# Essays in Microeconomics: <br> Opportunities of Competitive Screening and Shortcomings of Labor Market Signaling 

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The faculty accepted this thesis on the 4. November 2021 at the request of the two reviewers Prof. Dr. Marc Möller and Prof. Dr. Heiko Karle as dissertation, without wishing to comment on the views expressed therein.

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## Introduction

This thesis consists of three essays in industrial organization and behavioral economics. The part in industrial organization, consisting of the first two chapters, studies advance selling in an oligopolistic market with heterogeneous firms. Chapter 1 introduces a theoretical model in which asymmetric firms use advance purchase discounts as a competition instrument. The analysis focuses on how the considered heterogeneity influences the firms' profit-maximizing price schemes. Chapter 2 uses the developed framework and applies it to the study of entry in markets with advance selling. Predictions are tested through an empirical analysis of Italian railway pricing. The third chapter analyzes a labor market signaling model under the lense of behavioral economics. The proposed model considers workers characterized by loss aversion who use their parents' income as a reference point in their education choice. All chapters have in common the concept of asymmetric information in the sense that one party (the consumers/the workers) is better informed about their characteristics than the other (the firms/the employers). In the first two chapters, the uninformed part offers a menu of choices from which the informed part self-selects according to their type. In the third chapter it is the informed side that reveals its type by choosing to invest (or not) in a costly signal.

Chapter 1, co-authored by Marc Möller, proposes a model of advance selling in differentiated product markets with heterogeneous firms and individual demand uncertainty. We consider two types of asymmetry; firms differ either in their efficiency, that is, in their marginal cost of production, or in
their prominence, that is, net of prices, a larger fraction of consumers prefer one product over the other. We distinguish two periods: the advance selling period, in which consumers have imperfect information about their preferred product, and the consumption period, in which uncertainty is resolved. In order to attract customers firms offer a discounted price for consumers buying in advance. This strategy, however, increases the competition between the two firms that gain lower profits than when price discrimination is not allowed. The firms' exact pricing strategy depends on which type of asymmetry is considered.

When firms differ in their efficiency, they offer the same advance purchase discount in absolute terms. However, because the more efficient firm charges lower prices, its relative discount is higher. This allows for the more efficient firm to sell a higher output in the advance selling period and increase its market share compared to a uniform, time-invariant, pricing schedule.

When firms differ in their prominence, the prominent firm sets higher prices, offers a higher advance purchase discount and a lower relative discount. The less prominent firm has an incentive to offer a high relative advance purchase discount to shift the competition to the advance selling period, where due to uncertainty, consumers view the products as more homogeneous. The increase in competition caused by advance selling harms the prominent firm more than the less prominent firm, meaning that the profit difference among the two is smaller under price discrimination than with uniform pricing.

In Chapter 2, the model is applied to an entry scenario by considering a more prominent incumbent facing a more efficient entrant. Consistent with the results of Chapter 1, the entrant sets lower prices, offers a smaller absolute advance purchase discount and a larger relative advance purchase discount. Thanks to advance selling, the entrant diverts competition to the first period, where its prominence disadvantage looms less heavily. Advance selling promotes entry in the sense that compared to uniform pricing, it allows the entrant to increase its market share and narrows the profit gap with the incumbent.

The model's predictions are then tested with prices from the Italian railway market. The market, liberalized in the early 2000s, currently features two companies active in intercity connections: Trenitalia (incumbent and former monopolist) and Italo (entrant). Prices were collected three, two and one weeks before as well as at the day of departure. The empirical analysis shows significant evidence for the theoretical results.

Chapter 3 introduces loss aversion to Spence's (1973) labor market signaling model and explores a new channel through which family background influences education choice. In the presented model, workers derive utility from their own income and from a comparison of their income with their parents' income (i.e. their reference point). Loss aversion implies that the negative effect of an income lower than the reference one is stronger than the positive effect from exceeding the reference income.

The workers' education choice depends on their ability level and their reference income. Loss aversion and the weight attached to the referencedependent utility establish the propensity to follow the parents' footsteps. When this propensity is low compared to the difference in the workers' ability, we obtain an equilibrium in which workers separate by ability. High ability workers choose a higher level of education than low ability workers, independently of their family income. When the propensity to follow the parents' footsteps is high, workers separate by the income of the household of origin. Workers from rich households, independently from their ability, choose a higher level of education than workers from low income households. This result establishes a link between cultural differences in loss aversion and higher (in case of separation by ability) or lower (in case of separation by income) intergenerational mobility.

Although higher intergenerational mobility is desirable in terms of equality of opportunity, it is not necessarily beneficial in terms of welfare. In Chapter 3 , we see that in a separation by ability equilibrium, the cost of education is better distributed, but separation by income requires a lower level of education. Thus, separation by ability is better in terms of welfare than separation by income only if the difference in ability among workers is high enough.

