_0) \geq 0$. The latter is the case if (2.15) is satisfied. The proof of proposition 2.1 shows that (2.15) follows from assumption $\underline{s} < \overline{s}$ together with equilibrium condition (2.20). If the latter is satisfied, firms offer $p_0^L(s_0)$ and $p_1^L(s_1)$ as the requirements for a Limit price equilibrium, $\rho_0 = 1$ and $\rho_1 \in (0,1)$, impose no further restrictions on its existence[21]: given the firms' Limit prices

[21]However, uniqueness at the limit where (2.20) holds as equality requires the assumption that firm 0 plays her Limit not her Duopoly pricing strategy if she is indifferent. Note that our definition of a Duopoly price equilibrium implies this assumption by the condition on firm 0's probability, $\rho_0^D < 1$.

(2.13) and (2.14), they expect to win with probabilities

$$\rho_0^L \equiv 1, \text{ and}$$
$$\rho_1^L \equiv 1 - \overline{s}. \tag{2.21}$$

Proposition 2.1 summarizes above considerations, and its proof formally completes them.

Proposition 2.1 (Price equilibria). *If and only if* $1 - \overline{s} < 1/2 + (-s_0 + s_1 - 2\underline{s})/6$ *and* $\underline{s} < (3 + s_0 - s_1)/4$*, there exists a unique price equilibrium in pure strategies characterized by the Duopoly prices according to* (2.10) *and* (2.11)*. Moreover, if and only if* $1 - \overline{s} \geq 1/2 + (-s_0 + s_1 - 2\underline{s})/6$ *, there exists another unique price equilibrium in which firms offer their Limit prices given in* (2.13) *and* (2.14)*.*

Expert knowledge and prices. The expert's Duopoly equilibrium price $p_0^D(s_0)$ is decreasing in her perceived unsuitability $\underline{s}$. Due to price competition, firm 1 also compensates the buyer for $\underline{s}$, however, to a lower extent: because firm 1 does not believe in unsuitable project specifications, his best response $p_1^D(p_0)$ is independent of $\underline{s}$. That is, firm 1 has no intrinsic incentive for a price discount. Therefore, $p_0^D(s_0) < p_1^D(s_1)$, for all admissible $\underline{s}, s_0, s_1$ and t.

The expert's Limit equilibrium price $p_0^L(s_0)$, in contrast, is independent of her perceived unsuitability $\underline{s}$. However, it is increasing in her perceived expertise, or, more precisely, in the expert's belief about firm 1's unsuitability $1-\overline{s}$. Due to price competition also firm 1 increases his Limit price $p_1^L(s_1)$ with respect to $1 - \overline{s}$. As in the Duopoly case, the marginal effect of $1 - \overline{s}$ on firm 1's price is half the effect on the expert's price: in contrast to the latter, firm 1 has no intrinsic incentive for a price increase. To see this, recall that his best response $p_1^D(p_0)$ is independent of $1 - \overline{s}$. It follows that $p_0^L(s_0) > p_1^L(s_1)$ for all admissible s_0, s_1 and t, if equilibrium condition (2.20) is satisfied.

Expert knowledge and existence condition. In the Limit case, the increase in prices due to firm 0's perceived expertise is the only consequence of the priors' heterogeneity. In Duopoly price equilibria, however, differing priors have two effects: as described above the expert's perceived unsuitability shortens her home turf, and thus, affects prices. Additionally, her perceived expertise f_0 enhances her expected winning probability ρ_0^D, which in turn determines the conditions for the existence of Duopoly and Limit price equilibria, (2.18) and (2.20).

The two components of ρ_0^D contrarily depend on $\underline{s}$. Firm 0's home turf is decreasing, whereas $f_0 = 1/(\overline{s} - \underline{s})$ is increasing in $\underline{s}$. The overall effect of $\underline{s}$ on the expert's winning probability is thus ambiguous: the negative first effect prevails if $1 - \overline{s} \leq (1 - s_0 + s_1)/4$, that is, for high values of $\overline{s}$, or, for moderate levels of perceived experience. In this case, ρ_0^D is decreasing in $\underline{s}$. However, for higher levels of perceived expertise, $1 - \overline{s} \geq (1 - s_0 + s_1)/4$, the positive second effect exceeds the first, and ρ_0^D is increasing in $\underline{s}$.

Nevertheless, when deriving equilibrium condition (2.18), i.e., when solving $\rho_0^D < 1$ for $1 - \overline{s}$, the positive effect of firm 0's perceived expertise on her winning probability outreaches the negative effect of her perceived unsuitability. To see this, rewrite (2.18) as $1 - \overline{s} < (3 + s_0 - s_1 + 4\underline{s} - 6\underline{s})/6$, and note that the lower term $| + 4\underline{s}/6|$ corresponds to firm 0's perceived unsuitability, while the higher term $|-\underline{s}|$ is expertise-induced. Therefore, the effect of firm 0's perceived expertise dominates, and the interpretation of the condition for the existence of Duopoly price equilibria is unambiguous: the left hand side of (2.18) is decreasing in $\overline{s}$, and its right hand side is decreasing in $\underline{s}$. Recall that, holding the other parameter constant, f_0 is decreasing in $\overline{s}$ and increasing in $\underline{s}$. Therefore, the lower firm 0's perceived expertise, the more relaxed is inequality (2.18), i.e., the greater is the range of specifications s_0 and s_1 for which Duopoly price equilibria exist.

Intuitively, if firm 0's perceived expertise is moderate, she believes that firm 1's unsuitability, $1 - \overline{s}$, constitutes only a small disadvantage. In this case, it is optimal to offer the Duopoly price $p_0^D(s_0)$ that leaves her competitor a chance to win. In the contrary case, where firm 0's perceived expertise is high enough, she is sure to win the contract if she offers $p_0^D(s_0)$. Then, firm 0 optimally increases her price up to the Limit price $p_0^L(s_0)$, a price that stifles competition.

Note that the conditions for existence can be rewritten in terms of firm 1's expected probabilities alone, as their left hand sides both correspond to (2.21) and their right hand sides to (2.16). Condition (2.18) for the existence of a Duopoly price equilibrium thus corresponds to $\rho_1^L < \rho_1^D$, and equivalently, condition (2.20) for a Limit price equilibrium corresponds to $\rho_1^L \geq \rho_1^D$. This gives rise to a slightly different interpretation of proposition 2.1: firm 0 optimally bids the best price response that results in a higher expected winning probability of its competitor.

By doing so, the expert exploits her perceived expertise and the thereby implied overconfidence of firm 1.[22] If firm 0 anticipates that firm 1 is more confident

[22]Note that firm 0 thereby considers the absolute probabilities ρ_1^D and ρ_1^L, not the

to win in a Duopoly than in a Limit price equilibrium, she offers her Duopoly price reaction $p_0^D(p_1)$, and firms are in a Duopoly price equilibrium. However, if firm 0 believes that her competitor heavily misjudges the buyer's need, and thus, seriously overestimates his winning probability in Limit price equilibria, ρ_1^L, her profit maximizing price reaction $p_0^L(p_1)$ stifles competition. Then, firms are in a Limit price equilibrium.

Proposition 2.1 summarizes the pricing strategies of the second stage, $(p_0^D(s_0), p_1^D(s_1))$ and $(p_0^L(s_0), p_1^L(s_1))$. In the next section, we analyze the firms' specifications given these prices.

2.3.3 Differentiated and Customized specifications

In the first stage of the game, firms choose their specifications. They do so anticipating the equilibrium prices of proposition 2.1 and maximizing their corresponding profit $\pi_i^k = p_i^k(s_i)\,\rho_i^k$, with $k = D, L$. In what follows, we separately analyze these two cases: first, we derive the firms' project specifications given the Duopoly price equilibrium, then given the Limit price equilibrium.

Specifications given the Duopoly prices. In the Duopoly case, the rationale for the firms' specification choices of the Hotelling line remains valid: on the one hand, by differentiating her or his project, firm $i = 0, 1$ gives up part of her or his home turf, and thus, reduces her or his expected winning probability. On the other hand, differentiated projects mitigate price competition. Conversely, when deciding whether to customize their projects, i.e., whether to offer strictly positive specifications, both firms face the same trade-off: improving their chance of winning and intensifying price competition.

To see this, note that both expected home turfs, $\hat{s} - \underline{s}$ and $1 - \hat{s}$, are unambiguously increasing in the respective choice variable s_i, and so are the expected winning probabilities ρ_i^D.

The effect of the specification s_i on the Duopoly price $p_i^D(s_i)$ is not that clear: customized projects are less costly for the buyer[23], and therefore, allow firms to charge higher prices. However, customized projects come along with a low degree of differentiation. Because the buyer is more price sensitive if differentiation is

perceived "degrees of overconfidence" $\rho_1^D - (1 - \rho_0^D)$ and $\rho_1^L - (1 - \rho_0^L)$.

[23]More precisely, for a given degree of differentiation $1 - s_0 - s_1$, the Duopoly price $p_i^D(s_i)$ of firm $i = 0, 1$ is strictly increasing in her or his specification s_i.

low, this forces firms to lower prices.

In Hotelling's setup, the latter effect of customized projects on prices exceeds the former, and firm i's price is thus decreasing in her or his specification s_i. Here, this is the case for firm 1's Duopoly price: his prior $S_1 \sim U[0,1]$ entails firm 1's belief that his maximally differentiated project $1 - s_1 = 1$ lies at the border of the need's the support. He thus assesses the buyer's cost for unsuitability to be of less importance than the degree of differentiation. Consequently, firm 1's price $p_1^D(s_1)$ is decreasing in s_1 for all parameter values.

Firm 0's price $p_0^D(s_0)$, however, is decreasing in s_0 if and only if $\underline{s} \leq (1+s_0)/2$, i.e., for low degrees of perceived unsuitability $\underline{s}$. Then, firm 0 expects the buyer's cost for unsuitable specifications to be less important than his price sensitivity due to low differentiation.[24]

It follows that in the Duopoly case both firms face the trade-off between improving their chance of winning and intensifying price competition when deciding on the degree of project customization–exactly as in Hotelling's framework. Here, however, firms have heterogeneous priors, and therefore, assess this trade-off differently. As a consequence, the firms' specifications may be asymmetric.

Firm 1 believes in the buyer's announcement that the latter's need has support $S_1 \sim U[0,1]$, i.e., that his maximally differentiated project $1 - s_1 = 1$ is not unsuitable. Therefore, firm 1's incentive to customize his project in order to expand his home turf is weak. Firm 1 compares this weak positive effect of customization with the negative effect on his price. Because the latter exceeds the former he offers his differentiated project $1 - s_1^* = 1$.

The expert, in contrast, is convinced that the buyer's need is drawn from $[\underline{s}, \overline{s}]$, i.e., that her differentiated project $s_0 = 0$ is unsuitable. Therefore, firm 0's incentive to customize is stronger the higher her perceived unsuitability $\underline{s}$. If $\underline{s}$ is high enough, the expert customizes her project because the expansion of her home turf seems more important than the induced price reduction.[25]

The proof of lemma 2.2 shows that, given the Duopoly price equilibrium, firm

[24]In the next footnote 25 we show that this is the case in equilibrium, i.e., given the Duopoly prices, firm 0 chooses a specification that satisfies $s_0 \geq 2\underline{s} - 1$.

[25]Recall that $p_0^D(s_0)$ is decreasing in firm 0's specification if and only if $\underline{s} \leq (1 + s_0)/2$. Substituting (2.22) and (2.23) shows that this is satisfied in (putative) Duopoly equilibrium.

0's profit maximizing specification is

$$s_0^{DD} \equiv 0 \text{ if } \underline{s} \leq 1/4, \text{ and} \tag{2.22}$$

$$s_0^{DC} \equiv (4\underline{s} - 1)/3, \quad \text{otherwise.} \tag{2.23}$$

If firm 0's perceived unsuitability is low, she offers her maximally differentiated project $s_0^{DD} = 0$. We refer to the related kind of putative Duopoly equilibrium as *Differentiated Duopoly equilibrium* and denote it by the superscript DD. If firm 0's perceived unsuitability is low, she offers the customized project $s_0^{DC} = (4\underline{s} - 1)/3$. We call the related kind of putative Duopoly equilibrium *Customized Duopoly equilibrium* and denote it by the superscript DC.

Specifications given the Limit prices. In the Limit case, firms face no trade-off between their chance of winning and their price because firm 0's Limit price reaction $p_0^L(p_1)$ implies that both expected winning probabilities are independent of specifications.

Actually, firm 1 faces no trade-off at all. His Limit price $p_1^L(s_1)$ is strictly decreasing in his choice variable s_1. Project customization reduces differentiation, which in turn induces firm 1 to lower his Limit price. This is the only effect of customization on $p_1^L(s_1)$ because his competitor's Limit price fully compensates for the buyer's unsuitability cost reflected in firm 1's price reaction $p_1^D(p_0)$. To see this note that in his price reaction (2.7), firm 1 considers customizing his project in order to lower the buyer's unsuitability cost. Firm 0's Limit price reaction $p_0^L(p_1)$ given in (2.8) stifles this incentive for customization, which thus is absent in firm 1's Limit price $p_1^L(s_1)$. Intuitively, if the expert's perceived expertise is deep, she offers a price that prevents her competitor from gaining advantage by customizing his project. Therefore, firm 1 offers his maximally differentiated project $1 - s_1^* = 1$ also in the Limit case.

Firm 0, however, faces the same trade-off concerning her price as in the Duopoly case when deciding on her Limit specification. Her Limit price is decreasing in her specification if her perceived expertise is high. More precisely, $p_0^L(s_0)$ is decreasing in s_0 if and only if $1 - \overline{s} \geq (1 - s_0)/2$, i.e., if the expert believes that her competitor faces a high unsuitability. Then, the support of the buyer's need S_0 lies relatively close to firm 0's maximally differentiated project $s_0 = 0$. In this case, the positive effect of customization on firm 0's price due to a reduced unsuitability is weaker than its negative effect due to a lower degree of differentiation. Consequently, firm

0 bids

$$s_0^{LD} \equiv 0, \text{ if } 1-\overline{s} \geq 1/2.$$

We call the related kind of putative Limit equilibrium *Differentiated Limit equilibrium* and denote it by the superscript LD.

In contrast, if firm 0's perceived expertise is high, she believes that firm 1's unsuitability is small (for a given $\underline{s}$). That is, firm 1's differentiated project $1-s_1 = 1$ is closer to the upper bound of the need's support $\overline{s}$ than $s_0^{LD} = 0$. Then, firm 0's incentive to accommodate the buyer's need exceeds her incentive to avoid differentiation. Consequently, firm 0 bids the customized specification

$$s_0^{LC} \equiv 1-2(1-\overline{s}), \text{ if } 1-\overline{s} < 1/2.$$

We call the related putative equilibrium *Customized Limit equilibrium* and denote it by the superscript LC.

Lemma 2.2 summarizes above results. Its proof is in the appendix.

Lemma 2.2 (Equilibrium specifications)**.** *Given the Duopoly prices,* (2.10) *and* (2.11)*, firm 0 offers* $s_0^{DD} = 0$ *if and only if* $\underline{s} \leq 1/4$*, and* $s_0^{DC} = (4\underline{s}-1)/3$ *otherwise. Given the Limit prices,* (2.13) *and* (2.14)*, firm 0 offers* $s_0^{LD} = 0$ *if and only if* $1-\overline{s} \geq 1/2$*, and* $s_0^{LC} = 1-2(1-\overline{s})$ *otherwise. In both price equilibria, firm 1 offers* $1-s_1^* = 1$*.*

Prices and specifications. The proof of proposition 2.1 shows that firm 0 offers her Duopoly price if its perceived expertise is moderate. This implies that, for any $\underline{s}$, firm 0 assesses the unsuitability of firm 1's project range, $1-\overline{s}$, to be a small disadvantage.[26] Consequently, firm 0 expects her competitor to win with strictly positive probability. Lemma 2.2 takes this as given and concludes that the expert only considers her own unsuitability $\underline{s}$ when deciding on her Duopoly specification.[27] Both results together imply that firm 0 offers her maximally

[26] Recall that the existence conditions on $1-\overline{s}$, (2.18) and (2.20), only depend on firm 0's perceived expertise: the positive effect of f_0 on firm 0's winning probability in the Duopoly case is unambiguous for both parameters $\underline{s}$ and $\overline{s}$. It dominates the opposite negative effect that the perceived unsuitability $\underline{s}$ has on ρ_0^D through shortening firm 0's home turf.

[27] Firm 0's Duopoly price $p_0^D(s_0)$ and her home turf $\hat{s}-\underline{s}$ are independent of the expected expertise f_0. However, both are functions of firm 0's perceived unsuitability $\underline{s}$ as well as

differentiated project in order to mitigate price competition if her perceived unsuitability is low. Then, the expert has a low expectation about the buyer's cost for unsuitability of her own project. Therefore, firm 0 focuses on the negative effect of customization on the buyer's price-sensitivity due to a lower degree of differentiation, and offers $s_0^{DD} = 0$. In contrast, if her perceived unsuitability $\underline{s}$ is high, firm 0 customizes her project in order to expand her home turf, and thus, increase her winning probability.

Another result of proposition 2.1 is the existence of the Limit price equilibrium conditional on a high perceived expertise $f_0 = 1/(\overline{s} - \underline{s})$, or equivalently, high $1 - \overline{s}$, for a given $\underline{s}$. That is, firm 0 assesses the unsuitability of firm 1's project range, $1 - \overline{s}$, to constitute a big disadvantage, and is thus certain of victory. The proof of lemma 2.2 shows that, in this case, the expert ignores her own unsuitability $\underline{s}$ when deciding on her Limit specification. Instead, firm 0's perceived expertise, or more precisely, her belief about firm 1's unsuitability, $1 - \overline{s}$, determines her specification s_0. Both results together imply that firm 0 offers her maximally differentiated project in order to mitigate price competition if she assesses firm 1's unsuitability to be high. Then, the expert has a high expectation about the buyer's cost for the unsuitability of her competitor's project. Therefore, firm 0 focuses on the negative effect of customization on her Limit price due to lower differentiation, and offers $s_0^{LD} = 0$. If $1 - \overline{s}$ is low, in contrast, firm 0 assesses firm 1's differentiated project as rather appropriate for the buyer's need. Consequently, she customizes her own project in order to keep the involved unsuitability cost low. This customization allows the expert to offer her profit-maximizing price without risking its–as certain perceived–victory due to a lower differentiation.

The proof of lemma 2.2 derives the firms project specifications given the price equilibria of proposition 2.1. In the next section, we combine these results.

2.4 Equilibrium types

To determine the equilibria of the entire game, we match the firms' first-stage specifications with their pricing strategies of the second stage. Because both subgames entail unique best responses, the equilibria of the entire specification-then-price game described in proposition 2.2, and proved in the appendix, are

of her specification s_0. The perceived expertise f_0, in contrast, is independent of s_0. Therefore, firm 0 ignores the latter when deciding on her Duopoly specification.

unique too.

Proposition 2.2 (Equilibrium types)**.** *In the specification-then-price game with differing priors there exist four equilibrium types. Each of these is the unique equilibrium in pure strategies given its respective range of parameters* $\underline{s}$ *and* $\overline{s}$*.*
(i) If and only if $\underline{s} \leq 1/4$ *and* $1-\overline{s} < 1/2 - \underline{s}/3$*, firms bid Differentiated Duopoly offers*
$\left(s_0^{DD},\, p_0^{DD}\right) = (0,\, 2\,t\,(1/2 - 2\underline{s}/3))$ *and* $\left(1,\, p_1^{DD}\right) = (1,\, 2\,t\,(1/2 - \underline{s}/3))$.
(ii) If and only if $\underline{s} > 1/4$ *and* $1-\overline{s} < (5-5\underline{s})/9$*, firms* $i = 0, 1$ *bid Customized Duopoly (DC) offers* $\left((4\underline{s}-1)/3,\, 32\,t\,(1-\underline{s})^2/27\right)$ *and* $\left(1,\, 40\,t\,(1-\underline{s})^2/27\right)$.
(iii) If and only if $1/2 \leq 1-\overline{s}$*, firms* $i = 0, 1$ *bid Differentiated Limit (LD) offers* $(0,\, t\,(4(1-\overline{s})-1))$ *and* $(1,\, 2\,t\,(1-\overline{s}))$.
(iv) If and only if

$$1/2 > 1-\overline{s} \geq \begin{cases} 1/2 - \underline{s}/3 \leq 1-\overline{s}, & \textit{for } \underline{s} \leq 1/4, \textit{ and,} \\ (5-5\underline{s})/9, & \textit{for } 1/4 < \underline{s}, \end{cases}$$

firms bid Customized Limit (LC) offers $\left(1-2(1-\overline{s}),\, 4\,t\,(1-\overline{s})^2\right)$ *and* $\left(1,\, 4\,t\,(1-\overline{s})^2\right)$.

Note that proposition 2.2 crucially relies on the assumption $s_0 < 1 - s_1$, which prevents strategies that aim at maximizing differentiation by offering specifications on the competitor's side of the line, and thus, jeopardize existence of customized equilibrium types: if the expert offers close enough to $1 - s_1 = 1$, firm 1 deviates to $1 - s_1 = 0$.

Figure 2.3 depicts the regions of existence in the space of perceived unsuitabilities $(\underline{s}, 1-\overline{s})$, and provides a basis for comparing the parameter ranges of the different equilibria. The mass of Duopoly exceeds the one of Limit equilibria. Moreover, the sum of regions where both firms maximally differentiate their project specifications is smaller than the entire area where the expert customizes her project.

Before we carefully analyze the different regions of existence, note that the two components of expert knowledge, expertise f_0 and the unsuitability ratio $(1-\overline{s})/\underline{s}$, are constant along the dotted lines in Figure 2.3. At a first glance, these dotted lines allow a rough intuition about the forces that drive existence of the different equilibrium types: for a given level of expertise, the expert rather customizes her project if the unsuitability ratio is low, i.e., if the unsuitability affecting her own special field is high compared to her competitor's. Otherwise, for high values of β, she rather offers her differentiated project. For a given unsuitability ratio, the expert leaves her competitor a chance to win, if her expertise is moderate. Otherwise, for high values of α, the expert offers her Limit price and believes to win the contract for sure.

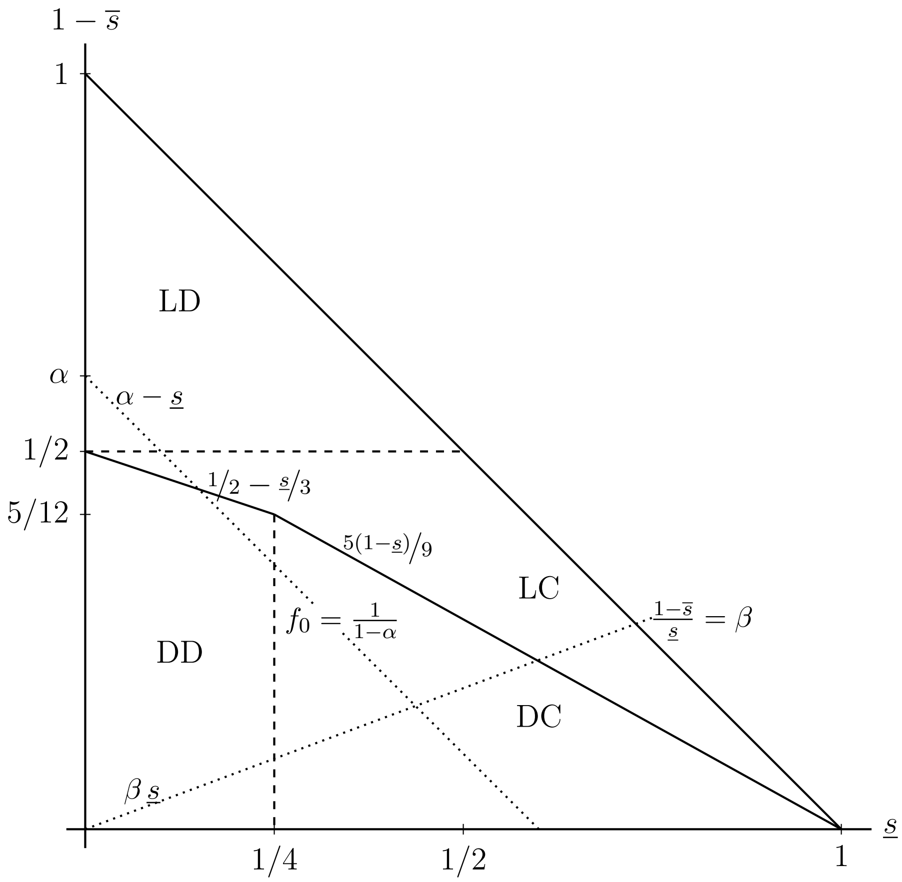

Figure 2.3: Regions of existence

More precisely, we may interpret the bound between Duopoly and Limit equilibria in two different ways: one interpretation concerns the experts' belief about her competitor's unsuitability $1-\overline{s}$ relative to her own unsuitability $\underline{s}$. The other interpretation is based on firm 1's expected winning probabilities. In the former, the expert assesses the unsuitability of her competitor's project $1-s_1^* = 1$ with regard to the buyer's need S_0. If she assesses firm 1's project as rather suitable, her equilibrium strategy lets the former a chance to win. Formally, if $1-\overline{s} < 1/2-\underline{s}/3$ and $1-\overline{s} < 5(1-\underline{s})/9$ there exist Duopoly equilibria. Otherwise, i.e., if firm 0 assesses her competitor's project as unsuitable she bids the Limit price that stifles competition.

For the alternative interpretation recall that condition (2.18) can be rewritten in terms of firm 1's expected winning probabilities only.[28] In this vein, Duopoly equilibria exist if firm 1 is more confident to win in Duopoly than in any Limit equilibrium. In the latter, firm 1 expects to win with probability $\rho_1^L = 1 - \overline{s}$. In his competitor's view, this is overconfident since firm 0 is convinced that her Limit price ensures winning. That is, $1 - \rho_0^L = 0 < \rho_1^L$. If this perceived overconfidence is high enough firm 0 bids her Limit price. More precisely, in the second stage with given specifications, firm 0 bids $p_0^L(s_0)$ if $\rho_1^L \geq \rho_1^D$. By doing so, firm 0 determines ρ_1^L, which is thus independent of specifications.