## Chapter 1

## Competition in Advance Purchase Markets

joint with Marc Möller

### 1.1 Introduction

There are several examples of markets in which firms offer advance purchase discounts. These include music and arts festivals, for which consumers can buy "early bird" tickets at a reduced price. Flights and train tickets can also be purchased at a lower price if booked in advance. These markets are characterized by the uncertainty of the consumers regarding their preferences. For example, when booking a flight in advance, travelers might have imperfect information about their future schedule, weather conditions, or other relevant information for their trip.

Advance purchase discounts have been shown to be profitable for monopolists (Nocke et al., 2011) and homogeneous firms in competitive markets (Gale, 1993; Möller and Watanabe, 2016).

So far, literature has focused on homogeneous firms. This chapter proposes a model of oligopolistic advance purchase markets with heterogeneous firms. Introducing firms' heterogeneity allows us to understand how advance purchase discounts depend on a firm's characteristics.

In our model we consider two firms selling two differentiated products. Consumers can make their purchases at two different times: the advance purchase period and the consumption period. Prior to the first period, the firms commit to a price schedule. The two periods distinguish themselves by the level of information the consumers have. In the advance purchase period consumers know the firms' prominence and receive a signal about their preferred good; however, the signal is correct only with a certain probability. In the consumption period, the preferred good is revealed and there is perfect information. Consumers are characterized by their choosiness level: more choosy consumers weigh the difference in the products' characteristics more heavily.

We assume that firms are heterogeneous by considering two types of asymmetry: firms can either differ in their efficiency, meaning that one firm has lower marginal costs than the other, or in their prominence, meaning that more consumers prefer one firm's product over the other.

When firms differ in their marginal costs, we find that the more efficient firm charges lower prices in both periods, and the advance purchase discount is the same for both firms in absolute terms. It follows that, in the first period, consumers whose favorite product is produced by the efficient firm have a higher expected discount. This results in the more efficient firm selling a higher output than the rival in the advance purchase period and a lower output in the consumption period. Advance selling is particularly profitable for the more efficient firm which, thanks to advance purchase discounts, can increase its market share with respect to the case with time-invariant pricing.

When firms differ in their prominence, we find that the prominent firm sets higher prices, offers a higher absolute advance purchase discount and a lower relative advance purchase discount. The less prominent firm offers a higher relative advance purchase discount to shift the competition to the advance selling period, where, due to uncertainty, consumers see the product as more homogeneous. Thanks to its lower prices, the less prominent firm can make up for its smaller ex-ante consumer base and sell the same output as the prominent firm in the advance selling period. In the consumption period and overall, the prominent firm sells a higher output. Market shares
are not affected by price discrimination.
The ability of price discriminating between periods is not beneficial for the firms. We show that for both types of asymmetry, when price discrimination is not allowed, firms earn higher profits because they can charge higher prices. If price discrimination is allowed, firms lower the prices in the advance purchase period to "lock-in" consumers before true preferences are revealed and to avoid losing customers to the competitor in the consumption period. As a consequence, second period prices decrease: a too high advance purchase discount increases the fraction of consumers who are willing to buy in the first period, and the firms cannot exploit the higher willingness to pay of choosier costumers.

This chapter is structured as follows: Section 1.2 introduces the model. In Section 1.3 and 1.4 we study the effects of efficiency and prominence asymmetry respectively. Section 1.5 discusses our results and concludes. Proofs can be found in Appendix A.

### 1.1.1 Related Literature

Advance selling in the presence of individual demand uncertainty falls under a broader literature on price discrimination (see Stole, 2007, for an overview) and it has been studied for various types of markets.

In a monopolistic set-up, Gale and Holmes (1993) show that when a monopoly airline faces capacity constraints during peak times, advance purchase discounts are a profitable strategy that shifts part of the demand to less requested times. Nocke et al. (2011) extend this result by proving the optimality of advance purchase discounts as a price discrimination device for monopolies with unlimited capacity constraints. Also in a monopolistic set-up, Möller and Watanabe (2010) derive conditions under which advance purchase discounts are preferable over clearance sales.

For perfectly competitive markets, Dana, Jr. (1998) shows the optimality of advance purchase discounts in the presence of aggregate demand uncertainty. When holding capacity is expensive, firms can reduce these costs by using advance selling to screen consumers over their uncertainty level. Ab-
stracting from aggregate demand uncertainty and capacity constraint, Gale (1993) shows that in a duopoly, an advance purchase discount is a profitmaximizing strategy when products are ex-ante perfect substitutes.

Our model is close to the one presented in Möller and Watanabe (2016).
In their model, two symmetric firms offer two differentiated products, selling in advance at a cheaper price is profitable because it allows the firms to steal consumers from their competitor. Karle and Möller (2020) confirm the profitability of advance purchase discounts in a similar set-up but with loss averse consumers.

So far, research on price discrimination with asymmetric firms focused on the case of third-degree price discrimination. Chen (2008) studies the effect of history-based price discrimination over multiple time periods on the consumers' welfare. He shows that if the weaker firm stays in the market, consumers can benefit from price discrimination. Customer poaching with asymmetric firms is analyzed in Carroni (2016). The author finds that asymmetry has an important effect on consumers' behavior, who are, in this case, harmed by price discrimination. In both cases, price discrimination might cause the smaller firm to exit the market. Advance purchase discounts are a form of second-degree price discrimination since consumers can decide when to purchase a product. We will see that in our case, the smaller firm is the one that can have an advantage under the ability of price discriminating.

Among the empirical literature, there are conflicting results regarding the correlation between asymmetry and price discrimination. Gerardi and Shapiro (2009) find that price discrimination increases with market share, while Borenstein and Rose (1994) and Asplund et al. (2008) find the opposite relationship. The difference between these papers is that Gerardi and Shapiro (2009) also find a negative correlation between price discrimination and competition, implying that price discrimination results from the firms' exploitation of consumers' heterogeneous elasticities of demand. On the contrary, Borenstein and Rose (1994) and Asplund et al. (2008) find a positive correlation between price discrimination and competition. The interpretation is that price discrimination is used as a competition instrument. In particular, Asplund et al. (2008) claim that small newspapers are more prone to
offer discounts to new readers than their larger competitors. Our model is in line with this interpretation since, in a duopoly, advance purchase discounts are used to steal consumers from the competitor.

### 1.2 Model

We consider a market in which two firms sell two differentiated products $i \in\{A, B\}$. Consumers can purchase one of the products in two periods: the advance purchase period $(t=1)$ and the consumption period $(t=2)$. Prices are established in advance, that is, each firm $i \in\{A, B\}$ commits to a price schedule $\left(p_{1, i}, p_{2, i}\right) \in \mathbb{R}_{+}^{2}$ prior to the sales periods. Firm $i \in\{A, B\}$ has unit production costs equal to $c_{i}$. Without loss of generality, we assume that $0 \leq c_{A} \leq c_{B}$. In Section 1.3, when we introduce asymmetry in the firms' efficiency, we do so by assuming that $c_{A}<c_{B}$. To simplify our analysis we abstract from discounting of future payoffs.

We consider a unit mass of consumers with unit demand. Consumers are characterized by their choosiness $\sigma \in[0,1]$, which is private information of the consumer. The utility that consumer $\sigma$ obtains from his preferred product is given by $s+\frac{\sigma}{2}$, while the utility he obtains from his least preferred product is given by $s-\frac{\sigma}{2}$. Therefore, for more choosy consumers the difference in the products' characteristics weighs more heavily. We assume that $\sigma$ follows a uniform distribution over $[0,1]$. Moreover, we assume that a fraction $\rho \in\left[\frac{1}{2}, 1\right)$ of consumers prefers product $A$ and a fraction $1-\rho$ prefers product $B$. When $\rho=\frac{1}{2}$ we consider firms to be symmetric in their prominence, while in Section 1.4 we consider firm A as the prominent firm by assuming $\rho>\frac{1}{2}$.

The key difference between the advance purchase period and the consumption period is the uncertainty in the consumers' preferences over the two products. In period 1 each consumer receives a signal $S \in\{A, B\}$ about his preferred product. If a consumer receives the signal $S=i, i \in\{A, B\}$, we refer to product $i$ as the consumer's favorite product. The signal is correct with probability $\gamma \in\left[\frac{1}{2}, 1\right)$. When $\gamma=\frac{1}{2}$, the signal is uninformative and consumers don't have any information about which one could be their


Figure 1.1: Signal's distribution.
preferred product other than their knowledge of the preferences' distribution $\rho$. On the other hand, $\gamma \rightarrow 1$ corresponds to complete certainty about their preferences. Figure 1.1 describes the distribution of the signal $S$.

We further assume that the parameters are such that each consumer buys exactly one of the two products, and each consumer consumes the purchased product, even if he bought his least preferred one. In the case of indifference between buying in the advance purchase period or the consumption period, consumers purchase the good in the second one.

### 1.3 Cost Asymmetry

In this section, we assume that $c_{A}<c_{B}$, so firm $A$ is more efficient than firm $B$. Moreover we let $\rho=\frac{1}{2}$ such that the only difference between the firms is given by their marginal costs. We also assume that $\gamma>\frac{1}{2}$, because otherwise, in the advance selling period, due to uncertainty, consumers would view the products as homogeneous and the model would reduce to perfect competition.

### 1.3.1 Benchmark: Uniform Pricing

We first consider the case in which price discrimination is ruled out by the requirement that $p_{1, i}=p_{2, i}$. This later allows us to better understand the effect of advance purchase discounts.