In Duopoly equilibria, in contrast, firm 1's expected winning probability is $\rho_1^{DD} \equiv 1/2 - \underline{s}/3$ if firm 0 maximally differentiates, and $\rho_1^{DC} \equiv 5(1 - \underline{s})/9$ if the expert customizes her project. The proof of lemma 2.2 shows that, in Duopoly equilibria, firm 0's decision whether to differentiate and avoid price competition or to customize and accept a lower price is determined by her own perceived unsuitability $\underline{s}$ only: if it is low enough, firm 0 maximally differentiates. If firm 0 faces a high unsuitability, she customizes her project. Thus, Differentiated Duopoly equilibrium exists if $\rho_1^L < \rho_1^{DD}$ and $\underline{s} \leq 1/4$, whereas Customized Duopoly equilibrium exists if $\rho_1^L < \rho_1^{DC}$ and $\underline{s} > 1/4$.

In Limit equilibria, firm 0's pricing strategy stifles competition. This allows the expert to ignore her own unsuitability when deciding on her specification. She maximizes her price, which–despite the specifications–solely depends on the expert's belief about her competitor's unsuitability $1 - \overline{s}$. Recall that this belief stems from firm 0's perceived expertise f_0. If it is high enough, or correspondingly, if $1 - \overline{s} \geq 1/2$, the expert differentiates. Otherwise, she customizes her project. Thus, Differentiated Limit equilibrium exists if $1 - \overline{s} \geq 1/2$ (implying $\rho_1^L \geq \rho_1^{DD}$ as well as $\rho_1^L \geq \rho_1^{DC}$ for $\underline{s} > 1/4$), whereas Customized Limit equilibrium exists if $1 - \overline{s} < 1/2$ and $\rho_1^L \geq \rho_1^{DD}$ for $\underline{s} \leq 1/4$ or $\rho_1^L \geq \rho_1^{DC}$ for $\underline{s} > 1/4$.

In what follows we separately analyze the different equilibria and their outcomes.

[28]For a description of this interpretation see page 85 in section 2.3.2.

2.4.1 Differentiated Duopoly equilibrium

If the expert's perceived unsuitability is low, i.e. if $\underline{s} \leq 1/4$, the negative effect customization has on her price due to a lower differentiation dominates the positive effect due to a reduction in the buyer's cost for unsuitability. Therefore, firm 0 offers her maximally differentiated project with $s_0^{DD} = 0$, and compensates the buyer for his cost by offering a price discount.

Firm 1 enters price competition and offers half the price discount of his competitor. The equilibrium prices in this case are

$$p_0^{DD} \equiv 2\,t\,(1/2 - 2\underline{s}/3)\,, \text{ and}$$
$$p_1^{DD} \equiv 2\,t\,(1/2 - \underline{s}/3)\,.$$

Recall that in Hotelling's symmetric model with quadratic unsuitability cost, uniformly distributed need, and constrained specifications firms also maximally differentiate. Consequently, their price offers in Differentiated Duopoly equilibrium are analogous: prices equal twice the cost parameter t times the respective firm's home turf. To see this, note that if Hotelling's line equals the unit interval, the buyer is indifferent at $\hat{s}^H = 1/2$, and firms offer identical prices $p_0^H = t = 2\,t\,\hat{s}^H$ and $p_1^H = t = 2\,t\left(1 - \hat{s}^H\right)$.[29] Here, asymmetric price discounts induce the buyer to be indifferent at $\hat{s}^{DD} = 1/2 + \underline{s}/3$. Therefore, firm 0's expected home turf is $h_0^{DD} = \hat{s}^{DD} - \underline{s} = 1/2 - 2\underline{s}/3$, while firm 1 expects to serve the longer interval $h_1^{DD} = 1 - \hat{s}^{DD} = 1/2 - \underline{s}/3$.

For the example of $t = 1$, $\underline{s} = .2$ and any arbitrary $\overline{s} > 1/2 + \underline{s}/3$ figure 2.4 depicts the unsuitability cost in DD equilibrium as a function of the buyer's need $s \in [0, 1]$.[30] The buyer's unsuitability cost function exhibits a gap at the specification $\hat{s}^{DD} = 17/30$ that sets him indifferent between projects (s_0^{DD}, p_0^{DD}) and $(1-s_1^*, p_1^{DD})$. Thus, the gap's length, $\left(\hat{s}^{DD} - s_0^{DD}\right)^2 - \left(1 - \hat{s}^{DD}\right)^2$, corresponds

[29] The proof of lemma 2.4 (on page 132 in the appendix) sketches existence and uniqueness in the standard Hotelling game (with quadratic unsuitability cost and uniformly distributed need) if firms share the common prior S_0, and with accordingly constrained specifications $s_i \in [\underline{s}, \overline{s}]$. The game if both firms share the prior $S_1 = [0, 1]$ is equivalent to the standard Hotelling game where the line's length is normalized to 1. Its solution $s_i^H = 0$ and $p_i^H = t$, $\forall i = 0, 1$ is thus equivalent to the equilibrium of lemma 2.4 for $\underline{s} = 0$ and $\overline{s} = 1$.

[30] Recall that we abstain from specifying whether the buyer's need has support $[\underline{s}, \overline{s}]$ or $[0, 1]$. Because the former is a subinterval of the latter, figure 2.4 shows the buyer's cost function for $s \in [0, 1]$.

to the difference in prices, $p_1^{DD} - p_0^{DD} = 2\,t\,\underline{s}/3$.

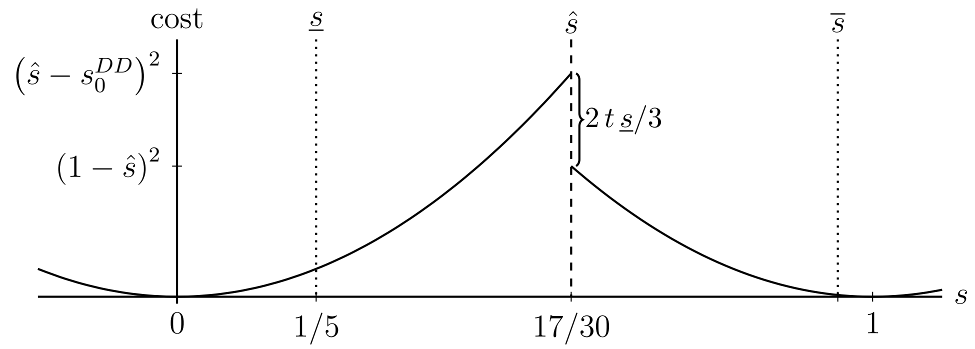

Figure 2.4: The buyer's cost for unsuitability in Differentiated Duopoly equilibrium

The Differentiated Duopoly offers (s_0^{DD}, p_0^{DD}) and $(1 - s_1^*, p_1^{DD})$ converge to the equilibrium of the Hotelling line if $\underline{s} \to 0$, i.e., if firm 0's perceived unsuitability is negligible. That is, in this limit case, prices converge to the buyer's unit cost of unsuitability t.[31] However, because of firm 0's perceived expertise $f_0 > 1$, only firm 1's expected winning probability converges to 1/2 if $\underline{s} \to 0$:

$$\rho_0^{DD} \equiv f_0\,(1/2 - 2\underline{s}/3)$$
$$\rho_1^{DD} \equiv 1/2 - \underline{s}/3.$$

Firm 1's winning probability ρ_1^{DD} equals his expected home turf.[32] It is strictly decreasing in firm 0's perceived unsuitability because the price difference $p_1^{DD} - p_0^{DD}$ is strictly positive and increasing in $\underline{s}$. This implies that the specification $\hat{s}^{DD}$, at which the buyer is indifferent, shifts to the right as $\underline{s}$ increases, and thus, shortens firm 1's expected home turf $1 - \hat{s}^{DD}$.

Firm 0's expected winning probability ρ_0^{DD}, however, is increasing in her unsuitability $\underline{s}$ for rather high unsuitability levels of her competitor, more precisely, for $1 - \overline{s} \geq 1/4$. In this case, firm 0's perceived expertise f_0 is high.[33] As

[31]The Hotelling prices t correspond to the ceiling in the Differentiated Duopoly case. Since prices are decreasing in firm 0's unsuitability, they reach their minima $p_0^{DD} = 2\,t/3$ and $p_1^{DD} = 5\,t/6$ in the other limit where $\underline{s} = 1/4$.

[32]Note that the sum of the firms' expected own home turfs equals $1 - \underline{s}$ in every equilibrium. This is a consequence of the firms' disagreement on the support of the buyer's need: while firm 1's expected home turf $h_1^k = 1 - \hat{s}^k$ is part of the unit interval, firm 0's home turf is shortened by her own unsuitability, i.e., $h_0^k = \hat{s}^k - \underline{s}$.

[33]A high expertise $f_0 \geq h_1^{DD}/h_0^{DD} = (3-2\underline{s})/(3-4\underline{s})$ also implies that firm 0's expected

mentioned before, a marginal increase in firm 0's unsuitability implies a price reduction. Because firm 0's winning probability positively depends on this price reduction weighted by the density f_0, she expects this compensation for the buyer's unsuitability cost of s_0^{DD} to be crucially beneficial for her winning probability in the case that f_0 is high enough. In other words, if firm 0's perceived expertise is high, she optimally overcompensates the buyer for his unsuitability cost. Conversely, firm 0's expected probability ρ_0^{DD} is decreasing in $\underline{s}$ for low unsuitability levels of her competitor, i.e., for $1-\overline{s} < 1/4$: because the positive effect the price difference has on the probability is weighted by a low density, firm 0 assesses the price discount as rather unimportant for the buyer's decision, and thus, abstains from full compensation.

The profits in Differentiated Duopoly equilibrium,

$$\pi_0^{DD} \equiv f_0\, 2\, t\, (1/2 - 2\underline{s}/3)^2 \,, \text{ and}$$
$$\pi_1^{DD} \equiv 2\, t\, (1/2 - \underline{s}/3)^2 \,,$$

however, are both strictly decreasing in $\underline{s}$. Thereby, firm 0's expected profit π_0^{DD} exhibits a higher variance than π_1^{DD}: if firm 0 faces a high unsuitability $\underline{s}$ and believes her competitor to face a low unsuitability $1-\overline{s}$, her expected profit is smaller than her competitor's, i.e., $\pi_0^{DD} - \pi_1^{DD} < 0$. In the limit of the opposite case, however, the expert's profit is twice as high as firm 1's expected profit since $\pi_0^{DD} \to t$ and $\pi_1^{DD} \to t/2$. Expressed in terms of perceived expertise, it holds that $\pi_0^{DD} \geq \pi_1^{DD} \Leftrightarrow f_0 \geq (3-2\underline{s})^2/(3-4\underline{s})^2$. Not surprisingly, if the firms' priors converge, i.e., if $\underline{s} \to 0$ and $\overline{s} \to 1$, not only the offers correspond to the Hotelling equilibrium, but also expected profits.[34]

winning probability exceeds firm 1's. Generally note that the expertise f_0 is constantly equal to $1/(1-a)$ along all lines $a - \underline{s}$ with intersections $a^k = 1 - h_0^k/h_1^k$. For $k = DD$ this intersect is $a^{DD} \equiv (3-2\underline{s})/(3-4\underline{s}) \in (0, 1/5]$. The line $a^{DD} - \underline{s}$ therefore partitions the DD parameter space for every $\underline{s} \in (0, 1/4]$. Differentiated Duopoly is the only equilibrium, in which firm 1's winning probability ρ_1^{DD} might exceed the expert's expectation ρ_0^{DD}. In all other equilibria $k = DC, LD, LC$, firm 0's perceived expertise satisfies $f_0 > h_1^k/h_0^k$, so her winning probability unambiguously exceeds her competitor's.

[34] The Hotelling profit $\pi^H = t/2$ corresponds to the supremum of firm 1's expected profit π_1^{DD}. This, and also firm 0's higher supremum profit $\pi_0^{DD} \to t$, result if $\underline{s} \to 0$ and $1-\overline{s} \to 1/2 - \underline{s}/3$. Consequently, the minimum of firm 1's profit, which equals $25\, t/72$, is smaller than π_H. However, it is higher than the infimum profit of firm 0, which is $8\, t/27$. These lowest values materialize if $\underline{s} = 1/4$ and $1-\overline{s} \to 0$.

2.4.2 Customized Duopoly equilibrium

Firm 0 customizes her project if her perceived unsuitability is large enough, i.e., if $\underline{s} > 1/4$. Her equilibrium specification $s_0^{DC} = (4\underline{s} - 1)/3$ is thus increasing in $\underline{s}$. In all equilibria, firm 1 maximally differentiates as he does not believe in unsuitability. Hence, the only effect of the expert's customized project on the firms equilibrium behavior is a tougher price competition. Consequently, firm 1's price

$$p_1^{DC} \equiv 40\,t\,(1 - \underline{s})^2\,/27$$

is decreasing in $\underline{s}$ only because of his competitor's price

$$p_0^{DC} \equiv 32\,t\,(1 - \underline{s})^2\,/27.$$

Firm 0's Customized Duopoly price p_0^{DC} is lower than firm 1's price p_1^{DC}, like in Differentiated Duopoly equilibrium.[35] In the latter, however, the marginal price discount firm 0 offers due to a increase in her unsuitability $\underline{s}$ exceeds the marginal price discount of firm 1. Here, in contrast, the negative impact $\underline{s}$ has on p_0^{DC} is weaker than on p_1^{DC}: due to the lower degree of differentiation in Customized Duopoly equilibrium firm 1 decreases his price to a higher extent than the expert, which thinks that customization partially compensates the buyer for his cost of unsuitability. Therefore, firm 0 believes that a high price discount is waste. In other words, the price reduction due to rather low differentiation in firm 0's case is alleviated by its–as appropriate perceived–customized project. Firm 1, however, thinks that customizing is out of place, and thus, incorporates the entire pressure of competition into his price.[36]

Figure 2.5 depicts this for the example of $t = 1$, $\underline{s} = .5$ and any arbitrary $\overline{s} > (4 + 5\underline{s})/9$ with equilibrium specifications $s_0^{DC} = 1/3$ and $1 - s_1^* = 1$. The length of the gap in the buyer's cost for unsuitability at $\hat{s}^{DC} = (4 + 5\underline{s})/9 =$

[35] Note that the composition of p_0^{DC} and p_1^{DC} is identical to the composition of Differentiated Duopoly prices, and thus, of the Hotelling prices. However, the lower degree of horizontal differentiation in Customized Duopoly equilibrium induces firms to give less weight to their expected home turfs $h_0^{DC} = 4(1 - \underline{s})/9$ and $h_1^{DC} = 5(1 - \underline{s})/9$. To see this, note that $p_0^{DC} = h_0^{DC}\,t\,8\,(1 - \underline{s})\,/3$, and analogous, $p_1^{DC} = h_1^{DC}\,t\,8\,(1 - \underline{s})\,/3$. The weight $8\,(1 - \underline{s})\,/3$ is strictly smaller than 2 (the weight in p_i^{DD} and p_i^{H}) if $\underline{s} > 1/4$, i.e., whenever Customized Duopoly equilibrium exists.

[36] In this vein, it is not surprising that the marginal effect of $\underline{s}$ on p_0^{DD} exceeds the one on p_0^{DC} for all $\underline{s} \in (0, \overline{s})$, whereas the effect on p_1^{DD} is higher than on p_1^{DC} only if $\underline{s} > 11/20$.

13/18 corresponds to the difference in prices, $p_1^{DC} - p_0^{DC} = 8\,t\,(1-\underline{s})^2/27$. This price difference is lower than in Differentiated Duopoly equilibrium. Moreover, in contrast to the price difference in the latter, the price difference in Customized Duopoly equilibrium is decreasing in the unsuitability $\underline{s}$. That is, by customizing her project, firm 0 induces a harmonization of prices.

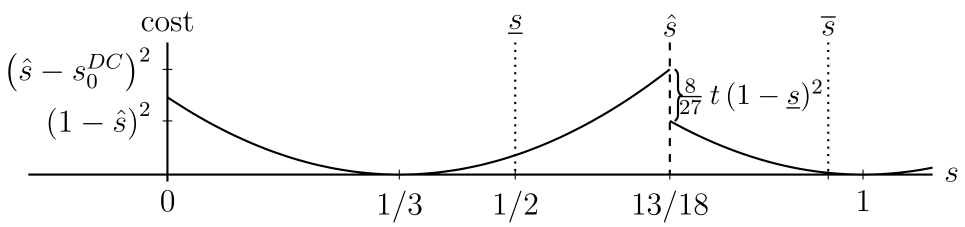

Figure 2.5: The buyer's cost for unsuitability in Customized Duopoly equilibrium

In the limit where $\underline{s} \to 1$, implying that $\overline{s} \to 1$, the game converges to a Bertrand competition: firms bid $s_0^{DC} \to 1$ and $1 - s_1^* = 1$, and this lack of differentiation pushes prices downwards marginal cost. In the other limit where $\underline{s} \to 1/4$, instead, the maximal prices in Customized Duopoly are achieved. They converge to the lowest prices in Differentiated Duopoly equilibrium, which are $2\,t/3$ and $5\,t/6$, respectively, and thus, lower than in the Hotelling's setup.

In contrast to firm 0's expected probability in DD equilibrium,

$$\rho_0^{DC} \equiv f_0\,4\,(1-\underline{s})/9$$

is strictly increasing in $\underline{s}$. While ρ_0^{DC} lies in the identical range as in Differentiated Duopoly, firm 1's probability

$$\rho_1^{DC} \equiv 5\,(1-\underline{s})/9$$

is strictly lower than in DD, and converges to the lowest value of ρ_1^{DD} for $\underline{s} \to 1/4$. The same is true for firm 1's expected profit

$$\pi_1^{DC} \equiv 200\,t\,(1-\underline{s})^3/243.$$

Also firm 0's profit

$$\pi_0^{DC} \equiv f_0 \, 128 \, t \, (1 - \underline{s})^3 \, /243$$

is strictly decreasing in $\underline{s}$ because the positive effect of her perceived unsuitability on her winning probability never compensates for the induced price discount. While firm 0's profit is higher than in Hotelling's setup for small levels of own unsuitability $\underline{s}$ and high beliefs about her competitor's unsuitability $1 - \overline{s}$, firm 1's profit is unambiguously lower.[37] Like in Differentiated Duopoly equilibrium firm 0's expected profit exceeds firm 1's if the former's perceived expertise is high. Formally, $\pi_0^{DC} - \pi_1^{DC} \geq 0 \Leftrightarrow f_0 \geq 25/16$.

2.4.3 Differentiated Limit equilibrium

If $1 - \overline{s} \geq 1/2$ firm 0's maximally differentiated project with specification 0 is close to her prior on the support of the buyer's need S_0, and thus, considered to be rather appropriate. Consistently, firm 0 perceives her competitor's specification $1 - s_1^* = 1$ as unsuitable. Therefore, her Differentiated Limit price

$$p_0^{LD} \equiv t \, (4(1 - \overline{s}) - 1)$$

is increasing in her competitor's unsuitability $1 - \overline{s}$. That is, the higher firm 1's unsuitability, the higher firm 0's price that ensures $\rho_0^L = 1$. This induces firm 1 to increase his price

$$p_1^{LD} \equiv 2 \, t \, (1 - \overline{s})$$

too. As in Differentiated Duopoly equilibrium the direct marginal effect of $1 - \overline{s}$ on p_0^{LD} is twice as high than its competition-induced effect on p_1^{LD}. In contrast to the Duopoly prices, firm 1's Limit price is thus lower than the expert's.

Firm 0's desirable prior allows maximal differentiation, and results in the highest prices of the entire game. That is, if $1 - \overline{s} \to 1$ implying $\underline{s} \to 0$ firms offer the supremum prices of all equilibria $3\,t$ and $2\,t$, respectively. Moreover, the

[37]Recall that the same is true in Differentiated Duopoly equilibrium. However, the parameter range for Customized Duopoly equilibrium contains the limit $\underline{s} \to 1$ implying $1 - \overline{s} \to 0$, in which firms compete solely in prices, and their profits converge to zero.

minimum prices in the Differentiated Limit case equal the Hotelling price t, and are thus higher than the infimum prices of all other equilibria.

For the example of $t = 1$, any arbitrary $\underline{s} < \overline{s}$, and $\overline{s} = .45$, figure 2.6 depicts the buyer's unsuitability cost. At $\hat{s}^L = \overline{s}$, this cost are lower for firm 0's differentiated project with $s_0^{LD} = 0$ than for her competitor's with $1 - s_1^* = 1$. The expert exploits that, and, compared to firm 1's price, charges a premium of $p_0^{LD} - p_1^{LD} = t\,(2(1-\overline{s}) - 1)$.

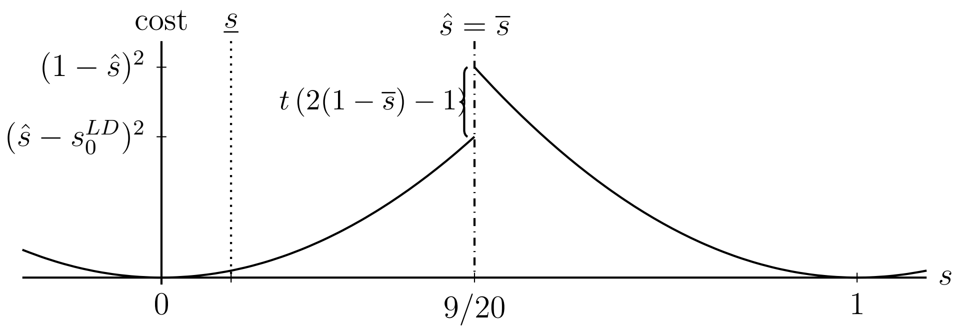

Figure 2.6: The buyer's cost for unsuitability in Differentiated Limit equilibrium

If $1-\overline{s} = 1/2$ both Differentiated Limit prices coincide with the Hotelling prices, t. At this lower limit of firm 1's unsuitability, his expected winning probability

$$\rho_1^L = 1 - \overline{s}$$

also coincides with the classic result. Consequently, so does his profit

$$\pi_1^{LD} \equiv 2\,t\,(1-\overline{s})^2 .$$

In contrast, $\rho_0^L = 1$ and firm 0's expected profit

$$\pi_0^{LD} \equiv t\,(4(1-\overline{s}) - 1)$$

is twice as high than in Hotelling's setup if $1-\overline{s} = 1/2$. Both profits are increasing in $1-\overline{s}$: firm 1 is convinced to face no unsuitability, and thus, that his competitor accordingly miscalculates his price p_0^{LD}. Therefore, firm 1 expects to win with probability equal to firm 0's "wrong" belief $1 - \overline{s}$. Together with the marginal effect of this belief on p_1^{LD} it follows that firm 1's profit exponentially increases in $1-\overline{s}$. As firm 0's expected winning probability in all Limit equilibria is equal to 1, her profit equals her price, and is linearly increasing in $1-\overline{s}$. Nevertheless, firm 0's profit is strictly higher than firm 1's for all $1-\overline{s} \in [1/2, 1)$.

2.4.4 Customized Limit equilibrium

For low unsuitability levels of firm 1, i.e., for $1-\overline{s} < 1/2$, firm 0 has an incentive to customize her project: she believes that the support of the buyer's need lies rather close to her competitor's project $1-s_1^* = 1$. So, she bids her price–and thus profit–maximizing specification $s_0^{LC} = 1-2(1-\overline{s})$. This is possible only in Customized Limit equilibrium: in contrast to the Differentiated Limit case, the call for bids does not confine firm 0's specification to be maximally differentiated. And, in contrast to both types of Duopoly equilibria, there is no need to take the specification's positive impact on the probability and its negative impact on the price into account. The optimal specification s_0^{LC} is decreasing in the unsuitability $1-\overline{s}$ since this is equivalent to a lower upper bound of the need's support, and therefore, provides a weaker incentive to bid a customized project. Note that also here, the equilibrium specification s_0^{LC} is lower than firm 0's prior on the support's lower bound $\underline{s}$.

For the example of $t = 1$, any $\underline{s} < \overline{s}$ that ensures $\rho_1^L \geq \rho_1^{DD}$ or $\rho_1^L \geq \rho_1^{DC}$, and $\overline{s} = .7$ firm 0 offers $s_0^{LC} = .4$. Figure 2.7 depicts this example and the equalizing effect of firm 0's customized project on the buyer's unsuitability cost, and therefore, prices.