When price discrimination is not allowed, given that a firm's prices are the same in both periods, consumers always wait and buy in the second period.

Purchasing in advance might result in buying the least preferred product without the benefit from a reduced price. Therefore, the problem is the same as the one of the standard Hotelling model, for which the equilibrium prices and profits are given by

$$
\begin{align*}
p_{i}^{U} & =\frac{2 c_{i}+c_{j}}{3}+1  \tag{1.1}\\
\Pi_{i}^{U} & =\frac{1}{18}\left[\left(c_{j}-c_{i}\right)^{2}+6\left(c_{j}-c_{i}\right)+9\right] \tag{1.2}
\end{align*}
$$

for $i \in\{A, B\}, j \neq i$.
Hence, given that $c_{A}<c_{B}$, without price discrimination firm $A$ sets lower prices and earns higher profits than firm $B$.

### 1.3.2 Advance Purchase Discounts

In this section we consider firms setting different prices in different periods. Firm $i \in\{A, B\}$ sells product $i$ and chooses a price schedule ( $p_{1, i}, p_{2, i}$ ) that maximizes its profits. It must hold that $p_{1, i} \leq p_{2, i}$ for $i \in\{A, B\}$, otherwise, given the uncertainty in utility in period 1 , all consumers would wait until the second period and buy their preferred product at a cheaper price. Firms could therefore decrease $p_{1, i}$ until $p_{1, i}=p_{2, i}$ without influencing their profits.

To define the profits of the two firms and derive the optimal price schedules, we first need to establish which consumers are buying from which firm and in which period.

Given that $\rho=\frac{1}{2}$, in the first period half of the consumers receives signal $S=A$ and the other half receives signal $S=B$.

Consider a consumer who, in the first period, receives a signal $S=A$. Given the signal $S=A$, the probability of preferring product $A$ is given by $\gamma$, while the probability of preferring product $B$ is given by $1-\gamma$. Purchasing product $A$ in advance gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma, A \mid 1, A)=s+\gamma \frac{\sigma}{2}-(1-\gamma) \frac{\sigma}{2}-p_{1, A} . \tag{1.3}
\end{equation*}
$$

If the consumer waits, he then buys his preferred product. Given that the price schedule is fixed, if the consumer knows that he will buy a certain
product no matter what the revealed preference is, he will buy it in the first period at a cheaper price. Hence, waiting for period 2 gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma, A \mid 2)=s+\frac{\sigma}{2}-\gamma p_{2, A}-(1-\gamma) p_{2, B} \tag{1.4}
\end{equation*}
$$

Therefore, a consumer with choosiness $\sigma$ who receives a signal $S=A$ is willing to wait if and only if

$$
\begin{align*}
U(\sigma, A \mid 2) & \geq U(\sigma, A \mid 1, A) \\
\sigma & \geq \sigma_{W}(A) \equiv \frac{(1-\gamma) p_{2, B}+\gamma p_{2, A}-p_{1, A}}{1-\gamma} \tag{1.5}
\end{align*}
$$

Analogously, a consumer who receives signal $S=B$ prefers product $B$ is willing to wait if and only if

$$
\begin{align*}
U(\sigma, B \mid 2) & \geq U(\sigma, B \mid 1, B) \\
\sigma & \geq \sigma_{W}(B) \equiv \frac{(1-\gamma) p_{2, A}+\gamma p_{2, B}-p_{1, B}}{1-\gamma} \tag{1.6}
\end{align*}
$$

The thresholds $\sigma_{W}(A)$ and $\sigma_{W}(B)$ define which consumers are buying in advance and which are waiting for the consumption period. Consumers whose favorite product is $S=i$ buy in the first period if and only if $\sigma \in\left[0, \sigma_{W}(i)\right)$, if $\sigma \in\left[\sigma_{W}(i), 1\right]$ they wait for the second period and buy their preferred product.

Given the firms' heterogeneity, we expect products' prices to differ. Thus, there might be consumers who are willing to buy their least favorite product because it is available at a lower price. Moreover, the least favorite product might end up to be the preferred one in the consumption period. Purchasing the favorite product is preferable if and only if the gain in consumption value is greater than the price difference, that is, if and only if

$$
\begin{align*}
s+\gamma \frac{\sigma}{2}-(1-\gamma) \frac{\sigma}{2}-\left[s-\gamma \frac{\sigma}{2}+(1-\gamma) \frac{\sigma}{2}\right] & \geq\left|p_{1, A}-p_{1, B}\right| \\
\sigma & \geq \bar{\sigma} \equiv \frac{\left|p_{1, A}-p_{1, B}\right|}{2 \gamma-1} . \tag{1.7}
\end{align*}
$$

We focus on price-discrimination equilibria, that is, profit-maximizing price systems $\left\{\left(p_{1, A}^{*}, p_{2, A}^{*}\right),\left(p_{1, B}^{*}, p_{2, B}^{*}\right)\right\}$ such that $p_{1, i}^{*}<p_{2, i}^{*}$, for $i \in\{A, B\}$,
with both firms selling strictly positive quantities in both periods. For this reason we assume that $\bar{\sigma}<\sigma_{W}(i), i \in\{A, B\}$. As long as the cost asymmetry is sufficiently small, this assumption is satisfied in our results. ${ }^{\top}$

For example, if $p_{1, A}<p_{1, B}$, consumers allocate in the following way: Consumers whose favorite product is $S=A$ and who have choosiness $\sigma \in$ $\left[0, \sigma_{W}(A)\right)$, buy product $A$ in the first period, and those with $\sigma \in\left[\sigma_{W}(A), 1\right]$ buy their preferred product in the second period. Consumers whose favorite product is $S=B$ and with choosiness $\sigma \in[0, \bar{\sigma})$ buy product $A$ in the first period, those with $\sigma \in\left[\bar{\sigma}, \sigma_{W}(B)\right)$ buy product $B$ in the first period, and those with $\sigma \in\left[\sigma_{W}(B), 1\right]$ buy their preferred product in the second period.

The profit of firm $A$ is given by $\Pi_{A}=\Pi_{1, A}+\Pi_{2, A}$, where the profit $\Pi_{1, A}$ from the first period is given by

$$
\Pi_{1, A}=\left\{\begin{array}{ll}
\frac{p_{1, A}-c_{A}}{2}\left(\sigma_{W}(A)-\bar{\sigma}\right) & \text { if } p_{1, A}>p_{1, B}  \tag{1.8}\\
\frac{p_{1, A}-c_{A}}{2}\left(\sigma_{W}(A)+\bar{\sigma}\right) & \text { if } p_{1, A} \leq p_{1, B}
\end{array},\right.
$$

while the profit of the second period is given by

$$
\begin{equation*}
\Pi_{2, A}=\frac{p_{2, A}-c_{A}}{2}\left[\gamma\left(1-\sigma_{W}(A)\right)+(1-\gamma)\left(1-\sigma_{W}(B)\right)\right] . \tag{1.9}
\end{equation*}
$$

Similarly, the profit of firm $B$ is given by $\Pi_{B}=\Pi_{1, B}+\Pi_{2, B}$ with

$$
\Pi_{1, B}= \begin{cases}\frac{p_{1, B}-c_{B}}{2}\left(\sigma_{W}(B)+\bar{\sigma}\right) & \text { if } p_{1, A}>p_{1, B}  \tag{1.10}\\ \frac{p_{1, B}-c_{B}}{2}\left(\sigma_{W}(B)-\bar{\sigma}\right) & \text { if } p_{1, A} \leq p_{1, B}\end{cases}
$$

and

$$
\begin{equation*}
\Pi_{2, B}=\frac{p_{2, B}-c_{B}}{2}\left[\gamma\left(1-\sigma_{W}(B)\right)+(1-\gamma)\left(1-\sigma_{W}(A)\right)\right] . \tag{1.11}
\end{equation*}
$$

The maximization problem of firm $i \in\{A, B\}$ yields the following firstorder conditions

$$
\begin{align*}
0= & \sigma_{W}(i)-\bar{\sigma}+\frac{1}{1-\gamma}\left[\gamma\left(p_{2, i}-c_{i}\right)-\left(p_{1, i}-c_{i}\right)\right]-\left(p_{1, i}-c_{i}\right) \frac{1}{2 \gamma-1}  \tag{1.12}\\
0= & \gamma\left[1-\sigma_{W}(i)\right]+(1-\gamma)\left[1-\sigma_{W}(j)\right] \\
& -\frac{\gamma}{1-\gamma}\left[\gamma\left(p_{2, i}-c_{i}\right)-\left(p_{1, i}-c_{i}\right)\right]-\left(p_{2, i}-c_{i}\right)(1-\gamma) \tag{1.13}
\end{align*}
$$

[^0]where $\sigma_{W}(i)$ and $\bar{\sigma}, i \in\{A, B\}, j \neq i$ are given by (1.5)-1.7), from which we obtain the following equilibrium prices
\[

$$
\begin{align*}
& p_{1, i}^{*}=p_{i}^{U}-\frac{6(1-\gamma) \gamma}{-1+7 \gamma-4 \gamma^{2}}  \tag{1.14}\\
& p_{2, i}^{*}=p_{i}^{U}-\frac{4(1-\gamma) \gamma}{-1+7 \gamma-4 \gamma^{2}} \tag{1.15}
\end{align*}
$$
\]

for $i \in\{A, B\}$.
Proposition 1.1. Suppose $c_{A}<c_{B}$. If a price-discrimination equilibrium exists, the prices are given by (1.14) - (1.15) and the following holds:

1. Prices are lower than in the uniform pricing benchmark.
2. In equilibrium the more efficient firm $A$ offers lower prices than the less efficient firm B in both periods. The absolute discount for an advanced purchase is the same for both firms, i.e. $p_{2, A}^{*}-p_{1, A}^{*}=p_{2, B}^{*}-p_{1, B}^{*}$.