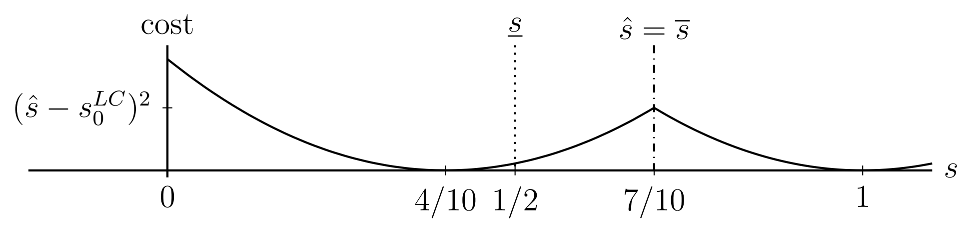

Figure 2.7: The buyer's cost for unsuitability in Customized Limit equilibrium

Generally, prices in Customized Limit equilibrium are

$$p_0^{LC} = p_1^{LC} \equiv 4\,t\,(1-\overline{s})^2\,.$$

Like in Differentiated Limit equilibrium, prices are increasing in firm 0's belief about her competitor's unsuitability. So, in the upper limit where $1-\overline{s} \to 1/2$, Customized Limit prices reach their maximum, which converges to the lowest value in the Differentiated Limit case, i.e., to the Hotelling prices t. Consequently, the same is true in the lower limit where $1-\overline{s} \to 1/2 - \underline{s}/3$ if firm 0's prior about her own unsuitability vanishes, i.e. if $\underline{s} \to 0$. Along the lower limit of $1-\overline{s}$, prices are strictly decreasing in $\underline{s}$, and reach their minimum where $\underline{s} \to 1$. This implies $1-\overline{s} \to 1$, and thus, firms enter Bertrand competition where prices are pushed down to marginal cost and profits are zero. The winning probabilities in all Limit equilibria are identical as already determined by firm 0's second-stage price reaction. However, here, firm 1's expected probability is lower than in Differentiated Limit equilibrium as the–as wrong assessed–belief of firm 0 is lower. Due to $\rho_1^L = 1-\overline{s}$ firm 1's profit is the $(1-\overline{s})$-multiple of firm 0's, and thus, strictly lower than the latter's:[38]

$$\begin{aligned}\pi_0^{LC} &\equiv 4\,t\,(1-\overline{s})^2 \\ \pi_1^{LC} &\equiv 4\,t\,(1-\overline{s})^3\,.\end{aligned}$$

Proposition 2.2 outlines the unique equilibria in pure strategies of the specification-then-price game with differing priors. That is, firm 0 has a more precise notion of the buyer's need than her competitor, and thus, considers herself an expert. This induces her deviation from the Hotelling offer $s_0^H = 0$ and $p_0^H = t$, which entails firm 1's price to differ from the Hotelling equilibrium too.[39] In this section, we described, amongst other things, how the expected profits given heterogeneous priors π_i^k, for $i = 0, 1$ and $k = DD, DC, LD, LC$ differ from the symmetric Hotelling profits $\pi_i^H = t/2$. These comparisons shed light on the consequences of extending Hotelling's line by an expert with prior $S_0 \sim U[\underline{s}, \overline{s}]$ on equilibrium offers and profits. However, they say nothing about the firms' valuations of expert knowledge given their differing beliefs S_0 and S_1.[40]

[38] All Limit profits are strictly increasing in $1-\overline{s}$. Therefore, π_0^{LC} and π_1^{LC} converge to their suprema t and $t/2$ at the upper limit $1-\overline{s} \to 1/2$. Due to continuity, these suprema in LC equilibrium correspond to the infima in LD equilibrium, i.e., in terms of Hotelling, they correspond to $2\pi_0^H$ and π_1^H.

[39] Recall that firm 1's price reaction function (2.7) as well as his equilibrium specification $1 - s_1^* = 1$ is analogous to the ones of Hotelling's line. Therefore, all deviations of his equilibrium prices from $p_1^H = t$ are reactions to the expert's deviations from (s_0^H, p_0^H).

[40] A proper welfare analysis in terms of the buyer's unsuitability cost requires an

We address this point in the next section. Thereby, we challenge the positive connotation of *expert knowledge* by comparing the equilibrium outcomes of proposition 2.2 with outcomes under alternative belief structures.

2.5 Profitability and market power

In this section, we study (i) whether the expert' knowledge is profitable, (ii) the expert discloses evidence to her competitor or the buyer, and (iii) whether firms assess expert knowledge as market power.

Profitability of expert knowledge. To answer question (i), we compare the expert's profit π_0^k in the equilibria $k = DD, DC, LD, LC$ of proposition 2.2 with her expected profit under the Hotelling strategy, but given her prior S_0. This comparison corresponds to analyzing an extension of the main game with differing priors (described in section 2.2), in which there is an additional ex ante stage to the specification-then-price subgames. In this ex ante stage, the expert decides whether to rely on her knowledge and prepare the offer herself (as in the main game) or to delegate the offer's preparation to a third party who shares the prior of the non-expert competitor. We assume that delegation is costless, for instance, because the non-expert third party is a "naïve" employee of the expert.

If the expert decides to prepare the offer by herself, i.e., apply her knowledge, the equilibria of proposition 2.2 arise. If, instead, the expert decides to delegate, i.e., to hand in an offer based on $S_1 \sim U[0,1]$, the firms' equilibrium strategies are identical as in Hotelling's line of length 1. That is, there exists a unique equilibrium in pure strategies with symmetric offers $s_i^H = 0$ and $p_i^H = t$, for both $i = 0, 1$. In this equilibrium, the buyer is indifferent at $\hat{s}^H = 1/2$.[41] While firm 1's expected profit equals the Hotelling profit $\pi_1^H = t/2$, firm 0's prior $S_0 \sim U[\underline{s}, \overline{s}]$

assumption on his need's true distribution, i.e., on the accuracy of the firms' priors. Chapter 2 uses such an assumption to compare welfare implications of tendering procedures with an expert and directly awarded contracts. Here, we abstain from that, and focus on the firms' expected benefits and cost of expert knowledge given their differing priors. However, note that the uniform distribution on $[0,1]$, with density 1, constitutes the limit of the joint distribution of S_0 and S_1 if they converge. For more heterogeneous priors, the density of their joint probability distribution within the unit interval is higher. Intuitively, the buyer's unsuitability cost is thus lower if the expert customizes her product than if she maximally differentiates.

[41] We outline the equilibrium in the classical formulation of Hotelling's line with quadratic transportation cost in lemma 2.4 on page 132 of the appendix.

gives rise to asymmetric expectations. We denote firm 0's expected home turf under the Hotelling strategy by $E_0\, h_0^H \equiv \hat{s}^H - \underline{s} = 1/2 - \underline{s}$, and her profit by $E_0\, \pi_0^H \equiv f_0 \cdot E_0\, h_0^H \cdot p_0^H = (\overline{s} - \underline{s})^{-1}(1/2 - \underline{s})\, t$.

The comparison of this profit with the equilibrium profits π_0^k corresponds to endogenizing the expert's decision whether to apply her knowledge for the offer preparation or not: if $\pi_0^k \geq E_0\, \pi_0^H$, the expert ex ante assesses her knowledge as profitable, and thus, offers (s_0^k, p_0^k) where $k = DD, DC, LD, LC$ as described in proposition 2.2. Otherwise, the expert delegates the offer's preparation to a non-expert party, and hands in the maximally differentiated Hotelling project $(0, t)$. Proposition 2.3 presents the equilibria of this extended specification-then-price game. Its proof is in the appendix, and its intuition below.

Proposition 2.3 (Profitability of knowledge). *If the expert decides whether to prepare the offer by herself or to delegate preparation to a non-expert party before the specification-then-price game starts, there exist the following unique types of pure strategy equilibria:*
(i) Hotelling (H) if and only if

$$1-\overline{s} < \begin{cases} 3/4-\underline{s}, & \textit{for } \underline{s} \leq 1/4, \\ \left(1-2\underline{s}+\sqrt{(5-2\underline{s})(1-2\underline{s})}\right)/4, & \textit{for } 1/4 < \underline{s} \leq (29-9\sqrt{6})/20, \\ (5-5\underline{s})/9, & \textit{for } (29-9\sqrt{6})/20 < \underline{s} \leq \tau, \end{cases}$$

with $\tau \approx .3653$;
(ii) Customized Duopoly (DC) if and only if $\tau < \underline{s}$ and $1-\overline{s} < (5-5\underline{s})/9$;
(iii) Differentiated Limit (LD) if and only if

$$1-\overline{s} \geq \begin{cases} 3/4-\underline{s}, & \textit{for } \underline{s} \leq 1/4, \\ 1/2, & \textit{for } 1/4 < \underline{s}; \end{cases}$$

(iv) Customized Limit (LC) if and only if

$$1/2 > 1-\overline{s} \geq \begin{cases} \left(1-2\underline{s}+\sqrt{(5-2\underline{s})(1-2\underline{s})}\right)/4, & \textit{for } 1/4 < \underline{s} \leq (29-9\sqrt{6})/20, \\ (5-5\underline{s})/9, & \textit{for } (29-9\sqrt{6})/20 < \underline{s}. \end{cases}$$

Figure 2.8 depicts the equilibria of propositions 2.2 and 2.3. In the shaded area, the expert assesses her knowledge-based strategies as unprofitable. Thus, in this parameter range the symmetric Hotelling equilibrium (where $s_i^H = 0$ and $p_i^H = t$, for both $i = 0, 1$) is the unique equilibrium of the extended game. In the white area, in contrast, the expert assesses her knowledge as profitable, and the respective equilibrium $k \in \{DC, LD, LC\}$ of the initial specification-then-price game carries over to its extension.

In what follows we outline the intuition of proposition 2.3 based on the equilibria of the initial specification-then-price game described in proposition 2.2. That is, we compare the expert's strategy in each equilibrium $k = DD, DC, LD, LC$ with the strategy $s_0^H = 0$ and $p_0^H = t$ in case of unprofitable knowledge, and describe the consequences for her expected home turfs, h_0^k and $E_0\, h_0^H$, as well as for her expected profits, π_0^k and $E_0\, \pi_0^H$.

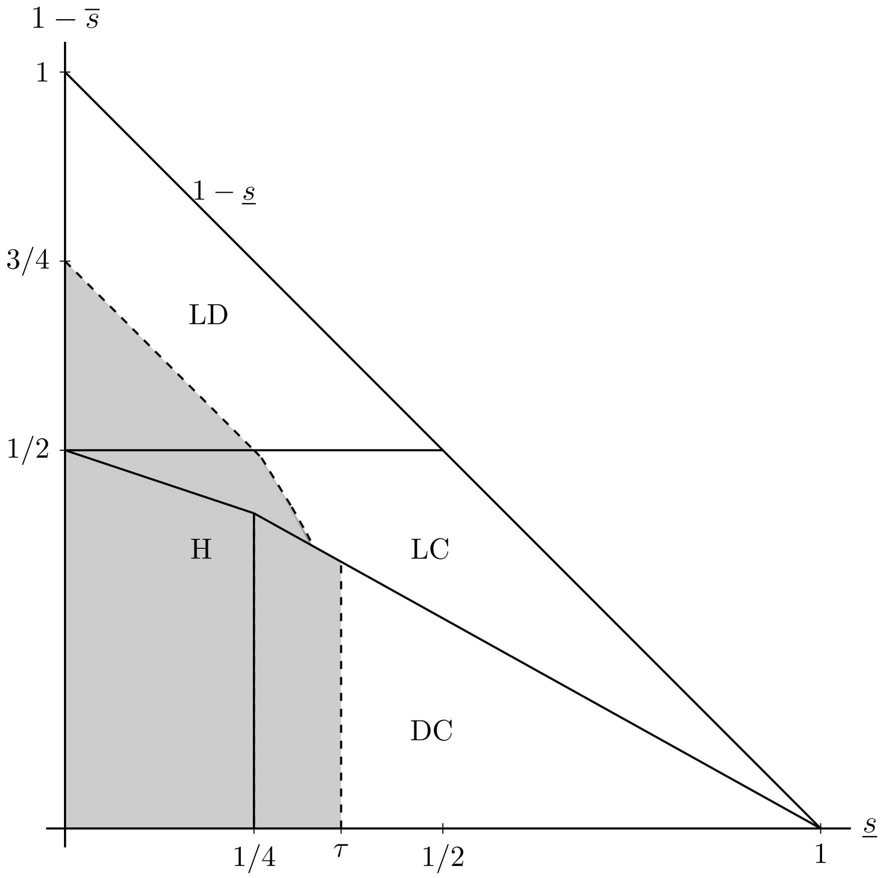

Figure 2.8: Profitability of expert knowledge (profitable in white, unprofitable in shaded area)

If priors are not too heterogeneous, i.e., if firm 0's perceived expertise f_0 is rather moderate and her perceived unsuitability $\underline{s}$ low, the expert's offer based on her knowledge S_0 lets her competitor a chance to win, and, abstains from customizing her project. Then, firms are in Differentiated Duopoly equilibrium. In DD, firm 0 grants a price discount to compensate the buyer for her unsuitability cost. Formally, $p_0^{DD} < p_0^H$, and consequently, $h_0^{DD} > E_0\, h_0^H$, $\forall \underline{s} \in (0, 1/4]$. The higher expected home turf in DD may not offset the price discount in terms of profit, i.e., $\pi_0^{DD} < E_0\, \pi_0^H$, $\forall \underline{s} \in (0, 1/4]$. In the DD region, $\underline{s} \leq 1/4$ and $1 - \overline{s} < 1/2 - \underline{s}/3$, the expert considers her knowledge as unprofitable: her perceived expertise f_0 plays no role in the comparison of profits, and the knowledge-based price discount not fully compensates for the unsuitable project specification $s_0^{DD} =$

0.

If the expert's perceived unsuitability is higher, i.e. if $1/4 < \underline{s}$, and if firm 0 considers this in her offer, she not only grants a price discount, but also customizes her project. In this case, firms are in Customized Duopoly equilibrium, in which $p_0^{DC} < p_0^H$ and $h_0^{DC} > E_0\, h_0^H$, $\forall \underline{s} \in (1/4, 1)$. However, in contrast to the DD case, the expert's knowledge in DC is profitable if firm 0's perceived unsuitability is high enough (roughly, for $\underline{s}$ higher than threshold $\tau \approx .3653$). Then, compared to the Hotelling strategy, the positive effect that customization has on h_0^{DC} outreaches the negative effect of the price discount on the expert's expected profit, and thus, $\pi_0^{DC} > E_0\, \pi_0^H$, $\forall \underline{s} \in (\tau, 1)$.

If the expert has high enough expertise f_0 and bases her offer on S_0 her pricing strategy stifles competition, i.e., there exists one of the Limit equilibria. In both, firm 0's expected home turf is $h_0^L = \overline{s} - \underline{s}$, which is smaller than $E_0\, h_0^H$ if and only if $\overline{s} \leq 1/2$, i.e., in Differentiated Limit equilibrium. Note that $\overline{s} \leq 1/2$ also implies that firm 0 assesses her competitor's specification $1 - s_1^* = 1$ as highly unsuitable. In this case, she thus offers her maximally differentiated project and charges a price premium without risking her (as certain assessed) victory. Formally, $p_0^{LD} > p_0^H$, $\forall \overline{s} \in (0, 1/2]$. In terms of the expert's expected profits, her knowledge is profitable if her home turf in LD is large enough. More precisely, $1/4 < h_0^L \Leftrightarrow \pi_0^{LD} > E_0\, \pi_0^H$.[42]

In Customized Limit equilibrium, in contrast, the expert not only customizes her project, but moreover, grants a price discount compared to $p_0^H = t$ in order to set the buyer indifferent at $\hat{s}^L = \overline{s} > 1/2$. This implies that $h_0^L > E_0\, h_0^H$, $\forall \underline{s} \in (0, 1)$. In this case, the expert assesses her knowledge as profitable if she believes that her competitor faces a relatively high unsuitability. More precisely, $\pi_0^{LC} \geq E_0\, \pi_0^H$ if and only if $1 - \overline{s} \geq (1 - 2\underline{s} + \sqrt{(5 - 2\underline{s})(1 - 2\underline{s})})/4$. Alternatively, for any given $\underline{s} \in (0, 1)$, firm 0's home turf $h_0^L = \overline{s} - \underline{s}$ is relatively low if this condition is satisfied. Since setting the buyer indifferent at a low $\hat{s}^L = \overline{s}$ requires only a low price discount, this does not strongly affect the LC profit. Thus, in this case knowledge is profitable.

In a nutshell, our answer to question (i) depends on the expert's perceived unsuitability levels: if her own is high enough, i.e., if the expert's assesses

[42] In Limit equilibria, a high home turf $h_0^L \geq 1/4$ is equivalent to a low expertise $f_0 \leq 4$. The latter, however, is an invalid interpretation as expertise cancels out when comparing firm 0's expected profits.

her special field as relatively unsuitable for the buyer's need, it is profitable to customize her offer, that is, apply her superior information and prepare the offer by herself. If it is low, i.e., if the expert's special field seems appropriate to satisfy the buyer's need, it is profitable to prepare the offer by herself if and only if her competitor's project seems unsuitable. In contrast, if both unsuitability levels are rather moderate, the expert benefits from delegating the offer's preparation to a non-expert party, i.e., from offering a maximally differentiated specification and abstaining from granting any price discount that launches competition.

No disclosure. Proposition 2.3 determines situations in which the expert assesses her knowledge as unprofitable. This by no means implies the answer to question (ii), whether the expert would persuade her competitor or the buyer to adopt knowledge. On the contrary, proposition 2.4 states that the expert is always better off if firms hold differing priors S_0 and S_1 than under the common prior $S_0 \sim U[\underline{s}, \overline{s}]$. Moreover, the expert is better off if the buyer asks for offers that allow the wider range of specifications $[0, 1]$ rather than $[\underline{s}, \overline{s}]$, that is, she benefits from the buyer's uncertainty. We discuss these result below, its proofs are in the appendix.

Proposition 2.4 (No disclosure)**.** *The expert never provides evidence for her knowledge in order to persuade her competitor or the buyer to adopt prior* $S_0 \sim U[\underline{s}, \overline{s}]$.

We prove this result by comparing the experts' equilibrium profits of the main game described in section 2.2 with her expected profits of two specification-then-price games with alternative belief structures.

First, we determine the equilibrium offers if both firms share the expert's prior $S_0 \sim U[\underline{s}, \overline{s}]$, while the buyer still believes that his need has support $[0, 1]$ and accordingly formulates the call for offers. We then use these equilibrium offers to show that the expert's expected profit is higher in the initial game with differing priors than if the duopolists share the common prior S_0. More precisely, we show that, firm 0's profit π_0^k in all equilibria $k = DD, DC, LD, LC$ is higher than her equilibrium profit if both firms agree on the expert's prior S_0.

Second, we compare the expert's expected profits in the equilibria of proposition 2.2 and in the equilibrium of Hotelling's classical line of length $\overline{s} - \underline{s}$ with accordingly constrained specifications. The latter corresponds to a specification-then-price game in which the buyer asks the firms for offers with specifications

$s_i \in [\underline{s}, \overline{s}]$, for both $i = 0, 1$. This constraint exogenously reduces horizontal differentiation, and consequently, intensifies price competition. Therefore, the expert's profit in this version of Hotelling's standard game with constrained specifications is lower than in the equilibria $k = DD, DC, LD, LC$ with differing priors. Consequently, the expert does not disclose evidence for her knowledge to the buyer.

Market power of expert knowledge. So far, we have analyzed the equilibria of proposition 2.2 only in terms of the expert's expectations, and ignored her competitor's view. In what follows we also consider the latter in order to answer question (iii), i.e., to determine whether firms assess expert knowledge as market power.

More precisely, for each equilibrium $k = DD, DC, LD, LC$, we compare firm i's home turf h_i^k with her or his expectation about the competitor j's home turf, which we denote $E_i\, h_j^k$, for $i \neq j$. This comparison is equivalent to a comparison of the expected winning probabilities $\rho_i^k = f_i\, h_i^k$ and $E_i\, \rho_j^k = f_i\, E_i\, h_j^k$ because, in the latter, firm i weights both home turfs with identical density f_i. We interpret a firm's winning probability as her or his market power, and say that firm 0 believes that *knowledge is power* if $h_0^k \geq E_0\, h_1^k$, and analogically, firm 1 believes that knowledge is power if $h_1^k \leq E_1\, h_0^k$. Moreover, we compare firm i's own profit π_i^k with her or his expectation about competitor j's profit $E_i\, \pi_j^k$. By doing so, we identify whether the expert's "superior" knowledge is a detriment or an advantage over her competitor. That is, if $\pi_0^k \geq E_0\, \pi_1^k$ or $\pi_1^k \leq E_1\, \pi_0^k$ the respective firm perceives knowledge as an advantage. Proposition 2.5 summarizes the results of these comparisons, which we discuss below. Simple rearrangements of the inequalities prove the results.

Proposition 2.5 (Market power of knowledge)**.** *In both Duopoly equilibria of proposition 2.2, firm 1 believes that knowledge is market power, i.e.,* $h_1^k \leq E_1\, h_0^k$ *holds for* $k = DD, DC$*. This is true for the expert if and only if her competitor's unsuitability is high enough, or formally,* $h_0^{DD} \geq E_0\, h_1^{DD} \Leftrightarrow 1 - \overline{s} \geq \underline{s}/3$ *and* $h_0^{DC} \geq E_0\, h_1^{DC} \Leftrightarrow 1 - \overline{s} \geq (1 - \underline{s})/9$*. In both Limit equilibria, the expert believes to be monopolist. Her competitor disagrees and believes that expert knowledge is power only in Customized, but not in Differentiated Limit equilibrium. That is,* $h_1^{LC} \leq E_1\, h_0^{LD}$ *and* $h_1^{LD} \geq E_1\, h_0^{LD}$ *in the according parameter ranges.*

In Differentiated Duopoly equilibrium, firm 0's price p_0^{DD} is lower than firm

1's price p_1^{DD}. As a consequence, firm 1 with prior $S_1 \sim U[0,1]$ expects his competitor's home turf $E_1\, h_0^{DD} = 1/2 + \underline{s}/3$ to exceed his own for all relevant $\underline{s}$. In firm 1's view, higher market power is not worth fiercer price competition.[43] Nevertheless, in terms of profits, his higher price does not offset his competitor's market power. Therefore, $\pi_1^{DD} < E_1\, \pi_0^{DD}$, $\forall \underline{s} \in (0, 1/4]$.

In the expert's view, in contrast, market power not only depends on the price difference, but additionally, on the relative unsuitability of the differentiated projects. Interestingly, firm 0's compensation for the buyer's unsuitability cost sustains her market power only if firm 1 faces a relatively high unsuitability. More precisely, $h_0^{DD} \geq E_0\, h_1^{DD} = \overline{s} - 1/2 - \underline{s}/3 \Leftrightarrow 1 - \overline{s} \geq \underline{s}/3$. Through increased market power, a price discount indirectly increases profit. However, the price discount's direct negative effect on profit dominates. Therefore, the parameter range in which $\pi_0^{DD} \geq E_0\, \pi_1^{DD} \Leftrightarrow 1 - \overline{s} \geq 1 - (3 - 4\underline{s} + 2\underline{s}^2)/(3 - 2\underline{s})$ is smaller than the parameter range in which knowledge is market power. Figure 2.9 depicts both bounds on $1 - \overline{s}$: in the white area, firm 0 not only perceives her knowledge as market power, but also expects $\pi_0^{DD} \geq E_0\, \pi_1^{DD}$. In the light shaded area, the latter does not hold in spite of firm 0's market power. In the dark shaded area, the expert perceives her knowledge as disadvantageous in both dimensions. These disadvantages increase in firm 0's unsuitability $\underline{s}$.

In Customized Duopoly equilibrium, the effects of knowledge on market power and profits are identical as in DD. That is, firm 1 sees his competitor's knowledge as an advantage for the latter, which is true in the expert's view only if her competitor faces high unsuitability. More precisely, $h_1^{DC} < E_1\, h_0^{DC} = 4/9 + 5\underline{s}/9$ and $\pi_1^{DC} < E_1\, \pi_0^{DC}$, $\forall \underline{s} \in (1/4, 1)$, whereas $h_0^{DC} \geq E_0\, h_1^{DC} = \overline{s} - (4 + 5\underline{s})/9 \Leftrightarrow 1 - \overline{s} \geq (1 - \underline{s})/9$ and $\pi_0^{DC} \geq E_0\, \pi_1^{DC} \Leftrightarrow 1 - \overline{s} \geq (1 - \underline{s})/5$. In contrast to the DD equilibrium, however, these lower bounds on $1 - \underline{s}$ are decreasing in $\underline{s}$ because product customization mitigates the disadvantages of firm 0's perceived unsuitability $\underline{s}$. Figure 2.9 depicts the expert's assessment of her knowledge in DC as described above.

[43] In DD equilibrium, project specifications are symmetric, i.e., $s_0^{DD} = 0$ and $1 - s_1^* = 1$. Suppose firm 1 offers a higher price discount than in equilibrium. Then, firm 0 undercuts according to its price reaction given in (2.6), and thus, $h_1 < E_1\, h_0$ still holds. If firm 1 exacerbates price competition to defend his home turf, firms enter Betrand competition and make zero profits. Therefore, in DD equilibrium, firm 1 leaves market power to the expert.

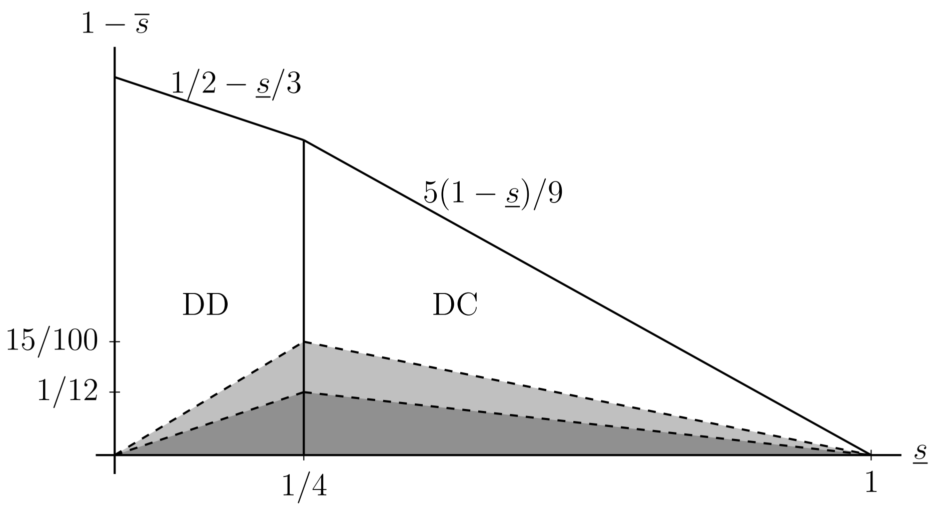

Figure 2.9: Advantage (white) and disadvantage (light shaded) of knowledge in terms of the expert's profit. In terms of her winning probability, knowledge is not market power in the dark shaded area.