The result follows directly from the equations (1.14) and 1.15). First, we notice that prices are lower than in the uniform, time-invariant, pricing benchmark. In the advance selling period consumers have a lower willingness to pay and firms offer advance purchase discounts to "lock-in" consumers before their true preferences are revealed. Advance selling increases competition, which is why prices in the consumption period are also lower than without price discrimination. Moreover, with a higher discount more consumers buy in the advance selling period and firms cannot profit from the higher willingness to pay of choosier consumers.

Firm $A$ takes advantage of the lower production costs and offers lower prices in both periods. However, the price difference generated by the cost asymmetry is the same with and without price discrimination. That is, cost asymmetry does not interact with dynamic pricing strategies. This is also reflected in the fact that, in absolute value, firms offer the same advance purchase discount.

The price difference can lead to firms selling only in the advance purchase period or only in the consumption period. The next proposition describes the parameter constellations for which a price discrimination equilibrium exists.

Proposition 1.2. Given $c_{A}<c_{B}$, a necessary condition for a price discrimination equilibrium to exist is

$$
\begin{equation*}
c_{B}-c_{A}<\Delta \tilde{c} \equiv \min \left\{\frac{3\left(-1+5 \gamma-4 \gamma^{2}\right)}{-1+7 \gamma-4 \gamma^{2}}, \frac{3(2 \gamma-1)}{-1+7 \gamma-4 \gamma^{2}}\right\} \tag{1.16}
\end{equation*}
$$

The proposition states that a price discrimination equilibrium is possible whenever the cost difference between the two firms is low enough ${ }^{2}$ From equations (1.14) and (1.15) it follows that a higher cost asymmetry translates into a higher price difference between the two firms. This difference affects the thresholds (1.5)-1.7). Therefore an interior solution might not exist if the asymmetry is too high. When the signal is rather uninformative ( $\gamma$ closer to $\frac{1}{2}$ ), consumers are less likely to buy in advance ( $\sigma_{W}(i), i \in\{A, B\}$, is low). At the same time, if the price difference is high, the fraction of consumers that, in the first period, receives signal $S=B$ but buy product $A$, increases (that is, $\bar{\sigma}$ is higher). Thus we might lose an interior solution because the more expensive firm $B$ ends up not selling anything in the advance purchase period (since $\left.\bar{\sigma}>\sigma_{W}(B)\right)$. On the other hand, when the signal is highly informative, that is $\gamma$ is close to 1 , consumers don't want to delay their purchases if their favorite product is substantially cheaper than the other, meaning that $\sigma_{W}(A)$ might be larger than one. Hence, the upper bound on cost difference $\Delta \tilde{c}$ is a concave function of $\gamma$. For $\gamma \in\left[\frac{1}{2}, \frac{3}{4}\right] \Delta \tilde{c}$ is increasing and defined by $\sigma_{W}(B)-\bar{\sigma}>0$ and for $\gamma \in\left[\frac{3}{4}, 1\right] \Delta \tilde{c}$ is decreasing and defined by $\sigma_{W}(A)<1$.

Corollary 1.1. Suppose $c_{A}<c_{B}$. Relative to its competitor, the more efficient firm ( $A$ ) sells a larger quantity in the advance purchase period but a smaller quantity in the consumption period. Overall, firm A sells a higher output than firm B. Advance selling increases firm A's market share compared to the uniform pricing benchmark.

[^1]

Figure 1.2: Consumers' expected discounts for $S=A$ and $S=B$.

There are two effects that result in firm $A$ selling a higher output than firm $B$ in period 1. The first comes from consumers whose favorite product is $S=B$, but who are almost indifferent between the two goods, that is, they have a low choosiness level $\sigma<\bar{\sigma}$. These consumers are almost indifferent between the two products and attracted by the low price, they prefer to buy product $A$, decreasing the fraction of consumers who would buy from firm $B$ in the first period. The second force is that, even though the actual discount for an advance purchase is the same for both firms, the share of consumers who receives signal $S=A$ and are willing to buy in the first period is higher than the corresponding one receiving signal $S=B$ (in equilibrium $\left.\sigma_{W}(A)>\sigma_{W}(B)\right)$. The reason behind this is that when consumers consider whether to buy in the first or second period, they consider the expected advance purchase discount and not the discount in absolute terms and the expected discount is higher for consumers whose favorite product is $S=A$ (see Figure 1.2). ${ }^{3}$

Intuitively, consumers who receive the signal $S=B$ are more likely to wait because if the signal is incorrect and they buy $B$ in the first period,

[^2]they end up buying their least preferred product at an expensive price. In contrast, if they wait, they buy their preferred good at a convenient price.

It follows that firm $A$ benefits more from the use of advance purchase discounts than firm $B$. By shifting the sales in the advance selling period, the efficient firm does not only sells a higher output than its competitor, but also increases its market share with respect to the benchmark without price discrimination.

Next, we turn to the profitability of advance purchase discounts. Replacing the equilibrium prices (1.14) and 1.15 in the profit functions (1.8)-(1.11), we obtain the following equilibrium profit

$$
\begin{equation*}
\Pi_{i}^{*}=\frac{1}{18}\left[\frac{-1+6 \gamma-4 \gamma^{2}}{2 \gamma-1}\left(c_{j}-c_{i}\right)^{2}+6\left(c_{j}-c_{i}\right)+\frac{1-10 \gamma+21 \gamma^{2}-8 \gamma^{3}}{\left(-1+7 \gamma-4 \gamma^{2}\right)^{2}} 9\right] \tag{1.17}
\end{equation*}
$$

for $i \in\{A, B\}, j \neq i$.
The next Corollary follows from $c_{A}<c_{B}$ and from the comparison with uniform pricing profit (1.2) for $\gamma \in\left(\frac{1}{2}, 1\right)$.

Corollary 1.2. Suppose $c_{A}<c_{B}$. With advance selling, the more efficient firm A earns higher profits than its competitor. Both firms' profits are lower when price discrimination is allowed and both firms' profits get reduced by the same (absolute) amount.

Due to is higher efficiency, firm $A$ earns higher margins and sells a higher output, which translates into higher profits than firm $B$.

For $\gamma \in\left(\frac{1}{2}, 1\right)$ price discrimination harms the firms' profits. This negative effect is stronger for lower values of $\gamma$, whereas as $\gamma$ gets closer to 1 , profits approach their respective uniform pricing benchmark's values. In the first period prices are driven down by the competition for less choosy costumers. Second period prices must in turn also decrease in order to avoid consumers with a high willingness to pay to buy in the first period at an even lower price and from the increase in competition. The result is that firms are worse off than when price discrimination is not allowed. The less informative the signal is, the stronger the prices decrease, amplifying the negative effect on the profits. The negative impact is the same for both firms.

### 1.4 Prominence Asymmetry

We now assume that firms are heterogeneous in their prominence $\rho$, where $\rho \in\left(\frac{1}{2}, 1\right)$ is the fraction of consumers who, net of prices, prefer product $A$ and $1-\rho$ the corresponding fraction for product $B$. As in the previous section, we consider firm $A$ to have an advantage over firm $B$. To highlight the effect of asymmetry in $\rho$ and to simplify the analysis, we let both firms have the same marginal costs and normalize them to zero, $c_{A}=c_{B}=0$. Because of prominence asymmetry, in the advance selling period the consumers view the products as heterogeneous also in the absence of an informative signal. To simplify our analysis we therefore set $\gamma=\frac{1}{2}$, that is, we consider an uninformative signal.

### 1.4.1 Benchmark: Uniform Pricing

As in the case with asymmetric efficiency, when price discrimination is not allowed and firms must set $p_{1, i}=p_{2, i}$, all consumers wait and buy in the second period. Assuming that the prominent firm $A$ sets a higher price ${ }^{4}$ a consumer who prefers product $B$ always buys product $B$ independently of $\sigma$ since it is cheaper. On the other hand, a consumer with choosiness $\sigma$ whose preferred product is $A$ buys product $A$ if and only if

$$
\begin{align*}
s+\frac{\sigma}{2}-p_{A} & \geq s-\frac{\sigma}{2}-p_{B} \\
\sigma & \geq \hat{\sigma} \equiv p_{A}-p_{B} . \tag{1.18}
\end{align*}
$$

Firm $A$ serves a fraction $\rho(1-\hat{\sigma})$ of the consumers, while firm $B$ serves a fraction $1-\rho+\rho \hat{\sigma}$. Hence, the firms' profits are given by

$$
\begin{align*}
& \Pi_{A}=p_{A} \rho(1-\hat{\sigma}),  \tag{1.19}\\
& \Pi_{B}=p_{B}(1-\rho+\rho \hat{\sigma}) . \tag{1.20}
\end{align*}
$$

Solving the system of equations generated by the first-order conditions of

[^3]the firms yields the following equilibrium prices
\[

$$
\begin{align*}
& p_{A}^{U}=\frac{1+\rho}{3 \rho}  \tag{1.21}\\
& p_{B}^{U}=\frac{2-\rho}{3 \rho} \tag{1.22}
\end{align*}
$$
\]

and profits

$$
\begin{align*}
\Pi_{A}^{U} & =\frac{(1+\rho)^{2}}{9 \rho}  \tag{1.23}\\
\Pi_{B}^{U} & =\frac{(2-\rho)^{2}}{9 \rho} \tag{1.24}
\end{align*}
$$

So for $\rho \in\left(\frac{1}{2}, 1\right)$ firm $A$ sets a higher price and earns a higher profit than its competitor. Intuitively, the firm with a higher consumer base can afford to set higher prices, as the gain from non-marginal consumers is higher than the loss from reduced demand.