In both Limit equilibria, the expert believes to entirely annex her competitor's home turf, i.e., $E_0\, h_1^L = \overline{s} - \overline{s} = 0$. Consequently, $h_0^L > E_0\, h_1^L$ and $\pi_0^L > E_0\, \pi_1^L = 0$. Firm 1, however, is convinced that the expert is wrong, and expects to face the strictly positive home turf $h_1^L = 1 - \overline{s}$. In Differentiated Limit equilibrium, $h_1^L \geq E_1\, h_0^L = \overline{s}$ since $\overline{s} \in (0, 1/2]$. Although firm 1 offers a lower price than his competitor, the positive effect of his home turf dominates his expected profit, and $\pi_1^{LD} > E_1\, \pi_0^{LD}, \forall \overline{s} \in (0, 1/2]$. That is, in Differentiated Limit equilibrium firm 0 assesses her knowledge as an advantage, while firm 1 assesses knowledge as a disadvantage.

In Customized Limit equilibrium, however, both firms assess the expert's knowledge advantageous: identical prices and $h_1^L < E_1\, h_0^L, \forall \overline{s} \in (0, 1/2)$ imply $\pi_1^{LC} < E_1\, \pi_0^{LC}$.

In a nutshell, our answer to question (iii) is that the expert assesses her knowledge as market power if either her competitor's project in Duopoly equilibria is unsuitable, or, if her expertise is deep enough to offer her Limit price. Firm 1, in contrast, assesses his competitor's knowledge as power in both Duopoly equilibria, while her perceives the expert's price premium in Differentiated Limit equilibrium as inappropriate, and thus, knowledge as a disadvantage.

2.6 Conclusion

This chapter studies the offers of specialized duopolists that compete for a complex contract in a one-shot specification-then-price game. In this context, it seems natural that the specialists ex ante have different experience, and thus, information at the moment the buyer asks them for offers.[44] We model the firms' different levels of experience as differing priors about the support of the buyer's horizontal need. Because priors are commonly known, this captures the notion of a self-proclaimed expert that believes to have superior information while her competitor assesses this "superior" information as wrong.

By extending Hotelling's seminal line with an expert we identify two components of expert knowledge that influence expected profits in opposite ways: on the one hand, deeper *expertise* increases the expert's winning probability. If expertise is high enough, the expert even is convinced to certainly win the contract. On the other hand, awareness of *unsuitable project specifications* may affect the expert's special field, and thus, force her to customize her project at the cost of less differentiation and fiercer price competition.

Indeed, there are parameter ranges in which the second component prevails, and thus, renders knowledge unprofitable. Nevertheless, the expert always profits from holding a differing prior, i.e., she never provides evidence in order to persuade her competitor or the buyer to adopt her knowledge. These insights stem from comparisons of the equilibrium outcomes in specification-then-price games with differing priors and alternative belief structures. While the latter equilibria are well-known, the fully characterized pure strategy equilibria of our main game with differing priors are a novelty in the literature on differentiated competition.

Future research might generalize our setup, for instance, by allowing for different distributions than the uniform, by randomizing the expert's prior such that it is no longer commonly known, by adding a revelation stage in which firms update their priors, or, by endogenizing the buyer's call for offers.

[44] Nevertheless, we tested robustness of our main result (presented in proposition 2.2) by replacing the differing priors with asymmetric information. Like welfare considerations, this requires further assumptions on the true support of the buyer's need. We therefore refer the interested reader to the robustness section in chapter 2 of this thesis.

2.A Appendices

A.1 Proof of lemma 2.1 (Price reactions)

Proof. This proof distinguishes two cases. In (i), we assume $\rho_i < 1, \forall i = 0, 1$ and derive the unique best price responses $p_i^D(p_j)$ with $j \neq i$. In (ii), we assume $\rho_i = 1$, $\forall i = 0, 1$ and derive the unique best price responses $p_i^L(p_j)$. In both parts, the specifications are given and satisfy $s_0 < 1 - s_1$.

(i) Let firm i's ex ante probability given by (2.4) for $i = 0$ and by (2.5) for $i = 1$ be strictly lower than 1, i.e., $\rho_i \in (0, 1)$, $\forall i = 0, 1$. Then, firm i's ex ante expected profit $\pi_i = p_i \cdot \rho_i$ exhibits a unique maximum in p_i: assumptions $\forall\, t > 0$, $0 < \underline{s} < \overline{s} < 1$, and $\forall s_i \in [0, 1), i = 0, 1$ with $s_0 < 1 - s_1 \Leftrightarrow \sum_i s_i < 1$ imply that both profits π_i are strictly concave in p_i since

$$\frac{\partial^2 \pi_0}{(\partial p_0)^2} = -\frac{1}{t\,(1 - s_0 - s_1)\,(\overline{s} - \underline{s})} < 0 \text{ and } \frac{\partial^2 \pi_1}{(\partial p_1)^2} = -\frac{1}{t\,(1 - s_0 - s_1)} < 0.$$

Moreover, profit π_i is increasing in p_i if and only if

$$\frac{\partial^2 \pi_0}{\partial p_0} \geq 0 \quad \Leftrightarrow \quad p_0 \leq \frac{1}{2}\left(p_1 + t\,(1 - s_0 - s_1)\,(1 + s_0 - s_1 - 2\underline{s})\right)$$

and

$$\frac{\partial^2 \pi_1}{\partial p_1} \geq 0 \quad \Leftrightarrow \quad p_1 \leq \frac{1}{2}\left(p_0 + t\,(1 - s_0 - s_1)\,(1 - s_0 + s_1)\right).$$

Therefore, taking the competitor's price p_j with $j \neq i$ as given, the unique profit maximizing price reactions if $\rho_i < 1$ are given by (2.6) and (2.7). (ii) Now, let firm i's ex ante probability be equal to 1, i.e., $\rho_i = 1$, $\forall i = 0, 1$. Then, firm i's expected profit is $\pi_i = p_i$, and her or his problem thus

$$\max_{p_i} p_i \quad \text{s.t.} \quad \rho_i = 1.$$

The objective function is strictly increasing in p_i, and the constraint determines the solution since

$$\begin{aligned} \rho_0 = 1 \quad &\Leftrightarrow \quad p_0 \leq p_1 + t\,(1 - s_0 - s_1)\,(-1 + s_0 - s_1 + 2(1 - \overline{s})) \\ \rho_1 = 1 \quad &\Leftrightarrow \quad p_1 \leq p_0 + t\,(1 - s_0 - s_1)\,(-1 - s_0 + s_1)\,. \end{aligned}$$

Therefore, taking the competitor's price p_j with $j \neq i$ as given, the unique profit maximizing price reactions if $\rho_i = 1$ are given by (2.8) and (2.9). □

A.2 Proof of proposition 2.1 (Price equilibria)

Proof. This proof completes the considerations on pages 81ff of the main text. There, we first state two equilibrium candidates, (i) and (ii), that satisfy the requirements of mutually best responses and positive prices. Here, we first show that the other two price pairs arising from equating the reaction functions of lemma 2.1 do not satisfy both requirements. Consequently, we exclude them from our further analysis. The other two equilibrium candidates, (i) and (ii), exist if they satisfy the conditions that lemma 2.1 imposes on expected winning probabilities. Moreover, these conditions imply uniqueness as the proof of lemma 2.1 shows that the firms' best response functions are unique if the expected winning probabilities are given. In the main text, we derive and analyze these conditions on firm 0's probability. Here, we complete this analysis for each price equilibrium separately. Besides the putative price equilibria (i) and (ii) that are described in the main text, equating the firms' reaction functions yields two other price pairs:

- Suppose firm 0 offers her Duopoly price reaction (2.6), and firm 1 his Limit price reaction (2.9). The candidate for a price equilibrium then is

$$p_0(s_0) = 2\,t\,\underline{s}\,(-1 + s_0 + s_1)\,, \tag{2.24}$$

$$p_1(s_1) = t\,(1 - s_0 - s_1)\,(-1 - s_0 + s_1 - 2\underline{s})\,. \tag{2.25}$$

  Assumption $s_0 < 1 - s_1 \Leftrightarrow s_0 + s_1 < 1$ implies $s_1 < 1$. Together with assumption $t > 0$, it follows that both prices, (2.24) and (2.25), are strictly negative. Because negative prices yield losses, they do not constitute a price equilibrium.

- Now, suppose both firms offer their Limit price reactions (2.8) and (2.9). This cannot be an equilibrium because the Limit price reactions are not mutually best responses: the inverse of firm 0's reaction, $\left(p_0^L(p_1)\right)^{-1}$, and firm 1's reaction $p_1^L(p_0)$ are parallels, and therefore, never cross. They are

$$\left(p_0^L(p_1)\right)^{-1} = p_0 - t\,(1 - s_0 - s_1)\,(1 + s_0 - s_1 - 2\underline{s})$$
$$p_1^L(p_0) = p_0 - t\,(1 - s_0 - s_1)\,(1 + s_0 - s_1)\,.$$

It follows that there exist no price equilibrium, in which firm 1 offers his Limit price reaction. We thus focus on the other two candidates (i) and (ii), characterized by prices ((2.10), (2.11)) and ((2.13),(2.14)), respectively.

(i) Note that, additional to the conditions on ρ_0^D, existence of Duopoly price equilibrium requires $\rho_1^D = 1/2 + (-s_0 + s_1 - 2\underline{s})/6 \in (0, 1)$. This holds for all admissible s_0, s_1 and $\underline{s}$ since

- $\rho_1^D > 0 \Leftrightarrow 3 - s_0 + s_1 - 2\underline{s} > 0$ holds by assumption $s_0 < 1 - s_1 \Rightarrow s_0 < 1$, $\forall s_1 \in [0, 1]$ together with assumption $\underline{s} < \overline{s} < 1$; and

- $\rho_1^D < 1 \Leftrightarrow 0 < 3 + s_0 - s_1 + 2\underline{s}$ holds by assumption $s_i \in [0,1], \forall i = 0,1$, together with assumption $0 < \underline{s}$.

It follows that the Duopoly prices (2.10) and (2.11) constitute a unique equilibrium in pure strategies if and only if the conditions on ρ_0^D are satisfied, i.e., if and only if $1 - \overline{s} < 1/2 + (-s_0 + s_1 - 2\underline{s})/6$ and $\underline{s} < (3 + s_0 - s_1)/4$ hold.

(ii) To see that condition (2.20) is equivalent to existence of a Limit price equilibrium, we now complete the considerations of the main text by showing that (2.20) implies (2.15). The latter condition, $1 - \overline{s} \geq (1 - s_0 + s_1)/4$, ensures that both Limit prices are positive. The former condition, $\rho_0 = 1 \Leftrightarrow 1 - \overline{s} \geq 1/2 + (-s_0 + s_1 - 2\underline{s})/6$ implies the latter if and only if

$$\begin{aligned} 1/2 + (-s_0 + s_1 - 2\underline{s})/6 &\geq (1 - s_0 + s_1)/4 \\ \Leftrightarrow \underline{s} &\leq (3 + s_0 - s_1)/4. \end{aligned} \tag{2.26}$$

In the Limit case, (2.26) is satisfied by assumption $\underline{s} < \overline{s} \Leftrightarrow 1 - \overline{s} < 1 - \underline{s}$, which together with (2.20) implies that

$$\begin{aligned} 1 - \underline{s} &> 1/2 + (-s_0 + s_1 - 2\underline{s})/6 \\ \Leftrightarrow \underline{s} &< (3 + s_0 - s_1)/4. \end{aligned}$$

It follows that the Limit prices (2.13) and (2.14) constitute a unique equilibrium in pure strategies if and only if $1 - \overline{s} \geq 1/2 + (-s_0 + s_1 - 2\underline{s})/6$. □

A.3 Proof of lemma 2.2 (Equilibrium specifications)

Proof. The proof is structured as follows. First, we derive the firms' reaction functions in Duopoly price equilibrium, $s_0^D(s_1)$ and $s_1(s_0)$. Then, in (ii), we calculate the resulting Duopoly equilibrium specifications s_0^D and s_1^D. Finally, in (iii), we determine the equilibrium specifications in Limit price equilibrium, s_0^L and s_1^L.

(i) Firm 0's profit in Duopoly price equilibrium, i.e., for all $1 - \overline{s} < 1/2 + (-s_0 + s_1 - 2\underline{s})/6$ and $\underline{s} < (3 + s_0 - s_1)/4 \Leftrightarrow s_0 - s_1 + 3 - 4\underline{s} > 0$, is $\pi_0^D = p_0^D(s_0)\,\rho_0^D$. It is increasing in her specification s_0 if and only if

$$\begin{aligned} \frac{\partial \pi_0^D}{\partial s_0} &\geq 0 \\ \Leftrightarrow 2(1 - s_0 - s_1)(s_0 - s_1 + 3 - 4\underline{s}) &\geq (s_0 - s_1 + 3 - 4\underline{s})^2 \\ \Leftrightarrow s_0 &\leq (4\underline{s} - 1 - s_1)/3. \end{aligned}$$

And, it is strictly concave if and only if $\partial^2 \pi_0^D/(\partial s_0)^2 < 0 \Leftrightarrow s_0 > (-5 + s_1 + 8\underline{s})/3$. The profit π_0^D is thus increasing and strictly concave for all $(-5 + s_1 + 8\underline{s})/3 <$

$s_0 \leq (4\underline{s} - 1 - s_1)/3$, and strictly decreasing for all $(4\underline{s} - 1 - s_1)/3 < s_0$. Note that $(-5 + s_1 + 8\underline{s})/3 < (4\underline{s} - 1 - s_1)/3 \Leftrightarrow \underline{s} < 1 - \frac{1}{2}s_1$ is true by assumption since $(3 + s_0 - s_1)/4 < 1 - s_1/2 \Leftrightarrow 0 < 1 - s_0 - s_1$. Given s_1, it follows that π_0^D has a unique maximum at $s_0(s_1) = (4\underline{s} - 1 - s_1)/3$. However, the government only considers bids with weakly positive specifications, so firm 0's unique best response is

$$s_0^D(s_1) \equiv \max\{0, (4\underline{s} - 1 - s_1)/3\},$$

which is strictly positive for $(1 + s_1)/4 < \underline{s}$. Note that this might hold in the entire parameter space of Duopoly price equilibrium since $(1 + s_1)/4 < (3 + s_0 - s_1)/4 \Leftrightarrow 0 < 2 + s_0 - 2s_1$ is true $\forall s_i \in [0, 1)$.

Firm 1's expected profit given the equilibrium prices $p_0^D(s_0)$ and $p_1^D(s_1)$ is $\pi_1^D = p_1^D(s_1)\,\rho_1^D$. It is increasing in s_1 if and only if

$$\frac{\partial \pi_1^D}{\partial s_1} \geq 0 \Leftrightarrow s_1 \leq \frac{1}{3}\left(2\underline{s} - 1 - s_0\right).$$

Thus, firm 1's profit has a unique maximum[45] at $s_1(s_0) = (2\underline{s} - 1 - s_0)/3$ since it is strictly concave in s_1, i.e., $\partial^2\pi_1^D/(\partial s_1)^2 < 0 \Leftrightarrow 5 - s_0 + 3s_1 - 4\underline{s} > 0$ is satisfied $\forall\, s_0 \in [0, 1)$ and $\forall \underline{s} < 1$. Because the government only considers offers with weakly positive specifications, firm 1's best response function is

$$s_1^D(s_0) \equiv \max\{0, (2\underline{s} - 1 - s_0)/3\}$$

, which is strictly positive for $(1 + s_0)/2 < \underline{s}$.

(ii) Let $s_1 = 0$. Then, firm 0's Duopoly specification is

$$s_0(0) = \begin{cases} 0, & \text{iff } \underline{s} \leq 1/4 \\ (4\underline{s} - 1)/3, & \text{iff } \underline{s} > 1/4. \end{cases}$$

For $s_0 = 0$, firm 1's Duopoly specification is

$$s_1(0) = \begin{cases} 0, & \text{iff } \underline{s} \leq 1/2 \\ (2\underline{s} - 1)/3, & \text{iff } \underline{s} > 1/2. \end{cases}$$

It follows that, in Duopoly price equilibrium, both firms offer their maximally differentiated projects with $s_0^{DD} \equiv s_1^* \equiv 0$ if and only if $\underline{s} \leq 1/4$.

Now, let firm 0 bid her interior best response $s_0^D(s_1) = (4\underline{s} - 1 - s_1)/3$. Indeed,

[45] Note that the first order condition $(\partial \pi_1^D)/(\partial s_1) = 0$ is also satisfied by $s_1 = -3 + s_0 + 2\underline{s}$. However, this solution contradicts a condition for the Duopoly price equilibrium.

firm 1's best response then is $s_1^* = 0$ since

$$\begin{aligned} s_1\left(s_0^D(s_1)\right) &= \frac{1}{3}\left(2\underline{s} - 1 - (4\underline{s} - 1 - s_1)/3\right) \\ \Leftrightarrow s_1 &= \frac{1}{4}(\underline{s} - 1) < 0, \ \forall \underline{s} \in (0,1). \end{aligned}$$

Therefore, given the Duopoly equilibrium prices, firm 0 offers the customized specification $s_0^{DC} \equiv (4\underline{s} - 1)/3$ if and only if $\underline{s} > 1/4$, and firm 1 offers $1 - s_1^* = 0$.

As this derivation holds for all $s_0(s_1)$ there is no Duopoly price equilibrium, in which firm 1 bids an interior specification. Together with the uniqueness of the reaction functions derived in (i), this guarantees that the Duopoly equilibrium specifications, s_0^{DD}, s_0^{DC} and $1 - s_1^*$, are unique.

(iii) Firm 0's expected profit given the Limit equilibrium prices $p_0^L(s_0)$ and $p_1^L(s_1)$ is $\pi_0^L = p_0^L(s_0)$, and increasing in s_0 if and only if

$$\frac{\partial \pi_0^L}{\partial s_0} \geq 0 \Leftrightarrow s_0 \leq 2\overline{s} - 1.$$

And, it is strictly concave since $\partial^2 \pi_0^L/(\partial s_0)^2 = -2b < 0$. It follows that firm 0 offers

$$s_0^L = \begin{cases} 0, & \text{iff } \overline{s} \leq 1/2 \\ 2\overline{s} - 1, & \text{iff } \overline{s} > 1/2. \end{cases}$$

Firm 1's expected profit given the prices $p_0^L(s_0)$ and $p_1^L(s_1)$ is $\pi_1^L = p_1^L(s_1)\,\rho_1^L$. It is strictly decreasing in s_1 since $\partial \pi_1^L/\left(\partial s_1\right) = -2b(1 - \overline{s})^2 < 0$. Thus, firm 1 offers $1 - s_1^* = 1$.

The proof of proposition 2.1 shows that the Limit equilibrium prices are unique. It follows that the Limit specifications are unique too given the respective parameter values. □

A.4 Proof of proposition 2.2 (Equilibrium types)

Proof. Given that prices are higher than marginal cost and expected winning probabilities are strictly positive, lemma 2.2 derives the unique equilibrium specifications given the prices of the second stage and proposition 2.1 proofs that these prices are unique given the parameters $\underline{s}$ and $\overline{s}$. For existence and uniqueness in the entire specification-then-price game described in section 2.2 we thus need to show that both prerequisites are satisfied. Moreover, we show that the assumption

$|p_0 - p_1| < t$ is satisfied in all equilibria.[46] Further, we rewrite the parameter ranges derived in proposition 2.1 by substituting the appropriate equilibrium specifications. In what follows, we do this separately for each equilibrium.

(i) The proof of proposition 2.1 shows that the range of parameters in which unique Duopoly price equilibrium exists is determined by $1-\overline{s} < \rho_1^D$ and $\underline{s} < (3+s_0-s_1)/4$. The proof of lemma 2.2 shows that, given the Duopoly prices, both firms offer maximally differentiated projects with $s_0^{DD} = 0$ and $1 - s_1^* = 1$ if and only if $\underline{s} \leq 1/4$. It follows that unique Differentiated Duopoly equilibrium exists if and only if $1 - \overline{s} < \rho_1^{DD} = 1/2 - \underline{s}/3$ and $\underline{s} \leq 1/4$ (implying $\underline{s} < 3/4$).

The prices in Differentiated Duopoly equilibrium are higher than marginal cost as $p_0^{DD} = t\,(1 - 4\underline{s}/3) \in [2t/3, t)$ and $p_1^{DD} = t\,(1 - 2\underline{s}/3) \in [5t/6, t)$. And, expected winning probabilities strictly lie within the unit interval as $\rho_0^{DD} = (3 - 4\underline{s})(6(\overline{s} - \underline{s}))^{-1} \in [4/9, 1)$ and $\rho_1^{DD} = (3 - 2\underline{s})/6 \in [5/12, 1/2)$. Moreover, $|p_0^{DD} - p_1^{DD}| < t \Leftrightarrow 2\underline{s}/3 < 1$ is satisfied for all $\underline{s} \in (0, 1/4]$.

(ii) The proof of lemma 2.2 shows that, given the Duopoly prices, firms offer $s_0^{DC} = (4\underline{s} - 1)/3$ and $1 - s_1^* = 1$ if and only if $\underline{s} > 1/4$. It follows that unique Customized Duopoly equilibrium exists if and only if $1 - \overline{s} < \rho_1^{DC} = 5(1 - \underline{s})/9$ and $\underline{s} > 1/4$. Note that the firms' specifications are such that there exist Duopoly equilibria for all possible parameter values since $\underline{s} < (3 + s_0^{DC} - s_1^*)/4 \Leftrightarrow \underline{s} < 1$.

The prices in Customized Duopoly equilibrium are higher than marginal cost as $p_0^{DC} = 32\,t\,(1 - \underline{s})^2/27 \in (0, 2t/3)$ and $p_1^{DC} = 40\,t\,(1 - \underline{s})^2/27 \in (0, 5t/6)$. And, expected winning probabilities strictly lie within the unit interval: $\rho_0^{DC} = 4(1 - \underline{s})(9(\overline{s} - \underline{s}))^{-1} \in (4/9, 1)$ and $\rho_0^{DC} = 5(1 - \underline{s})/9 \in (0, 5/12)$. Moreover, $|p_0^{DC} - p_1^{DC}| < t \Leftrightarrow 8(1 - \underline{s})^2/27 < 1$ is satisfied for all $\underline{s} \in (1/4, \overline{s})$.

(iii) The proof of proposition 2.1 shows that the range of parameters in which unique Limit price equilibrium exists is determined by $1 - \overline{s} \geq \rho_1^D$. The proof of lemma 2.2 shows that, given the Limit prices, both firms offer maximally differentiated specifications $s_0^{LD} = 0$ and $1 - s_1^* = 1$ if and only if $1 - \overline{s} \geq 1/2$. Since $1/2 > 1/2 - \underline{s}/3, \forall \underline{s} \in (0, 1/4]$ and $1/2 > 5(1 - \underline{s})/9, \forall \underline{s} \in (1/4, \overline{s})$ there exists unique Differentiated Limit equilibrium if and only if $1 - \overline{s} \geq 1/2$.

The probabilities in all Limit equilibria by construction are $\rho_0^L = 1$ and $\rho_1^L = 1-\overline{s}$, and therefore satisfy the prerequisite of strictly positive winning probabilities. The prices in Differentiated Limit equilibrium are $p_0^{LD} = t\,(3 - 4\overline{s}) \in [t, 3t)$ and $p_1^{LD} = 2t(1 - \overline{s}) \in [t, 2t)$, and hence, higher than marginal cost 0. Moreover, $|p_0^{LD} - p_1^{LD}| < t \Leftrightarrow 1 - 2\overline{s} < 1$ is satisfied for all $\overline{s} \in (0, 1/2]$.

(iv) The proof of lemma 2.2 shows that, given the Limit prices, the firms offer $s_0^{LC} = 2\overline{s} - 1$ and $1 - s_1^* = 1$ if and only if $1 - \overline{s} < 1/2$. Since $1/2 > 1/2 - \underline{s}/3, \forall \underline{s} \in$

[46] Since firm 1 offers $1 - s_1^* = 1$ it is trivial, and therefore omitted, to show that the assumption $s_0 < 1 - s_1$ is satisfied in all equilibria.

$(0, 1/4]$ and $1/2 > (5 - 5\underline{s})/9, \forall \underline{s} \in (1/4, 1)$ there exist unique Customized Limit equilibrium if and only if either $1/2 - \underline{s}/3 \leq 1 - \overline{s} < 1/2$ and $\underline{s} \leq 1/4$, or $5(1 - \underline{s})/9 \leq 1 - \overline{s} < 1/2$ and $\underline{s} > 1/4$.

The Customized Limit prices are higher than marginal cost since $p_0^{LC} = p_1^{LC} = 4\,t\,(1 - \overline{s})^2 \in (0, t)$. Moreover, $|p_0^{LC} - p_1^{LC}| < t \Leftrightarrow 0 < t$ is satisfied by assumption.

The parameter ranges for the different equilibria are derived in steps (i) to (iv). Thereby, it is shown that all prerequisites of previous lemmata 2.1 and 2.2 as well as of proposition 2.1 are satisfied, and hence, that the described unique equilibria exist. $\square$

A.5 Proof of proposition 2.3 (Profitability of knowledge)

Proof. The equivalence between comparing the equilibrium profits π_0^k with $E_0\,\pi_0^H$ and endogenizing the expert's ex ante decision whether to delegate the offer preparation to a non-expert party with prior $S_1 \sim U[0,1]$ or not is described in the text. Here, we compare π_0^k, for each equilibrium $k = DD, DC, LD, LC$, with $E_0\,\pi_0^H$ to determine the parameter ranges in which the expert assesses her knowledge as profitable. In this case, the equilibrium types of proposition 2.2 also exist in the extended game. If, in contrast, the expert assesses her knowledge as unprofitable, and thus, delegates the offer preparation to a non-expert party, firms offer maximally differentiated projects at identical prices $p_i^H = t$. For more details concerning this well established result, see the proof of lemma 2.4 on page 132.

(i) In the extended game, DD equilibrium does not exist. In its initial range, $\underline{s} \leq 4$ and $1 - \overline{s} < 1/2 - \underline{s}/3$, the unique equilibrium in pure strategies is the Hotelling equilibrium since

$$\begin{aligned} \pi_0^{DD} < E_0\,\pi_0^H &\Leftrightarrow 2\left(1/2 - 2\underline{s}/3\right)^2 < 1/2 - \underline{s} \\ &\Leftrightarrow 1/2 - 4\underline{s}/3 + 8\underline{s}^2/9 < 1/2 - \underline{s} \\ &\Leftrightarrow 0 < \underline{s}/3 - 8\underline{s}^2/9 \;\Leftrightarrow\; \underline{s} < 3/8 \end{aligned}$$

is satisfied $\forall \underline{s} \in (0, 1/4]$.

(ii) In the DC parameter range of proposition 2.2, there exist Hotelling and DC equilibria. To see this, we show that there exists a unique threshold τ such that $\underline{s} > \tau \in (1/4, 1)$ is equivalent to

$$\begin{aligned} \pi_0^{DC} > E_0\,\pi_0^H \;&\Leftrightarrow\; h_0^{DC} p_0^{DC} - E_0 h_0^H p_0^H > 0 \\ \Leftrightarrow 128\left(1/3 - \underline{s} + \underline{s}^2 - \underline{s}^3/3\right)/81 - (1/2 - \underline{s}) &> 0 \\ \Leftrightarrow \left(13/6 - 47\underline{s} + 128\underline{s}^2 - 128\underline{s}^3/3\right)/81 &> 0 \end{aligned} \tag{2.27}$$

Note that the LHS of (2.27) is strictly increasing in $\underline{s}$ if

$$\frac{\partial LHS}{\partial \underline{s}} = -\frac{1}{81}\,t\left(128\underline{s}^2 - 256\underline{s} + 47\right) > 0,$$

which holds if

$$128\underline{s}^2 - 256\underline{s} + 47 < 0. \tag{2.28}$$

Inequality (2.28) indeed holds $\forall \underline{s} \in (1/4, 1)$ because it is strictly decreasing, and negative at $\underline{s} = 1/4$:

$$\frac{\partial(128\underline{s}^2 - 256\underline{s} + 47)}{\partial \underline{s}} = 256(\underline{s} - 1)$$
$$128\underline{s}^2 - 256\underline{s} + 47\bigg|_{\underline{s}=1/4} = -9.$$

Hence, the LHS of (2.27) is strictly increasing. Existence of threshold τ therefore requires the LHS of (2.27) to be negative at $\underline{s} = 1/4$, and positive at $\underline{s} = 1$:

$$\left(13/6 - 47\underline{s} + 128\underline{s}^2 - 128\underline{s}^3/3\right)/81\bigg|_{\underline{s}=1/4} = -1/36$$
$$\left(13/6 - 47\underline{s} + 128\underline{s}^2 - 128\underline{s}^3/3\right)/81\bigg|_{\underline{s}=1} = 1/2.$$

There thus exists a unique threshold τ such that $\underline{s} > \tau \in (1/4, 1) \Leftrightarrow \pi_0^{DC} > E_0\, \pi_0^H$. Numerical approximation of the three roots of the LHS in (2.27) shows that only one of them is real, and roughly $\tau \approx .3653498367$.

(iii) In the LD parameter range of proposition 2.2, there exist Hotelling if and only if $h_0^L = \overline{s} - \underline{s} \leq 1/4$, and LD equilibrium otherwise:

$$\begin{aligned}\pi_0^{LD} \leq E_0\, \pi_0^H &\Leftrightarrow (\overline{s} - \underline{s})(4(1-\overline{s}) - 1) \leq 1/2 - \underline{s}\\ &\Leftrightarrow 0 \leq 1/2 + 2\underline{s} - 3\overline{s} - 4\underline{s}\overline{s} + 4\overline{s}^2\\ &\Leftrightarrow 0 \leq (1-2\overline{s})(1 - 4(\overline{s} - \underline{s}))/2 \end{aligned} \tag{2.29}$$

Because the first bracket of (2.29)'s is positive $\forall \overline{s} \in (0, 1/2]$, there exist Hotelling equilibrium if and only if $\pi_0^{LD} \leq E_0\, \pi_0^H \Leftrightarrow 1 - 4(\overline{s} - \underline{s}) \Leftrightarrow h_0^L \leq 1/4$, and LD equilibrium otherwise.

(iv) In the LC parameter range of proposition 2.2, there exist Hotelling if and only if $1 - \overline{s} \leq (1 - 2\underline{s} + \sqrt{(5-2\underline{s})(1-2\underline{s})})/4$, and LC equilibrium otherwise:

$$\pi_0^{LC} \leq E_0\, \pi_0^H \Leftrightarrow (\overline{s} - \underline{s})4(1-\overline{s})^2 - 1/2 + \underline{s} \leq 0 \tag{2.30}$$

If $\underline{s} \leq 1/4$ the relevant parameter range is determined by $\overline{s} \in (1/2, 1/2 + \underline{s}/3]$. To see that, in this case, knowledge is always unprofitable, first note that the LHS of (2.30) is strictly decreasing in $\overline{s}$ and strictly increasing in $\underline{s}$:

$$\begin{aligned}\frac{\partial LHS}{\partial \overline{s}} &= -8(\overline{s} - \underline{s})(1-\overline{s}) + 4(1-\overline{s})^2 < 0\\ &\Leftrightarrow 1 - \overline{s} < 2(\overline{s} - \underline{s}) \Leftrightarrow 3\overline{s} - 2\underline{s} > 1,\end{aligned}$$

which is satisfied $\forall \underline{s} \leq 1/4$ since, in LC, $\overline{s} > 1/2$. This also implies

$$\frac{\partial LHS}{\partial \underline{s}} = 1 - 4(1-\overline{s})^2 > 0.$$

Note that, therefore, (2.30) is satisfied for all relevant parameters if it is satisfied for $\underline{s} = 1/4$ and $\overline{s} \to 1/2$, which is the case:

$$\lim_{\overline{s}\to 1/2} (\overline{s} - \underline{s})4(1-\overline{s})^2 - 1/2 + \underline{s}\Big|_{\underline{s}=1/4} = 0.$$

Thus, knowledge is unprofitable for all $\underline{s} \leq 1/4$ and i.e., there exist Hotelling equilibrium.

For higher levels of own unsuitability $\underline{s} > 1/4$ the initial LC range is determined by $\overline{s} \in (1/2, 4/9 + 5\underline{s}/9]$. For this range, we show that $\pi_0^{LC} \geq E_0\,\pi_0^H$ is true if and only if $1 - \overline{s} \geq (1 - 2\underline{s} + \sqrt{(5-2\underline{s})(1-2\underline{s})})/4$ in what follows. First, note that $\pi_0^{LC} \geq E_0\,\pi_0^H$ can be written as

$$(2\overline{s} - 1)(1 + (\overline{s} - \underline{s})(4\overline{s} - 6)) \geq 0$$

since $1/2 < \overline{s}$, this inequality is satisfied if and only if

$$1 + (\overline{s} - \underline{s})(4\overline{s} - 6) \geq 0 \tag{2.31}$$

Solving for $\overline{s}$ yields $\overline{s} \leq (3+2\underline{s}-\sqrt{(5-2\underline{s})(1-2\underline{s})})/4$ and $(3+2\underline{s}+\sqrt{(5-2\underline{s})(1-2\underline{s})})/4 \leq \overline{s}$. These roots are real for $\underline{s} \in (1/4, 1/2]$, i.e., for higher values of $\underline{s}$, (2.31) is satisfied with strict inequality. The latter condition cannot be satisfied because its LHS exceeds 1. To see this, note that it is decreasing in $\underline{s}$ and equal to 1 if $\underline{s} = 1/2$:

$$\frac{\partial(2\underline{s} + \sqrt{(5-2\underline{s})(1-\underline{2s})})/4}{\partial \underline{s}} = \frac{1}{4}\left(2 - \frac{12 - 8\underline{s}}{2\sqrt{(5-2\underline{s})(1-\underline{2}s)}}\right)$$

is negative for all $\underline{s} \in (1/4, 1/2]$, since

$$\begin{aligned}
&\frac{12 - 8\underline{s}}{2\sqrt{(5-2\underline{s})(1-\underline{2}s)}} \geq 2 \\
&\Leftrightarrow (12 - 8\underline{s})^2 \geq 16(5-2\underline{s})(1-\underline{2}s) \\
&\Leftrightarrow 144 - 192\underline{s} + 64\underline{s}^2 \geq 80 - 192\underline{s} + 64\underline{s}.
\end{aligned}$$

Because

$$(3+2\underline{s}+\sqrt{(5-2\underline{s})(1-2\underline{s})})/4\Big|_{\underline{s}=1/2}=1,$$

only the first root is of interest. It lies in the LC parameter range as it converges to 1/2 (from the right) in the limit where $\underline{s}\to 1/4$, and

$$\begin{aligned}\left(3+2\underline{s}-\sqrt{(5-2\underline{s})(1-2\underline{s})}\right)/4 &\le 4/9+5\underline{s}/9\\ \Leftrightarrow 27+18\underline{s}-9\sqrt{(5-2\underline{s})(1-2\underline{s})} &\le 16+20\underline{s}\\ \Leftrightarrow (11-2\underline{s})^2 &\le 81(5-2\underline{s})(1-2\underline{s})\\ \Leftrightarrow 0 &\le 284-928\underline{s}+320\underline{s}^2 \end{aligned} \tag{2.32}$$

Solving (2.32) for $\underline{s}$ yields $\underline{s}\le(29-9\sqrt{6})/20\approx .3477$ and $2.5523\approx(29+9\sqrt{6})/20\le\underline{s}$. Thus, only the former is relevant. It follows that in the initial LC range $\pi_0^{LC}\ge E_0\,\pi_0^H\Leftrightarrow 1-\overline{s}\ge(1-2\underline{s}+\sqrt{(5-2\underline{s})(1-2\underline{s})})/4$ if $\underline{s}\in(1/4,(29-9\sqrt{6})/20]$. Otherwise, there exist Hotelling equilibrium. □

A.6 Proof of proposition 2.4 (No disclosure)

If the expert provides evidence for her prior to (a) her competitor or (b) the buyer, the latter party adopts firm 0's prior. In both cases, equilibrium offers are as if firms shared the common prior $S_0\sim U[\underline{s},\overline{s}]$:

In case (a), both firms indeed share the prior S_0. However, the buyer asks them for offers that allow for more differentiated specifications within the unit interval. Because $[\underline{s},\overline{s}]$ may be located asymmetrically in $[0,1]$, it depends on the respective firm's unsuitability whether she customizes her specification or is confined to maximally differentiate, i.e., to offer $s_0=0$ and $1-s_1=1$.

In case (b), in contrast, the buyer directly constraints project specifications to lie within the smaller interval $[\underline{s},\overline{s}]$. That is, the buyer calls for offers $(\underline{s}+s_0,p_0)$ and $(\overline{s}-s_1,p_1)$ with $s_i\in[0,1)$. As a consequence, firm 1's equilibrium offer is independent of his prior: his price reaction is according to our main setup, and thus, to a symmetric Hotelling line. Therefore, his expected profit is strictly decreasing in s_1 for all admissible values whether his prior is S_0 or $S_1\sim U[0,1]$, and he offers his maximally differentiated project with $s_1=0$.

In the specification-then-price game (a) there exist four types of pure strategy equilibria that are unique in mutually exclusive parameter ranges, see the proof of lemma 2.3. In the specification-then-price game (b), the unique equilibrium in pure strategies is identical to the equilibrium in Hotelling's line of length $\overline{s}-\underline{s}$ with accordingly constrained specifications, see lemma 2.4. To prove proposition 2.4 we compare π_0^k where $k=DD,DC,LD,LC$, i.e., the expert's expected profit in the main game with differing priors

(a) with her expected profits in the symmetric equilibria of lemma 2.3, π_0^b where $b = BDD, BCD, BDC, BCC$, and

(b) with her expected profit in the equilibrium of the constrained Hotelling game, π_0^{HC}.

(a) The expert never persuades her competitor to adopt prior S_0

Suppose both firms $i = 0, 1$ share the common prior $S_0 \sim U[\underline{s}, \overline{s}]$, with $0 < \underline{s} < \overline{s} < 1$, on the buyer's horizontal need. Denote the unsuitability of firm i's maximally differentiated project (with specification $s_i = 0$) by

$$\sigma_i \equiv \begin{cases} \underline{s}, & \text{for } i = 0; \\ 1 - \overline{s}, & \text{for } i = 1. \end{cases}$$

Let the buyer ask both firms for offers (s_0, p_0) and $(1 - s_1, p_1)$ with $s_i \in [0, 1)$, and firms simultaneously first choose specifications, and then prices. This game is equivalent to Hotelling's seminal line on the interval $[\underline{s}, \overline{s}]$ if specifications outside the unit interval $[0, 1]$ by assumption yield 0 profit.

Lemma 2.3 (Equilibria with common prior S_0)**.** *The buyer asks two firms $i = 0, 1$ with common prior $S_0 \sim U[\underline{s}, \overline{s}]$ for offers (s_0, p_0) and $(1-s_1, p_1)$ with $s_i \in [0, 1)$. In this specification-then-price game there exist 4 equilibrium types. The equilibrium offers of firms $i = 0, 1$ and $j \neq i$ are:*
(i) $s_i^{BDD} \equiv 0$ and $p_i^{BDD} \equiv t\,(1 - 2\,(2\sigma_i + \sigma_j)\,/3)$
iff $\sigma_i \leq 1/4 - \sigma_j/2$ and $\sigma_i \leq 1/2 - 2\sigma_j$;
(ii) $s_i^b \equiv (4\sigma_i + 2\sigma_j - 1)\,/3$ and $p_i^b \equiv 32\,t\,(1 - \sigma_i - \sigma_j/2)^2\,/27$, and
$s_j^b \equiv 0$ and $p_j^b = 40\,t\,(1 - \sigma_i - \sigma_j/2)\,(1 - \sigma_i - 7\sigma_j/5)\,/27$
iff $1/4 - \sigma_j/2 < \sigma_i \leq 1 - 5\sigma_j$.
(We write $b = BDC$ ($b = BCD$) if firm 0 maximally differentiates (customizes), while
firm 1 customizes (differentiates).)
(iii) $s_i^{BCC} \equiv (5\sigma_i + \sigma_j - 1)\,/4$ and $p_i^{BCC} = 2\,t\,(1 - \sigma_i - \sigma_j)^2\,/3$
iff $1 - 5\sigma_j < \sigma_i$ and $(1 - \sigma_j)/5 < \sigma_i$.

Proof. **Ex ante winning probability.** According to firm 0's winning probability (2.4) on page 76, firm $i = 0, 1$ ex ante expects to win with

$$\rho_i^b \equiv \frac{1}{2(1 - \sigma_i - \sigma_j)} \left(\frac{p_j - p_i}{t\,(1 - s_i - s_j)} + 1 + s_i - s_j - 2\sigma_i \right).$$

Note that, in contrast to the game with differing priors, firms that share a common prior agree on their winning probabilities, i.e., $1 - \rho_i^b = \rho_j^b$. Thus, focussing on strictly positive probabilities excludes probabilities of 1. In equilibrium, the prerequisite $\rho_i^b > 0$ is satisfied.

Price equilibrium. In the second stage, firms simultaneously choose prices given the specifications s_i of the first stage that by assumption satisfy $s_0 + s_1 \in [0, 1)$. If $\rho_i^b > 0$ for both $i = 0, 1$, there exist a unique price equilibrium in pure strategies where both firms $i \neq j$ bid

$$p_i^b(s_i) \equiv t\,(1 - s_i - s_j) \left(1 + \frac{1}{3}\,(s_i - s_j - 4\sigma_i - 2\sigma_j) \right). \tag{2.33}$$

In this price equilibrium, symmetric prices correspond to the expert's Duopoly price $p_0^D(s_0)$ given in (2.10) on page 82. However, since firm 1 adopts prior S_0 he considers $\sigma_1 = 1 - \overline{s}$ in his reaction function, and therefore, firm i's equilibrium price (2.33) not only compensates the buyer for σ_i, but due to price competition, also decreases in her or his competitor's unsuitability σ_j. Thus, given the specifications firm $i = 0, 1$ expects to win with probability

$$\rho_i^b(s_i) \equiv \frac{1}{2(1 - \sigma_i - \sigma_j)} \left(1 + \frac{1}{3}\,(s_i - s_j - 4\sigma_i - 2\sigma_j) \right).$$

Equilibrium specifications. Maximization of firm i's profit $\pi_i(s_i) =$

$p_i^b(s_i)\,\rho_i^b(s_i)$ yields the unique best response

$$s_i^b(s_j) \equiv (1/3)\left(4\sigma_i + 2\sigma_j - s_j - 1\right). \tag{2.34}$$

Given $s_j = 0$, firm i thus offers

$$s_i^b(0) = \begin{cases} 0, & \text{if } \sigma_i \leq 1/4 - \sigma_j/2 \\ \left(4\sigma_i + 2\sigma_j - 1\right)/3, & \text{else.} \end{cases} \tag{2.35}$$

Given $s_j^b(s_i) > 0$, substituting the reaction functions (2.34) results in firm i's putative offer

$$s_i^b = \left(5\sigma_i + \sigma_j - 1\right)/4, \tag{2.36}$$

which is strictly positive for $\sigma_i > (1 - \sigma_j)/5$. Its competitor j's putative equilibrium specifications are symmetric. Intuitively, the firm that faces the higher unsuitability rather customizes her or his project.

Because $s_i^b(s_j)$ is the unique eligible solution to the first order condition, strict concavity of the expected profit at above specifications (2.35) and (2.36) establishes uniqueness in the admissible parameter ranges.

Existence and uniqueness of equilibria in the entire game described in lemma 2.3 follows from the fact that the prerequisites for the price equilibrium, $\rho_i^b > 0 \Rightarrow \rho_j^b < 1$ and $p_i^b \geq 0$, are satisfied for both $i = 0, 1$ and all $b = BDD, BDC, BCD, BCC$. We leave the appropriate substitutions as well as the algebra to the reader. □

In what follows we prove statement (a) of proposition 2.4, the expert never persuades her competitor to adopt prior $S_0 \sim U[\underline{s}, \overline{s}]$. Specifically, we show that the expert's expected profits in the equilibria with differing priors of proposition 2.2 are higher than in the equilibria with common prior S_0 of lemma 2.3. That is, we show that $\pi_0^k \geq \pi_0^b$, where $k = DD, DC, LD, LC$ and $b = BDD, BDC, BCD, BCC$. We run these comparisons under consideration of the respective parameter ranges, which are depicted in figure 2.10, i.e. we distinguish ten cases. In each, we compare the expert's home turfs and prices. Note that $h_0^k \geq h_0^b$ and $p_0^k \geq p_0^b$ together imply $\pi_0^k = p_0^k\, f_0\, h_0^k \geq p_0^b\, f_0\, h_0^b = \pi_0^b$.

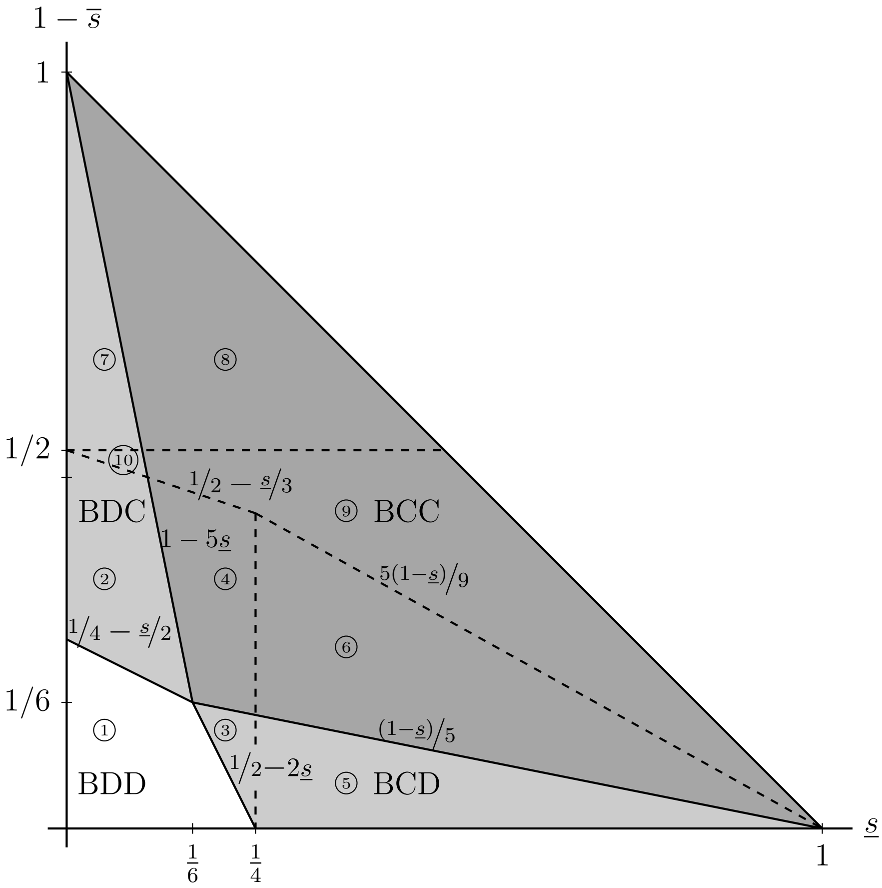

Figure 2.10: Regions of existence with common prior

Proof.

① $\pi_0^{DD} > \pi_0^{BDD}$ is satisfied for all admissible $\underline{s}, \overline{s}$ since

$$\text{(i) } h_0^{DD} > h_0^{BDD} \Leftrightarrow 1/2 - 2\underline{s}/3 > 1/2 - 2\underline{s}/3 - (1-\overline{s})/3,$$

and,

$$\text{(ii) } p_0^{DD} > p_0^{BDD} \Leftrightarrow 2\,t\,h_0^{DD} > 2\,t\,h_0^{BDD}.$$

② $\pi_0^{DD} > \pi_0^{BDC}$ since

$$\begin{aligned}\text{(i) } h_0^{DD} > h_0^{BDC} &= (5\overline{s} + 2\underline{s})/9 - \underline{s} \\ &\Leftrightarrow 1/2 - 2\underline{s}/3 > (5\overline{s} - 7\underline{s})/9 \Leftrightarrow 9/2 + \underline{s} > 5\overline{s}\end{aligned} \tag{2.37}$$

is satisfied in the respective parameter range where $1 - \overline{s} > 1/4 - \underline{s}/2$, and thus, $\overline{s}$ maximally converges to $3/4 + \underline{s}/2$. Therefore, (2.37) holds if $9/2 + \underline{s} > 5(3/4 + \underline{s}/2) \Leftrightarrow 3/4 - 3\underline{s}/2 > 0$, which is clearly satisfied since $\underline{s} \leq 1/6$. And,

$$\begin{aligned}\text{(ii) } p_0^{DD} > p_0^{BDC} &\Leftrightarrow 2\,t\,(1/2 - 2\underline{s}/3) > 40\,t\,(\overline{s} - \underline{s}/2)\,(\overline{s} - 7\underline{s}/5)/27 \\ &\Leftrightarrow 1/2 - 2\underline{s}/3 > 20\,(\overline{s}^2 - 19\,\overline{s}\underline{s}/10 + 7\underline{s}^2/10)/27 \qquad (2.38)\end{aligned}$$

The RHS in (2.38) is strictly increasing in $\overline{s}$ since $\partial RHS/(\partial\overline{s}) = 20(2\overline{s} - 19\underline{s}/10)/27 > 0$. If (2.38) thus holds at the infinum of $\overline{s}$, it holds for all admissible parameters. This indeed is the case:

$$\begin{aligned}1/2 - 2\underline{s}/3 &> 20\,((3/4 + \underline{s}/2)^2 - 19\,(3/4 + \underline{s}/2)\underline{s}/10 + 7\underline{s}^2/10)/27 \\ &\Leftrightarrow 1/2 - 2\underline{s}/3 > 20\,(9/16 - 27\underline{s}/40)/27 \\ &\Leftrightarrow 1/12 - \underline{s}/6 > 0,\end{aligned}$$

which is satisfied since $\underline{s} \in (0, 1/6)$.

3 $\pi_0^{DD} > \pi_0^{BCD}$ since

$$\begin{aligned}\text{(i) } h_0^{DD} > h_0^{BCD} &= (8\overline{s} - 7\underline{s} - 1)/9 - \underline{s} \\ &\Leftrightarrow 11/2 + 10\underline{s} > 8\overline{s} \qquad (2.39)\end{aligned}$$

is satisfied in the respective parameter range where $1-\overline{s} > 1/2-2\underline{s}$, and thus, $\overline{s}$ maximally converges to $1/2 + 2\underline{s}$. Therefore, (2.39) holds if $11/2 + 10\underline{s} > 8(1/2 + 2\underline{s}) \Leftrightarrow 3/2 - 6\underline{s} > 0$, which is satisfied since $\underline{s} \leq 1/4$. And,

$$\begin{aligned}\text{(ii) } p_0^{DD} \geq p_0^{BCD} &\Leftrightarrow 2\,t\,(1/2 - 2\underline{s}/3) \geq 32\,t\,(1/2 - \underline{s} + \overline{s}/2)^2/27 \\ &\Leftrightarrow 1/2 - 2\underline{s}/3 \geq 16\,(1/4 - \underline{s} + \overline{s}/2 + \underline{s}^2 - \overline{s}\underline{s} + \overline{s}^2/4)/27 \\ &\qquad (2.40)\end{aligned}$$

The RHS in (2.40) is strictly increasing in $\overline{s}$ because clearly $\partial RHS/(\partial\overline{s}) = 16(1/2 - \underline{s} + \overline{s}/2)/27 > 0$. If (2.40) thus holds at the infinum of $\overline{s}$, it holds for all admissible parameter values. This indeed is the case:

$$\begin{aligned}1/2 - 2\underline{s}/3 &\geq (16/27)(9/16) \\ &\Leftrightarrow \underline{s} \leq (3/2 - 1)/2 = 1/4.\end{aligned}$$

4 $\pi_0^{DD} > \pi_0^{BCC}$ since

$$\begin{aligned}\text{(i) } h_0^{DD} > h_0^{BCC} &= (\underline{s} + \overline{s})/2 - \underline{s} \\ &\Leftrightarrow 1 - \underline{s}/3 > \overline{s} \qquad (2.41)\end{aligned}$$

is satisfied in the respective parameter range where $1 - \overline{s} > (1 - \underline{s})/5$, and thus, $\overline{s}$ maximally converges to $4/5 + \underline{s}/5$. Therefore, (2.41) holds if $1 - \underline{s}/3 > 4/5 + \underline{s}/5 \Leftrightarrow 1/5 - 8\underline{s}/15 > 0$, which is satisfied since $\underline{s} \leq 1/4$. And,

$$\begin{aligned}\text{(ii) } p_0^{DD} > p_0^{BCC} &\Leftrightarrow 2\,t\,(1/2 - 2\underline{s}/3) > 2\,t\,(\overline{s} - \underline{s})^2/3 \\ &\Leftrightarrow 3/2 - 2\underline{s} > (\overline{s} - \underline{s})^2 \end{aligned} \tag{2.42}$$

The RHS in (2.42) is strictly increasing in $\overline{s}$ since $\partial RHS/(\partial \overline{s}) = 2(\overline{s} - \underline{s}) > 0$. If (2.42) thus holds at the maximum of $\overline{s}$, it holds for all admissible parameter values. This indeed is the case:

$$\begin{aligned}&3/2 - 2\underline{s} > (4(1 - \underline{s})/5)^2 \\ &\Leftrightarrow (43/2 - 18\underline{s} - 16\underline{s}^2)/25 > 0,\end{aligned}$$

which is satisfied since $\underline{s} \leq 1/4$.

5 $\pi_0^{DC} > \pi_0^{BCD}$ since

$$\begin{aligned}\text{(i) } h_0^{DC} > h_0^{BCD} &\Leftrightarrow 4(1 - \underline{s})/9 > (8\overline{s} - 16\underline{s} - 1)/9 \\ &\Leftrightarrow 4 + 12\underline{s} + 1 > 8\overline{s}\end{aligned}$$

is satisfied in the respective parameter range where $\overline{s} < 1$ and $\underline{s} > 1/4$. And,

$$\begin{aligned}\text{(ii) } p_0^{DC} > p_0^{BCD} &\Leftrightarrow 32\,t\,(1 - \underline{s})^2/27 > 32\,t\,(1/2 + \overline{s}/2 - \underline{s})^2/27 \\ &\Leftrightarrow 1 > (1 + \overline{s})/2.\end{aligned}$$

6 $\pi_0^{DC} > \pi_0^{BCC}$ since

$$\begin{aligned}\text{(i) } h_0^{DC} > h_0^{BCC} &\Leftrightarrow 4(1 - \underline{s})/9 > (\overline{s} - \underline{s})/2 \\ &\Leftrightarrow (8 + \underline{s})/9 > \overline{s}\end{aligned} \tag{2.43}$$

is satisfied in the respective parameter range where $1 - \overline{s} > (1 - \underline{s})/5$, and thus, $\overline{s}$ maximally converges to $4/5 + \underline{s}/5$. Therefore, (2.43) holds since $(8 + \underline{s})/9 > 4/5 + \underline{s}/5 \Leftrightarrow \underline{s} < 1$. And,

$$\begin{aligned}\text{(ii) } p_0^{DC} > p_0^{BCC} &\Leftrightarrow 32\,t\,(1 - \underline{s})^2/27 > 2\,t\,(\overline{s} - \underline{s})^2/3 \\ &\Leftrightarrow (16/9)^{1/2}(1 - \underline{s}) > \overline{s} - \underline{s} \\ &\Leftrightarrow (4 - \underline{s})/3 > \overline{s}\end{aligned}$$

is satisfied because $(4 - \underline{s})/3 > 4/5 + \underline{s}/5 \;\Leftrightarrow\; 8(1 - \underline{s})/15 > 0$.

7 $\pi_0^{LD} > \pi_0^{BDC}$ since

$$\begin{aligned}\text{(i) } h_0^{LD} > h_0^{BDC} &\Leftrightarrow \overline{s} - \underline{s} > (5\overline{s} + 2\underline{s})/9 \\ &\Leftrightarrow \overline{s} > 7\overline{s}/4 \qquad (2.44)\end{aligned}$$

is satisfied in the respective parameter range where $1 - \overline{s} < 1 - 5\underline{s}$, and thus, $\overline{s}$ minimally converges to $5\underline{s}$. Therefore, (2.44) holds if $5\underline{s} > 7\underline{s}/4$. And,

$$\begin{aligned}\text{(ii) } p_0^{LD} > p_0^{BDC} &\Leftrightarrow t\,(4(1-\overline{s}) - 1) > 40\,t\,(\overline{s} - \underline{s}/2)\,(\overline{s} - 7\underline{s}/5)/27 \\ &\Leftrightarrow 3 - 4\overline{s} > 40(\overline{s}^2 - 19\overline{s}\underline{s}/10 + 7\underline{s}^2/10)/27 \\ &\Leftrightarrow 3 - 28\underline{s}^2/27 > \overline{s}(4 + 40\overline{s}/27 - 76\underline{s}/27) \qquad (2.45)\end{aligned}$$

is satisfied because the RHS in (2.45) is increasing in $\overline{s}$, and holds at its maximum 1/2:

$$\begin{aligned}&3 - 28\underline{s}^2/27 > 2 + 10/27 - 28\underline{s}/27 \\ &(17 + 38\underline{s} - 28\underline{s}^2)/27 > 0.\end{aligned}$$

8 $\pi_0^{LD} > \pi_0^{BCC}$ since

$$\text{(i) } h_0^{LD} > h_0^{BCC} \Leftrightarrow \overline{s} - \underline{s} > (\overline{s} - \underline{s})/2,$$

and,

$$\begin{aligned}\text{(ii) } p_0^{LD} > p_0^{BCC} &\Leftrightarrow t\,(4(1-\overline{s}) - 1) > 2\,t\,(\overline{s} - \underline{s})^2/3 \\ &\Leftrightarrow 9 - 2\underline{s}^2 > 2\overline{s}(6 + \overline{s} - 2\underline{s}) \qquad (2.46)\end{aligned}$$

holds because the RHS in (2.46) is increasing in $\overline{s}$, and satisfied at its maximum 1/2:

$$\begin{aligned}&9 - 2\underline{s}^2 > 13/2 - 2\underline{s} \\ &\Leftrightarrow 5/2 + 2\underline{s}(1 - \underline{s}) > 0.\end{aligned}$$

9 $\pi_0^{LC} > \pi_0^{BDC}$ since

$$\begin{aligned}\text{(i) } h_0^{LC} > h_0^{BDC} &\Leftrightarrow \overline{s} - \underline{s} > (5\overline{s} - 7\underline{s})/9 \\ &\Leftrightarrow (4\overline{s} - 2\underline{s})/9 > 0.\end{aligned}$$

And,

$$\text{(ii) } p_0^{LC} > p_0^{BDC} \Leftrightarrow 4\,t\,(1-\overline{s})^2 > 40\,t\,(\overline{s}-\underline{s}/2)\,(\overline{s}-7\underline{s}/5)/27$$
$$\Leftrightarrow 1-2\overline{s}+\overline{s}^2 > 10(\overline{s}^2-19\overline{s}\underline{s}/10+7\underline{s}^2/10)/27$$
$$\Leftrightarrow 1-7\underline{s}^2/27 > \overline{s}(54-17\overline{s}-19\underline{s})/27 \tag{2.47}$$

holds because (ii.a) the RHS in (2.47) is increasing in $\overline{s}$ as $\partial RHS/(\partial\overline{s}) = (54-34\overline{s}-19\underline{s})/27 > 0 \Leftrightarrow 54+19\underline{s} > 34\overline{s}$, which is satisfied at the maximal admissible $\overline{s}$, i.e., at $1/2+\underline{s}/3$ because $54+19\underline{s} > 17+34\underline{s}/3 \Leftrightarrow 37+23\underline{s}/3 > 0$. And moreover, (ii.b) the inequality (2.47) clearly holds at $\overline{s} = 1/2+\underline{s}/3$:

$$1-7\underline{s}^2/27 > (1/2+\underline{s}/3)(91/2-74\underline{s}/3)/27$$
$$\Leftrightarrow 17/108+11\underline{s}^2/243-17\underline{s}/162 > 0.$$

10 Finally, $\pi_0^{LC} > \pi_0^{BCC}$ since

$$\text{(i) } h_0^{LC} > h_0^{BCC} \Leftrightarrow \overline{s}-\underline{s} > (\overline{s}-\underline{s})/2,$$

And,

$$\text{(ii) } p_0^{LC} > p_0^{BCC} \Leftrightarrow 4\,t\,(1-\overline{s})^2 > 2\,t\,(\overline{s}-\underline{s})^2/3$$
$$\Leftrightarrow 6-12\overline{s}+6\overline{s}^2 > \overline{s}^2-2\overline{s}\underline{s}+\underline{s}^2$$
$$\Leftrightarrow 6-\underline{s}^2 > \overline{s}(12-5\overline{s}-2\underline{s}) \tag{2.48}$$

is satisfied because (ii.a) the RHS in (2.48) is increasing in $\overline{s}$: since $\partial RHS/(\partial\overline{s}) = 12-10\overline{s}-2\underline{s} > 0$ is satisfied at maximum of $\overline{s} = 4/9+5\underline{s}/9$:

$$12-40/9-50\underline{s}/9-2\underline{s} = 68(1-\underline{s})/9 > 0.$$

It follows that (2.48) is satisfied if it (ii.b) holds at the maximal admissible $\overline{s}$. This indeed is the case:

$$6-\underline{s}^2 > (4+5\underline{s})(108-5(4+5\underline{s})-18\underline{s})/81$$
$$\Leftrightarrow 134(1+\underline{s}^2-2\underline{s})/81 > 0 \Leftrightarrow (1-\underline{s})^2 > 0.$$

These 10 comparisons show that the expert's profit is higher with differing priors than with the common prior S_0. She therefore never persuades her competitor to adopt knowledge. □

In what follows we prove part (b) of proposition 2.4 by comparing π_0^k in each equilibrium $k = DD, DC, LD, LC$ of the main game with differing priors and firm 0's expected profit π_0^{HC} in the equilibrium of the constrained Hotelling line.

(b) The expert never persuades the buyer to adopt prior S_0

By the "constrained Hotelling line" (HC) we mean the classical specification-then-price game, where both duopolists know that the buyer's need is uniformly distributed on $[\underline{s}, \overline{s}]$, project specifications outside this interval yield 0 profit, and transportation cost are quadratic. Lemma 2.4 presents the equilibrium of this game. Its proof is thus equivalent to the ones described in textbooks, see e.g., Tirole (1988) or Anderson et al. (1992).[47] However, we denote the offers by $(\underline{s} + s_0, p_0)$ and $(\overline{s} - s_1, p_1)$ with $s_i \in [0, 1)$, and accordingly assume $\underline{s} + s_0 < \overline{s} - s_1$.

Lemma 2.4 (Equilibria in Hotelling's constrained line)**.** *In the specification-then-price game on Hotelling's line of length $\overline{s} - \underline{s}$ where specifications are constrained to lie in the interval $[\underline{s}, \overline{s}]$ there exists a unique equilibrium in pure strategies. In this equilibrium, firms $i = 0, 1$ maximally differentiate, i.e. $s_i = 0$, and offer $(\underline{s}, t(\overline{s} - \underline{s})^2)$ and $(\overline{s}, t(\overline{s} - \underline{s})^2))$, respectively.*

Proof. **Ex ante winning probability.** The buyer is indifferent at $\hat{s}$ that solves

$$p_0 + t(\hat{s} - s_0 - \underline{s})^2 = p_1 + t(\overline{s} - s_1 - \hat{s})^2$$
$$\Leftrightarrow \hat{s}^{HC} \equiv \underline{s} + s_0 + \frac{\overline{s} - s_1 - (\underline{s} + s_0)}{2} + \frac{p_1 - p_0}{2\,t\,(\overline{s} - s_1 - (\underline{s} + s_0))}.$$

The firms' expected home turfs are $h_0^{HC} \equiv \hat{s}^{HC} - \underline{s}$ and $h_1^{HC} \equiv \overline{s} - \hat{s}^{HC}$. Hence, firm $i = 0, 1$ expects to win with probability

$$\rho_i^{HC} \equiv f_0\, h_i^{HC}, \text{ where } f_0 = \frac{1}{\overline{s} - \underline{s}}.$$

Note that, in contrast to the game with differing priors, firms that share a common prior agree on their winning probabilities, i.e., $1 - \rho_i^{HC} = \rho_j^{HC}$ for $i \neq j$. Thus, focussing on strictly positive probabilities excludes probabilities of 1. In equilibrium, the prerequisite $\rho_i^b > 0$ is satisfied.

Price equilibrium. In the second stage, firms simultaneously choose prices given the specifications s_i of the first stage, which by assumption satisfy $\underline{s} + s_0 < \overline{s} - s_1 \Rightarrow s_0 + s_1 \in [0, \overline{s} - \underline{s})$. If $\rho_i^{HC} > 0$ for both $i = 0, 1$, there exist a unique price equilibrium in pure strategies in which both firms $i \neq j$ bid

$$p_i^{HC}(s_i) \equiv t\,(\overline{s} - \underline{s} - s_i - s_j)\left(\overline{s} - \underline{s} + \frac{s_i - s_j}{3}\right).$$

Equilibrium specifications. It is simple to check that firm i's first stage profit $\pi_i^{HC}(s_i) \equiv p_i^{HC}(s_i)\,\rho_i^{HC}(s_i)$ is strictly decreasing in s_i, $\forall s_i \in [0, \overline{s} - \underline{s})$. It follows that, in the pure strategy equilibrium, both firms $i = 0, 1$ offer their

[47] For primary sources, see the example in d'Aspremont et al. (1979), and the proof of Anderson (1988) who proves existence of pure strategy equilibrium for sufficiently convex transportation cost.

maximally differentiated projects with $s_i^{HC} \equiv 0$ at prices $p_i^{HC} \equiv t\,(\overline{s} - \underline{s})^2$, and the buyer is indifferent at $\hat{s}^{HC} = (\underline{s} + \overline{s})/2$. Thus, expected profits equal $\pi_i^{HC} \equiv t\,(\overline{s} - \underline{s})^2/2$. $\square$

In the remainder of this proof we show that the expert's expected profits in the four equilibrium types with differing priors presented in proposition 2.2 are higher than π_0^{HC}.

Proof. (i) $\pi_0^{DD} > \pi_0^{HC}$ since it is equivalent to

$$\begin{aligned} &f_0\, 2\,t\,(1/2 - 2\underline{s}/3)^2 > t\,(\overline{s} - \underline{s})^2/2 \\ &\Leftrightarrow 4\,(1/2 - 2\underline{s}/3)^2 - (\overline{s} - \underline{s})^3 > 0, \end{aligned} \tag{2.49}$$

which holds as the LHS in (2.49) is strictly decreasing in $\overline{s}$ and still positive at $\sup \overline{s} = 1$:

$$\begin{aligned} \partial LHS/(\partial \overline{s}) &= -3(\overline{s} - \underline{s})^2 < 0 \\ LHS|_{\overline{s}=1} &= 4(1/2 - 2\underline{s}/3)^2 - (1 - \underline{s})^3 \\ &= (\underline{s}/9)\,(9\underline{s}^2 - 11\underline{s} + 3) > 0 \\ &\Leftrightarrow 9\underline{s}^2 - 11\underline{s} + 3 > 0, \end{aligned}$$

which is satisfied since $3 > 11\underline{s}$ holds even at $\arg\max \underline{s} = 1/4$.

(ii) $\pi_0^{DC} > \pi_0^{HC}$ since it is equivalent to

$$\begin{aligned} &f_0\, 128\,t\,(1 - \underline{s})^3/243 > t\,(\overline{s} - \underline{s})^2/2 \\ &\Leftrightarrow 256\,(1 - \underline{s})^3/243 > (\overline{s} - \underline{s})^3, \end{aligned}$$

which is obviously satisfied.

(iii) $\pi_0^{LD} > \pi_0^{HC}$ since it is equivalent to

$$\begin{aligned} &t\,(4(1 - \overline{s}) - 1) > t\,(\overline{s} - \underline{s})^2/2 \\ &\Leftrightarrow 6 - \underline{s}^2 - \overline{s}(8 + \overline{s} - 2\underline{s}) > 0, \end{aligned} \tag{2.50}$$

which holds as the LHS in (2.50) is strictly decreasing in $\overline{s}$ and positive at $\sup \overline{s} = 1/2$:

$$\begin{aligned} \partial LHS/(\partial \overline{s}) &= -(8 + 2(\overline{s} - \underline{s})) < 0 \\ LHS|_{\overline{s}=1/2} &= 7/4 - \underline{s}^2 + \underline{s} > 0 \Leftrightarrow 7/4 > \underline{s}^2 - \underline{s}, \end{aligned}$$

which is satisfied since $\underline{s}^2 - \underline{s} < 0$.

(iv) $\pi_0^{LC} > \pi_0^{HC}$ since it is equivalent to

$$\begin{aligned} & 4\,t\,(1-\overline{s})^2 > t\,(\overline{s}-\underline{s})^2/2 \\ \Leftrightarrow\, & 8\,(1-\overline{s})^2 - (\overline{s}-\underline{s})^2 > 0 \\ \Leftrightarrow\, & 8 - 16\overline{s} + 7\overline{s}^2 + 2\overline{s}\underline{s} - \underline{s}^2 > 0, \end{aligned} \tag{2.51}$$

which holds as the LHS in (2.51) is strictly decreasing in $\overline{s}$ and positive at the supremum of $\overline{s}$, which equals $1/2 + 2\underline{s}/3$ if $\underline{s} < 1/4$, and $(5+4\underline{s})/9$ else:

$$\begin{aligned} \partial LHS/(\partial\overline{s}) &= -16 + 14\overline{s} + 2\underline{s} < 0 \\ \partial LHS/(\partial\overline{s})|_{\overline{s}=1/2+2\underline{s}/3} &= -16 + 7 + 20\underline{s}/3 < 0, \text{ holds as } \underline{s} \leq 1/4. \text{ And,} \\ \partial LHS/(\partial\overline{s})|_{\overline{s}=(5+4\underline{s})/9} &= -74(1-\underline{s})/9 < 0, \text{ for } \underline{s} < 1. \end{aligned}$$

Therefore, if LHS in (2.51) is positive at both suprema of $\overline{s}$, then $\pi_0^{LC} > \pi_0^{HC}$:

$$LHS|_{\overline{s}=1/2+2\underline{s}/3} = 7/4 + 4\underline{s}^2/9 - 2\underline{s} > 0 \tag{2.52}$$

the LHS in (2.52) is clearly decreasing as $\partial LHS2/(\partial\underline{s}) = 8\underline{s}/9 - 2 < 0$. Therefore, we evaluate it at $\underline{s} = 1/4$ to see that

$$LHS2|_{\overline{s}=1/2+2\underline{s}/3,\underline{s}=1/4} = 7/4 + 1/36 - 1/2 > 0.$$

And similarly,

$$LHS2|_{\overline{s}=(5+4\underline{s})/9} = 103(1-\underline{s})^2/81 > 0.$$

Above comparisons (i) to (iv) show that (b) the expert never persuades the buyer to adopt her knowledge. Together with part (a), this proves proposition 2.4. □

A.7 Proof of proposition 2.5 (Market power of knowledge)

All comparisons are mentioned in the main text on pages 109ff. We omit the simple algebra.

Chapter 3

Prejudice, Quotas, and Wages

3.1 Introduction

About 250 years ago Voltaire said that "prejudices are what fools use for reason". Evidence suggests that people still hold prejudice, that is, preconceived (explicit or implicit) opinions on a group (defined, e.g., by ethnicity, race, or gender) and its members: Fershtman and Gneezy (2001) conduct experiments to study ethnic discrimination in Israeli Jewish society, and identify systematical mistrust against men of Eastern origin. Their results indicate that this mistrust stems from mistaken ethnic stereotypes and not from a *taste for discrimination*. The latter type of discrimination emerges if utility functions include emotions (in form of preferences) toward a certain group, whereas stereotypes are sets of perceived qualities that characterize a group and its members. Arnold et al. (2018) analyze court records from Philadelphia and Miami-Dade on the release tendencies of bail judges, and find a bias against black defendants. By considering race-specific differences as, for instance, the probability of being arrested for certain types of crime, they control for *statistical discrimination*. This type of discrimination stems from updating imperfect signals about an individual's characteristics based on statistics about her or his group membership. Their findings suggest that the racial bias emerges from inaccurate stereotypes that exaggerate the danger of releasing black defendants. The majority of economic studies on the sources of discrimination distinguishes between the mentioned two types: taste-based (early discussed in Becker, 1957) and statistical discrimination (established, amongst others, in Arrow, 1973 and Phelps, 1972). Few articles, however, identify inaccurate prior beliefs and mistaken stereotypes as another root of discrimination.[1] Bohren et al. (2019) provide a survey of such articles, and outline possible sources of prejudice.

This chapter introduces inaccurate priors on the ability of potential job candidates in a simple principal agent model. The prejudiced principal wants

[1]For a discussion on the taste-based and statistical discrimination see, e.g., Guryan and Charles (2013), and for a review of field experiments see Bertrand and Duflo (2017). While these two well studied types of discrimination focus on situations in which the discriminator makes conscious decisions, discrimination due to mistaken prejudice and stereotypes may be implicit. Bertrand et al. (2005) outline evidence on implicit discrimination (as it was early established in the field of psychology), and reasons for its importance in economic research.

Readers who question existence of implicit prejudice may take an Implicit Association Test as available, e.g., under https://implicit.harvard.edu/implicit/takeatest.html.

to hire a team of two agents who may differ concerning one single attribute as, for instance, ethnicity, race, or gender. Throughout the whole chapter we stick to the example of gender bias, and assume that the principal underestimates the women's abilities while he overestimates those of men. More precisely, he wrongly believes that women bear higher cost of performing demanding work than men. Thus, without regulation the principal would hire two male agents to minimize his wage cost (for any given effort levels). To focus on the interesting case of a gender-mixed team we therefore assume that the principal has to implement a quota.

Indeed, women are significantly underrepresented in demanding jobs. In 2019, they constitute almost half of the workforce at the entry level in the US, while they only make up a fifth of senior executive roles as CEO, CFO, COO, CIO and their direct subordinates (Huang et al., 2019). Women are also underrepresented in the top management of the largest listed companies in the EU: in 2018, women held 29.3% of non-executive positions in the highest decision-making bodies, 16.6% of executive positions, but only 6.7% and 6.5% of board chairs and CEO positions, respectively. The share of female managers is increasing and steadily reduces the gender gap in decision-making functions. In the EU this trend is mainly driven by member states that apply legally enforceable gender quotas for listed companies, which currently are France, Italy, Belgium, Germany, Austria and Portugal (European Commission, 2019a). Gender quotas directly tackle the underrepresentation of females in certain positions, at least if underrepresentation originates from the demand side of the labor market.[2]

Besides taste-based and statistical discrimination, prejudice and stereotypes

[2]Gender quotas are controversially disputed and politically difficult to implement (see, e.g., He and Kaplan, 2017). A widespread concern is that gender quotas allow for less competent women to hold positions over more adequately qualified men. However, Besley et al. (2017) show that the opposite occurred after the Swedish Social Democratic Party implemented a "zipper quota" that ensures gender-alternating listing on the ballot: highly skilled women displaced mediocre men. A thorough study on labor market outcomes of the Norwegian board quota for public limited liability companies finds little impact on business women beyond the direct effect of more female board members (Bertrand et al., 2019). Besides quotas, concealing the applicant's gender also seems to increase the proportion of women in demanding positions. Goldin and Rouse (2000) analyze data from symphony orchestras that switched from classical hiring auditions (in which the jury knows the applicants' identities) to blind auditions (in which the identities are concealed behind a screen), and find evidence that the latter procedure increases the probability that a female musician is hired.

may influence hiring and promotion decisions in manifold ways. Studies in several fields illustrate that ex ante both genders perceive women as less able than men, especially in male-dominated jobs and leadership positions.[3] Fleischmann et al. (2016) not only find that computer skills of women wearing a dress and some makeup are judged lower than skills of the same women wearing trousers and no makeup, but also, that the success in computational tasks of the former rather is attributed to luck, while the latter's rather is attributed to skill. In an initial public offering game Bigelow et al. (2014) show that participants rate investments in otherwise identical firms led by female instead of equally qualified male CEOs as less attractive because women are perceived to be less capable. A possible consequence of this ex ante underestimation of women's abilities is the relative lack of support before individuals apply for jobs, for instance, concerning educational opportunities or in building networks. Milkman et al. (2015) contact professors posing as fictional students seeking 10 minute appointments to discuss their research proposals before applying to a doctoral program. They find that faculty members are significantly more responsive to presumably Caucasian males than to other ethnicities and females. Evidence suggests that gender bias goes beyond the first hurdle of being perceived as less competent: ex post identical performance demonstrations women are evaluated worse than men. An experiment by Gygax et al. (2019) reveals that the audience of a magician assumed to be male perceives the quality of the same magic trick as higher than that of a supposedly female magician. This is in line with a wide range of studies that identify significant gender bias in teaching evaluations (see, e.g., Mengel et al., 2019) and evaluations of leaders (see Eagly et al., 1992, for a meta-analysis in psychology). Wold and Wennerås (1997) analyze peer-review scores for postdoctoral fellowship

[3]Bayer and Rouse (2016) as well as Lundberg and Stearns (2019) provide overviews of different reasons for the underrepresentation of minorities in the economics professions. Ridgeway (2011) reviews literature in sociology on gender stereotypes and its consequences.

Koch et al. (2015) run a meta-analysis of studies in psychology on gender stereotypes and bias in employment decision making, and find that men are preferred for male-dominated jobs, whereas there is no significant gender preference for female-dominated or neutral jobs. The *role congruity theory* provides a canonical explanation for this finding. It states that prejudice toward female leaders stems from the perceived incongruity between characteristics attributed to the female stereotype and the leadership role. Eagly and Karau (2002) identify two forms of such prejudice: ex ante men are favored as potential leaders, and, ex post the performance of female leaders is evaluated worse. They conclude that "it is more difficult for women to become leaders and to achieve success in leadership roles".

applications in the biomedical field, and find that the competence evaluations of research proposals of female applicants are systematically lower than those of their male colleagues with the same scientific productivity. Concerning economic research, Sarsons et al. (2021) find that, controlling for quality (journal ranking and citations) of papers, women who solo-author have similar tenure rates than men. However, an additional coauthored paper yields a 7.4% increase in tenure probability for men but only a 4.7% increase for women. Since this gap is significantly less pronounced for women who coauthor with women, it seems that credit for an article is more likely to be attributed to male than female coauthors. In a microeconomic approach Bordalo et al. (2019) experimentally show that both genders not only overestimate their own but also a partner's performance in a quiz. They thereby identify gender gaps in the assessment of others as well as in self-confidence: the exaggerated performance expectation concerning the performance of others is more pronounced for male than for female partners, and moreover, men's overconfidence in their own performance exceeds the women's. Indeed, men tend to have higher self-esteem than women (for a review see, e.g., Casale, 2020).

On the supply side of the labor market, low self-esteem affects performance, and thus, educational as well as occupational attainment as thoroughly examined by Heckman et al. (2006) and similarly concluded by psychologists.[4] Bénabou and Tirole (2002) as well as Compte and Postlewaite (2004) theoretically support the reverse of this finding by showing that self-confidence enhances motivation, and consequently, performance. Gneezy et al. (2003) conjecture that the lower self-esteem of women may explain why their performance in competitive environments

[4]For a meta-study in psychology on self-esteem and job performance see, e.g., Judge and Bono (2001). A low self-esteem may stem from stereotypes, which in turn may reduce performance through other mediator variables. This effect is called *stereotype threat* and widely studied by psychologists. For a review of this phenomenon and its mediator variables see, e.g., Pennington et al. (2016) or Schmader (2012).

Note that an increasing share of female role models as, e.g., professors or leaders intuitively reduces the gender bias through convergence of perceived gender congruent characteristics, enhancement of women's self-esteem, and mitigation of stereotype threat. Besides inspiring potential female successors (in economics there is evidence that more female professors attract more female students (Porter and Serra, 2019) and establish a higher proportion of female PhD students (Hale and Regev, 2014)), changing social norms as suggested in Fernández (2013), women in top positions might change institutions such that women benefit, e.g., promote appropriate mentoring programs, more flexible working hours, and better child care. However, Bagues and Esteve-Volart (2010) suggest that recruiting committees with female majorities are not more likely to hire a women, yet conversely, overestimate the quality of male candidates.

is significantly lower than that of men, which is true only in gender-mixed competitions such as typical promotion tournaments. Azmat and Ferrer (2017) identify other key determinants for lower performance of young female lawyers, which explains roughly 40% of the gender gap in the likelihood of becoming partner: the presence of preschool-aged children and a lower aspiration to be promoted. Evidence suggests that women's preferences for family-friendly job characteristics indeed explains a major part of their underrepresentation at higher hierarchy levels. For instance, Cortés and Pan (2019) analyze the impact of low-skilled immigration as a proxy for child care, and find a positive correlation with high-qualified women's likelihood to move up the occupational hierarchy to positions with longer working hours and higher wages. Wiswall and Zafar (2018) estimate that women have a higher willingness to pay for flexible working hours and job stability, while men are rather willing to compensate negative job attributes with higher earnings growth. All these papers conjecture that discrimination–apart from social norms that make women responsible for the household production–is not the main factor that constrains women's career opportunities.[5]

We contribute to this literature by proposing a new rationale for the underrepresentation of women in demanding jobs. Our main result suggests that a prejudiced principal cannot hire a gender-mixed team in which both agents collaborate as hard-working peers. However, he can hire a man for demanding work and a woman for trivial tasks. As we rule out any demand side reasons for the woman's underrepresentation in higher positions by assuming a quota, this results from the female agent's decisions to reject the contract as a peer but to accept the contract as a subordinate. Our agents are rational and–except for their gender–identical. The prejudiced principal nonetheless believes that the female candidate's cost of performing demanding work exceeds the male's, for instance, due to presumably lower ability or higher opportunity cost of rival household production. The principal wants to hire a team to carry out a project, which either fails or succeeds. The probability of success is higher if agents perform demanding work in the sense of going a costly extra mile as, e.g., investing in networking. Because effort is unobservable, the principal offers wage contracts

[5]The empirical distinction between discrimination and job-related preferences is rather challenging, among other problems, because of labor market frictions as well as omitted variable bias. Cortés and Pan (2018) review recent advances and outline gender differences in attitudes toward risk and competition, and in preferences for social contribution, success, money, as well as workplace flexibility.

contingent on the project's outcome. If his expected net revenue is high enough he wants to hire two peers who both go the extra mile. Due to his prejudice the principal in this case offers excessive incentive pay to the underestimated female candidate but not enough to compensate the overestimated male. Therefore, the former prefers to work hard, while the latter shirks. As a consequence, the actual success probability of the project is lower than intended by the principal, and the female candidate's wage offer does not reimburse her for her male colleague's free riding. Thus, she rejects the contract as a peer. However, if the principal's expected net revenue is rather low he forgoes incentivizing the "costlier" woman, and offers her a fixed wage for a trivial job. Because this wage is independent of the project's success probability, she does not care about her male coworker's shirking, and consequently, accepts the contract.

Shirking of an overestimated coworker as a rationale for the underrepresentation of women in demanding jobs has–to the best of our knowledge–not been discussed so far. Note that, in contrast to the vast majority of literature, our model conjectures that the underestimated woman's expected wage as a peer exceeds the man's, and nevertheless, she rejects the job offer.[6] We propose a simple and obvious remedy for this problem: wage equality.

Policies that aim to achieve *equal pay for equal work* for men and women are widespread and have been implemented for a long time. Since 1951, the Equal Remuneration Convention establishes this principle as an international human rights law (ILO, 1996). The Treaty of Rome, signed in 1957, also stipulates pay equality, and the EU adopted it over for all its member states (EUR-Lex, 2006). US Congress passed the Equal Pay Act in 1963, which dictates that "employers may not pay unequal wages to men and women who perform jobs that require substantially equal skill, effort and responsibility, and that are performed under similar working conditions within the same establishment" (EEOC, 1997).[7]

Vast evidence on the gender wage gap[8] indicates that, at least in developed countries, equal work indeed is paid almost equally. The Economist (2017) points

[6]Leslie et al. (2017) review some evidence for a "female premium" and suggest that the typical gender gap may reverse if firms set diversity goals because these yield a demand surplus for highly qualified women.

[7]For characterizations and cost-benefit analyses of different equal pay policies see Chicha (2006).

[8]For a recent overview on the wage gap in developed countries, see e.g., Kunze (2018) and for a comprehensive meta-analysis Weichselbaumer and Winter-Ebmer (2005).

out that the major portion of the raw wage gap (in 2016, an average women in Britain earned 29% less than an average men, while she earned 15% less in Germany) is explained by lower hierarchical ranks at lower-paying organisations. Accounting for jobs at the same level, company and function, the pay gaps shrink to 1% and 3%, respectively. Similarly, an online salary survey with roughly 1.6 million self-reporting participants between 2018 and 2020 concludes that the uncontrolled median salary for women as percentage of the median salary for men is 81%. Controlling for industry, occupation, location, qualification and other compensable factors as maternity leaves, as well as demographic information substantially reduces the pay gap: for the same job and with identical qualifications women earn 98% of the median men's wage (PayScale, 2020). In a report on the sources for the gender wage disparity for the US Department of Labor, scholars summarize research on the observed differences and present results from analyzing Current Population Survey data from 2007. Besides human capital development, work experience, career interruptions, and wage adjustments that have different gender-specific impacts as, e.g., health insurance and the duty to work overtime, they identify occupational segregation (with respect to hierarchical ranks as well as industries) as the main explanatory factor for the raw gender wage gap (CONSAD, 2009).[9]

We reverse this line of argument, and propose that an equal pay policy causes the underestimated woman to accept the contract for a demanding job, even though it has no impact on her own wage but raises her potential coworker's who therefore decides to work hard instead of shirking. At first glance, imposing wage equality may seem harmful to the prejudiced principal because it prevents him from offering contracts that consider the gender-specific cost of working. More precisely, if the principal is allowed to offer individual contracts these set both agents' participation as well as incentive compatibility constraints binding–at least according to the principal's biased beliefs. Wage equality, however, forces him to leave a rent to one agent. He offers both agents either the contract designed to

[9] Using Panel Study of Income Dynamics data, however, Blau and Kahn (2017) find that the unexplainable wage gap in the US remains in a range of 8%-18% from the 1990 to 2010, after it sharply fell in the 80ties. Currently, the gap is largest at the top of the wage distribution.

Moreover, evidence suggests that the unexplainable part of the pay disparity between blacks and whites is higher than between genders. Charles and Guryan (2008), e.g., estimate that around 25% of the racial wage gap stems from prejudice.

incentivize the "costlier" woman in case he wants to hire a team of hard-working peers, or, the contract to incentivize the man if he wants the woman to do a trivial job. Thereby, the principal ignores that, in contrast to individual contracts, the agents in both cases accept equal wage contracts. Moreover, equal wage contracts actually might induce them to go the extra mile while they unambiguously shirk with individual wages (given that both accept in the first place).

This chapter contributes to the literature on inaccurate beliefs and mistaken stereotypes as sources of discrimination in the workplace by proposing a new rationale on the supply side of the labor market for the underrepresentation of women at higher hierarchy levels. Above we review some articles on this discussion. To rule out discrimination from the demand side, we impose a gender quota, which may be due to legal requirements or reputational concerns and self-imposed diversity goals. However, the underestimated woman rejects the employment contract if she has to work as a peer with an overestimated man. This is not due to a lower wage, self-esteem, stereotype threat, time constraints or other gender differences: she rejects the peer position because her male coworker shirks instead of working. Our results suggest that the implementation of a gender quota alone does not mitigate the gender gap in hierarchical ranking but an additional equal pay policy does.

We comply with vast evidence on the prevalence of gender prejudice and simply assume the principal to believe that women are less competent than men. Morgan and Várdy (2009) provide a more sophisticated theory that incorporates sequential search with challenging communication between potential employers and job candidates who belong to minority groups. As a consequence, the former remain unsure about the ability of the latter. Our model, in contrast, introduces inaccurate priors in a static principal agent model with discrete effort choice as outlined in any textbook on incentive contracts (see, e.g., Laffont and Martimort (2009) or Bolton and Dewatripont (2005)), which we apply to two agents. We thereby focus on the inconsistency of biased contracts and the agents' decisions on effort and participation.[10]

[10]For a comprehensive review on principal agent models that covers the problem of free-riding in teams and solution approaches as tournaments and relational contracts see Prendergast (1999).

Note that in our model, the principal is the only biased player. Models on overconfidence, in contrast, typically assume the agents to hold mistaken beliefs or perceptions (see, e.g., Van den Steen (2004) or De la Rosa (2011)).

The remainder of this chapter is organized as follows. In the next section we describe the model. In section 3.3 we derive the optimal contracts as well as the agents' effort and participation decisions if the principal is free to offer individual wages. In section 3.4 we consider the case of wage equality. In section 3.5 we outline some insights on profits. In section 3.6 we discuss a modification of the main model with prejudice concerning productivity instead of working cost. Section 3.7 concludes.

3.2 Model

A principal hires a team of two agents to carry out a project. The agents are identical. However, the principal holds prejudice: based on an arbitrary attribute such as, for instance, ethnicity, race or gender, he underestimates one agent's ability, while he overestimates the other's.[11] Throughout the whole chapter we use the example of a principal with gender bias who underestimates the female agent $i = f$, while he overestimates her male coworker $i = m$. We assume that the principal has to implement a quota that ensures the team to consist of a female and a male worker.[12]

Each agent i exerts effort $e^i \in \{0, 1\}$ to implement a common project. That is, agent i either shirks ($e^i = 0$) or works ($e^i = 1$). Contingent on both agents' effort decisions the outcome of the project is binary too: either it fails and generates low revenue $y_L > 0$ or it succeeds and generates strictly higher revenue y_H. Working increases the probability of success. We denote the conditional probability of success given both effort levels by $\Pr(y_H|e^f, e^m) \equiv p_{e^f e^m}$, and assume it to be increasing, i.e., $p_{00} < p_{01} = p_{10} < p_{11}$.

The principal cannot observe whether the agents shirk or work. However, he observes whether the project fails or succeeds. Therefore, wage contracts are contingent on the project's success. We compare two different wage policies: while

[11] Assuming prejudice (or *differing priors*) drives our results: without this assumption the model coincides with a simple principal agent framework, as e.g., in Laffont and Martimort (2009, pp 158ff), applied to two agents rather than a single one. Extending the baseline model to two agents does not generically change the optimal wage contract. Due to the principal's prejudice, however, the agents' participation and effort decisions may be inconsistent with the former's priors, and thus, contracts.

[12] Without mandatory quota the principal would hire two agents of the same sex; the one that minimizes his expected wage cost for given effort levels.

the principal may offer individual contracts (w_L^i, w_H^i) in section 3.3, he is restricted to paying equal wages (w_L^e, w_H^e) in section 3.4.

The principal is risk neutral and maximizes his expected profit, which equals the difference between expected revenue and wage payments, $\pi = p\,(y_H - \sum w_H) + (1-p)\,(y_L - \sum w_L)$. The agents are identical: their preferences are additively separable, i.e., their utility is $u(w) - c(e)$. They both have the same utility function $u(w)$, with $u(0) = 0$, $u' > 0$ and $u'' < 0$, as well as the same cost function $c(e)$, with $c(0) = 0$ and $c(1) = c$. Loosely speaking, the agents are risk averse, and bear identical cost of effort. We now specify the principal's prejudice. He wrongly believes that the female agent's cost for high effort exceeds the male's: we denote the principal's priors by c^i, and assume $c^f > c > c^m$, where c is both agents' true cost.[13] Both agents are aware of the principal's prejudice as well as of their true cost.

We focus on the interesting cases in which the principal wants at least one agent to work rather than shirk.[14] For the sake of clarity we stick to the wording "work" and "shirk", however, propose interpreting the former as performing a demanding job that calls for high effort, and the latter as doing a trivial job.

The timing is as follows. The principal offers each agent $i = f, m$ a wage contract.[15] Each agent either rejects or accepts. If one agent rejects both receive their reservation utility, normalized to zero. In case that both agents accept, they simultaneously decide whether to shirk (exert no effort $e^i = 0$) or work (exert effort $e^i = 1$). Then, the project either fails or succeeds, which is observed by all parties. Contingent on that, the fully committed principal pays the wages.

We focus on pure strategies. The principal offers profit maximizing contracts (according to his beliefs). And, both agents maximize their utilities given their

[13]Alternatively, we may assume that the principal underestimates the female's contribution to the project's success probability, i.e., that his (wrong) prior is $p_{10}^f < p_{01}^m$. The results given this productivity bias are similar, however, the agents' participation and effort decisions of the main model hold for interchanged genders. We briefly outline these results in section 3.6.

[14]More precisely, we assume that his net revenue is high enough to compensate for the occurring incentive cost, and that contracting a gender-mixed team yields positive profit. Formally, $(p_{01} - p_{00})(y_H - y_L) \geq p_{01} \sum w_H + (1 - p_{01}) \sum w_L$ and $(p_{11} - p_{00})(y_H - y_L) \geq p_{11} \sum w_H + (1 - p_{11}) \sum w_L$ ensure that the principal wants at least one agent to work. And, either $p_{01}\big(y_H - \sum w_H\big) + (1 - p_{01})\big(y_L - \sum w_L\big) \geq 0$ or $p_{11}\big(y_H - \sum w_H\big) + (1 - p_{11})\big(y_L - \sum w_L\big) \geq 0$ implies that contracting a gender-mixed team is beneficial.

[15]Because the principal offers the wage contracts before profit realization, he does not update his priors. Consistently, we use the terms *prior* and *belief* as synonyms.

wage contract. Solutions to our one-shot games are characterized by the contracts and the agents' participation as well as effort choices. Note that the latter two may be inconsistent with the principal's beliefs. Therefore, our solution concept is sensitive to repetitions of the game, and we avoid the term "equilibrium" in what follows.[16]

3.3 Individual wage contracts

In this section we derive the wages if the principal is free to offer each agent $i = f, m$ an individual contract as well as the agents' effort and participation decisions. To do so, we determine the principal's profit maximizing contracts by first solving his cost minimization problems given that he either offers both agents demanding jobs (i.e., wants them both to work) or he offers one agent a demanding and the other a trivial job (i.e., wants a single agent to work). Then, we compare the two cases in terms of expected profits. Because these profits are biased, the agents' effort and participation choices may be inconsistent with the contracts. We thus close this section with an explicit description of the agents' choices.

Recall that the principal has prejudice $c^m < c^f$, and is obliged to hire a gender-mixed team. In case he wants both agents $i = f, m$ to work, i.e., to hire them as hard-working peers, the optimal contracts (w_L^{ib}, w_H^{ib}) are solution to

$$\begin{aligned}
\min_{w_L^i, w_H^i} \quad & p_{11} \sum_i w_H^i + (1 - p_{11}) \sum_i w_L^i \\
\text{s.t.} \quad & p_{11}\, u(w_H^i) + (1 - p_{11})\, u(w_L^i) - c^i \geq p_{01}\, u(w_H^i) + (1 - p_{01})\, u(w_L^i) \qquad (IC^i) \\
& p_{11}\, u(w_H^i) + (1 - p_{11})\, u(w_L^i) - c^i \geq 0. \qquad (PC^i)
\end{aligned}$$

In words, the principal minimizes total wage cost, where p_{11} denotes the probability that the project succeeds if both agents work, and p_{01} the success probability if one agent shirks ($e^i = 0$) and the other works ($e^j = 1$). The principal's expectation is thus consistent with his beliefs if and only if both agents $i = f, m$ decide to work given the other agent $j \neq i$ works and they both accept their contract, i.e., (IC^i) and (PC^i) are satisfied for both $i = f, m$.

After a change of variables where we define $u(w_k^i) \equiv u_k^i$ and $w_k^i \equiv h(u_k^i)$ for

[16] For a treatise on differing priors and their convergence to a common prior see page 62 in chapter 2 of this thesis.

both $k \in \{L, H\}$, and denote the inverse utility function by $h = u^{-1}$, we rewrite the principal's problem as

$$\begin{aligned}
\min_{u_L^i, u_H^i} \quad & p_{11} \sum_i h(u_H^i) + (1 - p_{11}) \sum_i h(u_L^i) & & \\
\text{s.t.} \quad & u_H^i - u_L^i \geq \frac{c^i}{p_{11} - p_{01}} & (IC^i) & \qquad (3.1) \\
& p_{11}\, u_H^i + (1 - p_{11})\, u_L^i \geq c^i & (PC^i) &
\end{aligned}$$

for both $i = f, m$. The objective function in (3.1) is increasing and strictly convex, and its constraints are linear in the wage-utilities u_L^i and u_H^i. Therefore, the first order conditions are necessary and sufficient. Moreover, the constraints are binding as shown in the proof of lemma 3.1 in the appendix. Intuitively, if (IC^i) were not to bind, the principal could lower u_L^i and raise u_H^i while keeping the participation constraint satisfied. Similarly, if (PC^i) were not to bind, the principal could lower both wage-utilities, u_L^i and/or u_H^i while keeping the incentive compatibility constraint satisfied. Thus, from (IC^i) it follows that $u_H^i = u_L^i + c^i/(p_{11} - p_{01})$, which substituted into the binding (PC^i) yields the wage contracts presented in lemma 3.1.

Lemma 3.1 (Individual contracts (I))**.** *If the principal is free to individually contract, and wants both agents to work he offers each agent $i \in \{f, m\}$*

$$\begin{aligned}
w_L^{ib} &= h\left(c^i\left(1 - \frac{p_{11}}{p_{11} - p_{01}}\right)\right) \text{ if the project fails, and} \\
w_H^{ib} &= h\left(c^i\left(1 + \frac{1 - p_{11}}{p_{11} - p_{01}}\right)\right) \text{ if the project succeeds.}
\end{aligned}$$

The female's wage in case of failure is strictly lower than the male's since $c^f > c^m$ and $1 - p_{11}/(p_{11} - p_{01}) < 0$ imply $u_L^{fb} < u_L^{mb} \Leftrightarrow w_L^{fb} < w_L^{mb}$. In case of success, however, the female's wage strictly exceeds the male's: the principal's prejudice induces him to offer higher incentives to the female than the male to compensate the former for her "higher" working cost, i.e., $w_H^{fb} - w_L^{fb} > w_H^{mb} - w_L^{mb}$. Compared to the male's wage contract, the principal thus raises w_H^{fb} and lowers w_L^{fb} such that the female still accepts the contract, i.e., (PC^f) still binds.

These insights also shape the contracts in the other case, in which the principal wants only one agent to work and the other to shirk. His problem then is

$$\begin{aligned}
\min_{u_L^i, u_H^i} \quad & p_{10} \sum_i h(u_H^i) + (1 - p_{10}) \sum_i h(u_L^i) \\
\text{s.t.} \quad & u_H^l - u_L^l \geq \frac{c^l}{p_{10} - p_{00}} && (IC^l) \qquad (3.2) \\
& p_{10}\, u_H^i + (1 - p_{10})\, u_L^i \geq c(e^i), && (PC^i)
\end{aligned}$$

for both $i = f, m$, and one working agent $l \in \{f, m\}$. While contracts that incentivize both agents satisfy all four constraints in (3.1), incentivizing a single agent as in (3.2) reduces the binding constraints to (PC^{-l}), (PC^l), and (IC^l). Note that (due to the agents' identical reservation utilities) the principal's expected wage cost that ensures participation of the shirking agent (who bears zero effort cost) is identical for both genders. Thus, if agent i shirks (at $c(e^i) = 0$) given her or his coworker $l \neq i$ works, the binding (PC^i) implies $u_H^i = -u_L^i\,(1 - p_{01})/p_{01}$. However, because there is no trade-off between incentives and risk sharing the principal offers the shirking agent of either gender a fixed wage-utility equal to her or his reservation utility, i.e., $u_L^{is} = u_H^{is} = 0$. Consequently, to decide whether to incentivize the female or male, the principal compares minimal expected wage cost necessary to make agent $l \neq i$ work given the other agent i shirks. According to his prior $c^m < c^f$ the principal conjectures the male as less costly to incentivize than the female. He therefore offers the male a contract such that, in addition to (PC^m), also (IC^m) binds.

Lemma 3.2 (Individual contracts (II))**.** *If the principal is free to individually contract and wants a single agent to work, he incentivizes the male by offering him*

$$w_L^{ms} = h\left(c^m\left(1 - \frac{p_{10}}{p_{10} - p_{00}}\right)\right) \text{ if the project fails, and,}$$
$$w_H^{ms} = h\left(c^m\left(1 + \frac{1 - p_{10}}{p_{10} - p_{00}}\right)\right) \text{ if the project succeeds.}$$

And, he lets the female shirk by offering her the fixed wage $w^{fs} = h(0)$.

Comparing the principal's expected profits shows that he wants both agents to work if and only if $(p_{11} - p_{01})(y_H - y_L) \geq p_{11} \sum_i w_H^{ib} + (1 - p_{11}) \sum_i w_L^{ib} - p_{01} \sum_i w_H^{is} - (1 - p_{01}) \sum_i w_L^{is}$. Because the right hand side of this condition may be positive the principal has no generally dominant strategy. That is, if the net revenue in case of success exceeds the incentive cost for the woman, the principal optimally induces both agents to work by offering them contracts for demanding jobs according to lemma 3.1. If the net revenue is low, in contrast, he lets the female shirk by offering her a trivial job according to lemma 3.2.[17] However, due to the principal's bias the contracts are not necessarily consistent with the agents' effort and participation decisions. In the remainder of this section, we thus explicitly derive these decisions if the principal is free to offer individual wages.

First, we consider the contracts (w_L^{ib}, w_H^{ib}) of lemma 3.1, and show that the underestimated female rejects the contract in anticipation that her male coworker shirks.[18] Given that coworker $j \neq i$ participates and works, agent $i = f, m$ works if and only if

$$u_H^{ib} - u_L^{ib} \geq \frac{c}{p_{11} - p_{01}} \quad \Leftrightarrow \quad c^i \geq c, \tag{3.3}$$

where c denotes the agents' true effort cost. Agent i's true incentive compatibility constraint (3.3) is thus satisfied with strict inequality for the underestimated female $i = f$ while being violated for the overestimated male $i = m$. It follows that the contracts (w_L^{ib}, w_H^{ib}) fail to simultaneously incentivize both team members. In fact,

[17] We focus on the interesting cases in which the principal wants at least one agent to work. Otherwise, he would offer a fixed wage equal to the reservation utility (normalized to zero) to both agents. Since the principal's bias concerns the agents' costs of working, both agents would accept the fixed wage contract and shirk.

[18] Recall that we assume the agents to know their true cost c as well as the principal's prejudice. Alternatively to the latter, we may assume the agents to observe not only their own, but also the contract offered to their potential coworker.

given individual contracts, the overestimated male never works. To see this, assume that his female coworker shirks. The male agent then would work if and only if

$$u_H^{mb} - u_L^{mb} \geq \frac{c}{p_{10} - p_{00}} \quad \Leftrightarrow \quad \frac{c^m}{c} \geq \frac{p_{11} - p_{01}}{p_{10} - p_{00}}. \tag{3.4}$$

But, given the male's true incentive compatibility constraint (3.4) is satisfied, the female's is equivalent to $c^f \geq c$, and thus, also satisfied. We know, that in this case, the overestimated male shirks. Given that, the female agent works if and only if

$$\frac{c^f}{c} \geq \frac{p_{11} - p_{01}}{p_{10} - p_{00}}. \tag{3.5}$$

Since $c^f/c > 1$ the female's true incentive compatibility constraint (3.5) is clearly satisfied if $p_{11} - p_{01} \leq p_{10} - p_{00}$, i.e., if the agents' effort choices are substitutes.[19] In case that working is complementary, however, the female works if the relative prejudice, c^f/c, exceeds the relative increase in the marginal productivity of her effort. Otherwise, she shirks. However, non of these effort decisions are implemented because agent i rejects the contract if her or his coworker $j \neq i$ shirks: if both agents shirk their true participation constraints are

$$p_{00}\, u_H^{ib} + (1 - p_{00})\, u_L^{ib} - 0 \geq 0 \quad \Leftrightarrow \quad c^i \left(\frac{p_{00} - p_{01}}{p_{11} - p_{01}} \right) \geq 0, \tag{3.6}$$

and thus, violated. If the female works, though, the male would participate since his true participation constraint $p_{01}\, u_H^{mb} + (1 - p_{01})\, u_L^{mb} - 0 \geq 0 \Leftrightarrow c^m \left(\frac{0}{p_{11} - p_{01}} \right) \geq 0$ is satisfied with equality. This situation never occurs due to the female who bears effort cost c, and thus, rejects the contract since

$$p_{01}\, u_H^{fb} + (1 - p_{01})\, u_L^{fb} - c \geq 0 \quad \Leftrightarrow \quad c^f \left(\frac{0}{p_{11} - p_{01}} \right) \geq c$$

is clearly violated. We conclude that the female rejects her individual contract if the principal wants both agents to work. That is, the individual wage contract as a hard-working peer does not reimburse the female agent for the free-riding of her overestimated male coworker. Therefore, she rejects the demanding job offer.

[19] We define the effort choices as substitutes if $(p_{00} + p_{11})/2 \leq p_{01}$. Otherwise, we call them complements.

In the other case in which the principal believes that his net revenue $(p_{11} - p_{01})(y_H - y_L)$ is not high enough to incentivize the "costlier" female, he offers her a fixed wage-utility equal to her outside option, i.e., $u^{fs} = 0$ (see lemma 3.2). That is, the principal offers the woman a trivial job. Because this trivial job causes no additional cost c for, e.g., networking or substitution of household production, she indeed shirks. Her true participation constraint then holds with equality no matter what her male coworker does. Given that, her male coworker's true incentive compatibility constraint

$$u_H^{ms} - u_L^{ms} \geq \frac{c}{p_{10} - p_{00}} \quad \Leftrightarrow \quad c^m \geq c$$

is violated, and he also shirks. Since his participation constraint then holds with equality[20], the agents' decisions are optimal given the offered contracts. Lemma 3.3 concludes.

Lemma 3.3 (Rejection and shirking with individual contracts)**.** *In case of individual wages, the female candidate rejects her contract if the principal wants both agents to work. If, however, he incentivizes only the male agent, both agents accept their individual contracts and shirk.*

Intuitively, if the biased principal wants to incentivize both agents he offers not enough to the overestimated male and more than necessary to the underestimated female. Therefore, the former shirks while the latter rather prefers to work. However, her individual contract (w_L^{fb}, w_H^{fb}) sets the participation constraint (PC^f) in the principal's problem (3.1) binding. In contrast to the female's true participation constraint, which takes the male's shirking into account, the constraint (PC^f) is based on the success probability given both agent's work, and thus, overstates u_H^{fb} while it insufficiently weights u_L^{fb}. Consequently, the female agent rejects the contract (w_L^{fb}, w_H^{fb}) even though it promises an excessive incentive pay $u_H^{fb} - u_L^{fb}$. Note that in case of rejection, both agents by assumption receive their reservation utility (normalized to zero). This coincides with their expected utilities in the standard model with an unbiased principal because the latter offers contracts that set the agents' true participation constraints binding. In this model

[20] Note that the constraints (PC^m) and (IC^m) in problem (3.2) bind. Formally, the contract (w_L^{ms}, w_H^{ms}) satisfies $p_{10}\, u_H^{ms} + (1-p_{10})\, u_L^{ms} - c^m = 0$ and $p_{10}\, u_H^{ms} + (1-p_{10})\, u_L^{ms} - c^m = p_{00}\, u_H^{ms} + (1-p_{00})\, u_L^{ms}$. Therefore, the male's true participation constraint if he shirks, $p_{00}\, u_H^{ms} + (1-p_{00})\, u_L^{ms} = 0$, is satisfied with equality.

the true success probabilities are known by the agents, however, might differ from the biased principal's prejudice. Therefore, the agents' expected utilities might exceed their reservation utilities. In what follows we call this difference a *rent*.

In case the principal wants only the male to work, he offers the female the fixed wage w^{fs}. Therefore, (PC^f) in problem (3.2) is equivalent to the female's true participation constraint that is independent of the male's effort, and she accepts. This is the only situation with individual contracts in which the principal is able to hire a team. To the best of his knowledge, this team consists of a male who goes an extra mile and a female who does a trivial job. However, he offers the male an insufficient incentive pay who thus shirks to avoid cost c. That is the reason why the male accepts contract (w_L^{ms}, w_H^{ms}) even though the principal overstates u_H^{ms} while he insufficiently weights u_L^{ms} in (PC^m). This overestimation of the project's success probability moreover implies that the principal's expected profit is higher than his true profit.

Before we study whether this also applies to the case with wage equality, we derive the according contracts as well as the agents' effort and participation decisions. That is, in the next section, the principal not only has to hire a gender-mixed team, but additionally has to offer equal wages.

3.4 Wage equality

If the principal wants both agents $i = f, m$ to work and has to pay them equal wages-utilities u_L^e and u_H^e, his problem is

$$\begin{aligned} \min_{u_L^e, u_H^e} \quad & p_{11}\, 2\, h(u_H^e) + (1 - p_{11})\, 2\, h(u_L^e) \\ \text{s.t.} \quad & u_H^e - u_L^e \geq \frac{c^i}{p_{11} - p_{10}} && (IC^i) \\ & p_{11}\, u_H^e + (1 - p_{11})\, u_L^e \geq c^i. && (PC^i) \end{aligned}$$

The principal's cost bias $c^m < c^f$ implies that he perceives the woman as costlier to incentivize, and also to ensure her participation. If he wants to hire both agents as hard-working peers he thus offers wage-utilities such that the female's constraints (PC^f) and (IC^f) bind. That is, he offers both agents the same contract as he individually offers the female for a demanding job in lemma 3.1. Thereby, he grants the "cheaper" male a rent.

In case that the principal wants only one agent to go an extra mile and the other to do a trivial job, he can save this rent since (IC^f) ceases.[21] Then, the principal's optimal wage-utilities set the male's incentive compatibility constraint (given his female coworker shirks) binding. This contract is identical to the male's contract in lemma 3.2. Lemma 3.4 summarizes.

Lemma 3.4 (Equal wage contracts)**.** *With wage equality the principal offers* (w_L^{fb}, w_H^{fb}) *if he wants both agents to work. However, if he wants a single agent to work he offers both agents* (w_L^{ms}, w_H^{ms})*.*

Comparing the principal's expected profits shows that he wants both agents to work if and only if

$$(p_{11} - p_{01})(y_H - y_L) \geq 2 \left(p_{11}\, w_H^{fb} + (1 - p_{11})\, w_L^{fb} - p_{01}\, w_H^{ms} - (1 - p_{01})\, w_L^{ms} \right).$$

Because the right hand side of this condition may be positive, the principal has no generally dominant strategy: if the net revenue in case of success is high the principal optimally offers both agents a demanding job. Otherwise, he saves wage cost by offering the female a trivial job.

[21] See page 148 for the discussion on the reason why the principal incentivizes the male if he wants a single agent to work and the other to shirk.

To see whether the contracts of lemma 3.4 are consistent with the agents' effort and participation decisions, we derive the latter in what follows. We thereby show that, in contrast to individual contracts, wage equality induces both agents to accept demanding jobs, and to actually go the extra mile. Moreover, they may also both do so in case the principal offers the female a trivial job only.

If the principal offers (w_L^{fb}, w_H^{fb}) and given that coworker $j \neq i$ participates and works, the other agent $i = f, m$ also works: both, her or his true incentive compatibility constraint

$$u_H^{fb} - u_L^{fb} \geq \frac{c}{p_{11} - p_{01}} \quad \Leftrightarrow \quad c^f \geq c, \tag{3.7}$$

as well as her or his true participation constraint, $p_{11}\, u_H^{fb} + (1 - p_{11})\, u_L^{fb} \geq c$, are satisfied with strict inequality.[22] This implies that if the equal wage contract is designed to incentivize the underestimated female too, each agent receives a rent in the amount of the bias, that is, $c^f - c$ utils.

Indeed, if the principal offers (w_L^{fb}, w_H^{fb}) both agents work and accept: given agent $j \neq i$ shirks, the other agent $i = f, m$ also shirks if and only if $u_H^{fb} - u_L^{fb} < c/(p_{10} - p_{00}) \Leftrightarrow c^f/c < (p_{11} - p_{01})/(p_{10} - p_{00})$. However, if they shirk, the agents' true participation constraints are equivalent to (3.6) with $i = f$, and thus, violated. We conclude that, in contrast to individual contracts, with wage equality both agents go the extra mile if the principal wants them to. By doing so they both receive a rent.

Also in the other case, in which the principal wants the male to perform a demanding job, the female may decide to support her coworker by going an extra mile too: given the principal offers (w_L^{ms}, w_H^{ms}) and coworker $j \neq i$ participates and works, the other agent $i = f, m$ also works if and only if

$$u_H^{ms} - u_L^{ms} \geq \frac{c}{p_{11} - p_{01}} \quad \Leftrightarrow \quad \frac{c}{c^m} \leq \frac{p_{11} - p_{01}}{p_{10} - p_{00}}. \tag{3.8}$$

Since the left hand side of (3.8) strictly exceeds 1 both agents work if $p_{10} - p_{00} < p_{11} - p_{10}$, i.e., if effort choices are complements. Actually, it requires a rather strong complementary and a rather moderate overestimation of the male, i.e., a

[22] Note that (3.7) further implies that no agent shirks given the other works.

high c^m. Given that both agents work their true participation constraints are

$$p_{11}\, u_H^{ms} + (1 - p_{11})\, u_L^{ms} \geq c \quad \Leftrightarrow \quad \frac{c}{c^m} \leq \frac{p_{11} - p_{00}}{p_{10} - p_{00}},$$

and thus, implied by their true incentive compatibility constraints (3.8). It follows that, even if the biased principal wants only the male to work, both team members may do so. However, given agent $j \neq i$ shirks, the other agent $i = f, m$ also shirks since

$$u_H^{ms} - u_L^{ms} < \frac{c}{p_{10} - p_{00}} \quad \Leftrightarrow \quad c^m < c \tag{3.9}$$

is satisfied, and, the true participation constraints $p_{00}\, u_H^{ms} + (1 - p_{00})\, u_L^{ms} \geq 0$ is also satisfied with equality.[23] It follows that, given equal wages (w_L^{ms}, w_H^{ms}), both agents may shirk and thereby receive no rent. Moreover, if and only if (3.8) is satisfied they may both work. In this case, each agent receives a rent equal to $c^m\,(p_{11} - p_{00})/(p_{10} - p_{00}) - c$. Lemma 3.5 summarizes.

Lemma 3.5 (Acceptance and ambiguous efforts with wage equality)**.** *With wage equality both agents accept the contracts. If the principal wants both to work, they indeed do so. If the principal wants only the male to work, both agents shirk if* $c/c^m > (p_{11} - p_{01})/(p_{10} - p_{00})$*. Otherwise, they may also both work.*

In contrast to individual contracts, wage equality induces both agents to accept demanding job offers. Moreover, they may actually work with equal wages whereas they unambiguously shirk with individual wages (given that both accept in the first place). With wage equality, the agents' effort choices also are unambiguous if the project's net revenue $y_H - y_L$ is high enough for the principal to incentivize the "costlier" female and offer (w_L^{fb}, w_H^{fb}). Then, both agents go the extra mile and receive a rent of $c^f - c$ each.

In the other case of rather low net revenue and insufficient incentive pay (w_L^{ms}, w_H^{ms}), effort choices also are unambiguous if they are substitutes, or formally, if $(p_{00} + p_{11})/2 < p_{10}$. That is, if the success probability given an agent works while her or his coworker shirks exceeds the average probability given both agents either shirk or work. This implies that each agent's own effort choice is decisive for her or his utility, and thus, independent of the coworker's behavior. Therefore, if

[23]Moreover, (3.9) implies that no agent works given the other shirks.

the incentive pay is insufficient and effort choices are substitutes, shirking is the dominant strategy. This is also true for relatively weak complementarity, i.e., if the principal's relative bias exceeds the relative degree of complementarity. More precisely, if $c/c^m > 1$ is higher than $(p_{11} - p_{01})/(p_{10} - p_{00}) > 1$, the shortfall of the incentive pay together with an agent's own behavior is decisive, and consequently, her or his dominant strategy is shirking. However, this is not true if effort choices are strong complements. Then, the agents depend on their coworker: given that coworker $j \neq i$ shirks, agent i also shirks because being the single one working only moderately increases the success probability (from p_{00} to p_{10}), and, due to the principal's bias, wages are insufficient to compensate for the cost of working. However, given that coworker $j \neq i$ works, agent i also works because strong complementarity implies a significant increase in the success probability (from p_{01} to p_{11}), which outweighs the (rather small) shortfall of incentive pay. That is, if c/c^m is lower than $(p_{11} - p_{01})/(p_{10} - p_{00})$ the agents' working behavior is ambiguous: either they both shirk or they both work. In the former case, their expected utility equals their reservation utility. However, if both agents go the extra mile each receives a positive rent of $c^m \, (p_{11} - p_{00})/(p_{10} - p_{00}) - c$.

In contrast to the agents' rents, the biased principal's profit evidently is lower than his profit in the standard formulation without prejudice: since unbiased contracts maximize (true) profit, every bias-induced deviation from the optimal contracts reduces the principal's true profit. However, the comparison between the principal's expectation and his true profit is less obvious. We thus describe some insights concerning profits in the next section.

3.5 Profits

The situations described in lemmata 3.3 and 3.5 do not satisfy belief consistency.[24] As a consequence, the principal's expected profit differs from his true profit. While he clearly overestimates his benefits from individual contracting, the effect of his misjudgment in case of an equal pay policy is ambiguous.

With individual contracts, either the principal fails to hire a team of hard-working peers because (at least) the female rejects or both agents accept and shirk. In the former case, the principal's expected profit exceeds his (not realizable) actual profit due to the assumption that the project's implementation is profitable. The latter case occurs if the principal wants to incentivize only the male agent, who nevertheless shirks. Therefore, the principal overestimates the project's success probability, and (due to well-known wage cost) also his profit.

With wage equality and high enough net revenue $y_H - y_L$, the principal offers both agents demanding jobs and they indeed go the extra mile. Consequently, his expected and his true profit coincide. With wage equality and rather low net revenue, however, the principal wants only the male candidate to work hard and offers equal contracts (w_L^{ms}, w_H^{ms}). In this case, the agents' working decisions as well as the principal's misjudgment are ambiguous: if the relative bias c/c^m is high both agents shirk, and thus, the principal's expected profit exceeds the actual profit. However, if his bias is low enough, they may both work, and consequently, the principal's expectation is lower than his true profit.

In a nutshell, the principal overestimates the benefits from individual contracting since he miscalculates the incentive pays while he underestimates the benefits of wage equality, which partly compensate for his bias.

The comparison between actual profits of the two wage policies is more complex. In case of high net revenue it is straightforward by assumption: because the principal fails to hire a team of two hard working peers with individual wages (w_L^{ib}, w_H^{ib}), his true profit is higher if he offers equal wage contracts (w_L^{fb}, w_H^{fb}) to both agents $i = f, m$. However, if net revenue is rather low, he might be better off with individual contracts. That is the case if both agents decide to shirk with equal wage contracts (w_L^{ms}, w_H^{ms}). Then, the success probability equals p_{00} under both policies, and the principal optimally bears the lower cost of individual wages

[24] This belief inconsistency is the reason why we mostly avoid the term "equilibrium".

(w_L^{is}, w_H^{is}).[25] This is not true if both agents go the extra with equal wage contracts, which might occur if their working decisions are strong complements. An explicit profit comparison in this case requires further assumptions on the functional form of the inverse utility $h = u^{-1}$. Intuitively, the profit with equal contracts exceeds the profit with individual wages if the difference in the success probabilities $p_{11} - p_{00}$ as well as the net revenue $y_H - y_L$ are high and the difference in wage cost is low.

3.6 Productivity bias

In this section we outline some insights from modifying the model of section 3.2 by assuming the principal to hold prejudice concerning gender-specific productivities instead of costs. That is, he knows the agents' true cost for performing demanding work c, however, wrongly believes that the female's contribution to the project's success probability given she works and her male coworker shirks is lower than vice versa. More precisely, his priors (denoted by the appropriate superscripts $i = f, m$) are $p_{10}^f < p_{10} = p_{01} < p_{01}^m$, where p_{10} and p_{01} represent the true success probabilities if either the female or the male agent works and her or his respective coworker shirks. Below we suppress the subscripts of the principal's priors and borrow the familiar notation from the previous sections. This simplifies our reasoning on why the results given a productivity bias are similar to those of the main model with cost bias except for the most important difference that the agents' participation and effort decisions carry over for interchanged genders.

Analogous to the proof of lemma 3.1 it is simple to show that the principal with productivity bias offers individual contracts

$$w_L^{ib} = h\left(c\left(1 - \frac{p_{11}}{p_{11} - p^i}\right)\right)$$
$$w_H^{ib} = h\left(c\left(1 + \frac{1 - p_{11}}{p_{11} - p^i}\right)\right)$$

if his net revenue is high enough such that he presumably benefits from incentivizing both team members. In contrast to the main model, the gender wage gap is reversed because, compared to the unbiased (and the female's) offer,

[25] More precisely, the principal's cost in case of individual wages is lower than with wage equality since $h(0) < p_{00}\, h(u_H^{ms}) + (1 - p_{00})\, h(u_L^{ms})$ holds. That is, with individual contracts the principal saves the (positive) difference between the risk premium necessary to incentivize the male and the fixed wage that induces the female to participate.

the male's offer with productivity bias rewards him for his "higher" marginal productivity instead of granting him a lower cost compensation. Thus, with productivity bias the principal's expected wage cost of incentivizing a man exceeds those of incentivizing a woman (since $w_L^{mb} < w_L^{fb} < w_H^{fb} < w_H^{mb}$). This insight from assuming a productivity bias is in line with the majority of empirical studies. In case of a cost bias, in contrast, we find that the principal's expected wage cost for incentivizing the female exceeds those of incentivizing the male. As a corollary thereof, lemma 3.2 concludes that the principal with cost prejudice (and low net revenue) offers the male candidate a demanding and the female a trivial job.

Similarly, this applies for a principal with productivity prejudice who offers the male the incentive contract

$$\begin{aligned} w_L^{ms} &= h\left(c\left(1 - \frac{p^m}{p^m - p_{00}}\right)\right) \\ w_H^{ms} &= h\left(c\left(1 + \frac{1 - p^m}{p^m - p_{00}}\right)\right), \end{aligned}$$

while he offers the female the fixed wage $w^{fs} = 0$. However, the rationale for this insight differs from the one in the main model: the principal's productivity bias implies that his expected wage cost of incentivizing a single agent is independent of gender (as weighted according to the gender-specific success probabilities). Therefore, he considers expected revenue, which is higher if the "more productive" male performs the demanding job.

Following the proof of lemma 3.3 in the main text shows that the agents' effort and participation decisions given individual contracts carry over to the modification with productivity bias, however, with interchanged genders. Intuitively, this is due to the reversed gender pay gap. Another consequence of the reversed pay gap is that, in contrast to lemma 3.4, the principal with productivity bias offers both agents equal wage contracts designed to optimally incentivize the man. That is, with productivity bias the male's ability determines the equal wage contracts. Wage equality implies that the agents' decisions are symmetric. Therefore, the results of lemma 3.5 carry over to this modification with productivity bias except for the uniqueness condition: in case of one demanding and one trivial job, both agents unambiguously shirk if $p_{00} + p_{11} < p_{01} + p_{01}^m$, which is more relaxed than in the case of cost prejudice.

3.7 Conclusion

We extend a static principal agent model with discrete effort choice by assuming the principal to have gender prejudice in form of differing priors on potential job candidates' working cost. More precisely, the prejudiced principal wrongly believes that women bear higher cost for performing demanding work than men, for instance, due to presumably lower ability or rival household production. We impose a quota that forces the principal to hire a gender-mixed team of two agents. Contingent on his net revenue he either offers both agents demanding jobs or he wants the man to perform demanding and the woman to do trivial work. The optimal contracts in the former case contradict the widely discussed gender pay gap: the female candidate's expected wage exceeds the male's. However, she rejects to work as a peer because her overestimated coworker shirks. He also shirks in the latter case, however, the woman nevertheless accepts the trivial job. This result constitutes a new rationale for the underrepresentation of women in higher hierarchy levels. Furthermore, we propose wage equality as a simple remedy for the free-riding problem because it induces the principal to increase the male agent's incentive pay, and consequently, mitigates the gender gap in the workplace.

Our exercise raises some questions that are, to the best of our knowledge, not well studied by economists. Can overestimation of abilities indeed reduce effort provision? May equally able women indeed refuse to work as peers with overestimated men? Future experimental research might try to shed light on such issues arising from gender prejudice.

3.A Appendices

A.1 Proof of lemma 3.1

Proof. By assumption $u'(w) > 0$ the inverse utility $h = u^{-1}$ is well defined on the interval $u(w)$. Since $u'' < 0$ the Inverse Function Theorem, $h'(u) = 1/u'(w)$, implies that $h(u)$ is strictly increasing and convex. Thus, the objective function in problem (3.1), a convex combination of h, is also strictly increasing and convex. Since the constraints are linear in the choice variables u_L^i and u_H^i the first order conditions are necessary and sufficient. They are

$$\begin{aligned}(1-p_{11})\,h'(u_L^i) + \lambda_{IC} - (1-p_{11})\,\lambda_{PC} &= 0\\ p_{11}\,h'(u_H^i) - \lambda_{IC} - p_{11}\,\lambda_{PC} &= 0\end{aligned}$$

for both $i = \{f, m\}$. Together, the first order conditions imply that

$$\lambda_{PC} = p_{11}\,h'(u_H^i) + (1-p_{11})\,h'(u_L^i), \text{ and} \tag{3.10}$$

$$\lambda_{IC} = p_{11}\,(1-p_{11})\left(h'(u_H^i) - h'(u_L^i)\right). \tag{3.11}$$

From (3.10) it directly follows that $\lambda_{PC} > 0$, and thus, that both participation constraints are binding. Note that (IC^i) implies $u_H^i > u_L^i$ since its right hand side, $c^i/(p_{11} - p_{10})$, is strictly positive. Thus, together with $h'' > 0$, it follows from (3.11) that $\lambda_{IC} > 0$, i.e., the incentive compatibility constraints are binding too. Substitution yields the contracts (w_L^{ib}, w_H^{ib}). □

A.2 Proof of lemma 3.2

Proof. To see that all three constraints of problem (3.2) are binding proceed according to the proof of lemma 3.1. Given that, the participation constraint of the shirking agent i is equivalent to $u_H^i = -u_L^i\,(1-p_{01})/p_{01}$. Substitution in the objective function and differentiation with respect to u_L^i yields the first order condition $h'(u_L^i) = h'\left(-u_L^i\,(1-p_{01})/p_{01}\right)$, and thus, $u_L^{is} = 0$. It follows that $u_H^{is} = 0$, i.e., the principal offers the shirking agent i a fixed wage equal to her (or his) reservation utility, which is identical for both genders.

It follows from lemma 3.1 that the principal's expected wage cost from incentivizing agent $l \in \{f, m\}$ to work (given $i \neq l$ shirks) is $p_{10}\,h(u_H^l) + (1-p_{10})\,h(u_L^l)$, with $u_L^l = c^l\,(1 - p_{10}/(p_{10} - p_{00}))$ and $u_H^l = c^l\,(1 + (1-p_{10})/(p_{10} - p_{00}))$. Since $u_L^f < u_L^m < u_H^m < u_H^f$ and h strictly increasing and convex, the principal expects the male to be less costly to incentivize (see below figure 3.1). He thus offers him the contract (w_L^{ms}, w_H^{ms}), while he offers the female the fixed wage w^{fs}. $\square$

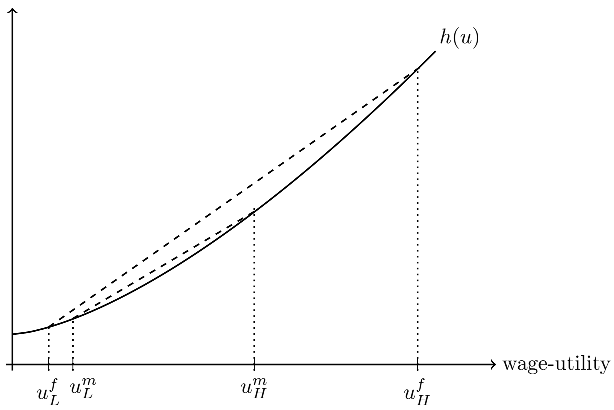

Figure 3.1: Expected wage cost for the working agent

A.3 Proofs of lemmata 3.3 to 3.5

The proofs of lemmata 3.3 to 3.5 are outlined in the main text.

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Bern, 24. Dezember 2020

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