Note that an increase in prominence reduces the equilibrium prices. The parameter $\rho$ is the mass of consumers who prefer product $A$. These are the consumers that the firms want to attract, since the consumers who prefer product $B$ are already secured at firm $B$ due to its lower price. We can therefore think of $\rho$ as the size of the contested market. As this market size increases, firms fight more aggressively for it by lowering their prices.

### 1.4.2 Advance Purchase Discounts

We now focus on dynamic pricing in which firms choose a price schedule $\left(p_{1, i}, p_{2, i}\right), p_{1, i}<p_{2, i} .^{5}$ for $i \in\{A, B\}$, that maximizes their profits. As in the cost asymmetric case, we focus on equilibria in which both products are sold in both periods.

To define the firms' profits we look at how consumers allocate over time and across firms. Because the signal is uninformative, it is also irrelevant

[^4]for the purchase choice. Still we need to define both thresholds $\sigma_{W}(A)$ and $\sigma_{W}(B)$ to establish the relevant one.

Consider a consumer purchasing product $A$ in the advance selling period. The probability of preferring product $A$ is given by $\rho$, while the probability of preferring product $B$ is given by $1-\rho$. Purchasing product $A$ in advance gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma \mid 1, A)=s+\rho \frac{\sigma}{2}-(1-\rho) \frac{\sigma}{2}-p_{1, A} . \tag{1.25}
\end{equation*}
$$

If the consumer waits, he then buys his preferred product. Given that the price schedule is fixed, if he knows that he is going to buy a certain product no matter what his revealed preference is, he will buy it in the first period at a cheaper price. Hence, waiting for period 2 gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma \mid 2)=s+\frac{\sigma}{2}-\rho p_{2, A}-(1-\rho) p_{2, B} \tag{1.26}
\end{equation*}
$$

Therefore, a consumer with choosiness $\sigma$ prefers waiting over purchasing product $A$ in the advance purchase period if and only if

$$
\begin{align*}
U(\sigma \mid 2) & \geq U(\sigma \mid 1, A) \\
\sigma & \geq \sigma_{W}(A) \equiv \frac{(1-\rho) p_{2, B}+\rho p_{2, A}-p_{1, A}}{1-\rho} \tag{1.27}
\end{align*}
$$

Analogously, a consumer with choosiness $\sigma$ prefers waiting over buying product $B$ in the advance purchase period if and only if

$$
\begin{align*}
U(\sigma \mid 2) & \geq U(\sigma \mid 1, B) \\
\sigma & \geq \sigma_{W}(B) \equiv \frac{\rho p_{2, A}+(1-\rho) p_{2, B}-p_{1, B}}{\rho} . \tag{1.28}
\end{align*}
$$

The thresholds $\sigma_{W}(A)$ and $\sigma_{W}(B)$ define which consumers are buying in advance and which are waiting for the consumption period. More precisely, a consumer of type $\sigma \in\left[0, \max \left\{\sigma_{W}(A), \sigma_{W}(B)\right\}\right)$ buys in the first period (independently if the received signal is $S=A$ or $S=B$ ). If $\sigma \in$ $\left[\max \left\{\sigma_{W}(A), \sigma_{W}(B)\right\}, 1\right]$ the consumer waits for the consumption period and


Figure 1.3: Consumers' purchase decisions with asymmetric prominence.
buys his revealed preferred product. In the following, we assume (and later confirm) that $\sigma_{W}(A)>\sigma_{W}(B)$.

Next, we determine the product choice for consumers who buy in the first period. Given the firms' heterogeneity, we expect products' prices to differ and we assume that $p_{1, A}>p_{1, B}$, which is confirmed in our results. A consumer buying in the first period therefore faces a tradeoff between buying the cheaper product $(B)$ and the product he is more likely to prefer $(A)$. Buying product $A$ is preferable if and only if the gain in consumption value is greater than the price difference, that is if and only if

$$
\begin{align*}
s+\rho \frac{\sigma}{2}-(1-\rho) \frac{\sigma}{2}-\left[s-\rho \frac{\sigma}{2}+(1-\rho) \frac{\sigma}{2}\right] & \geq p_{1, A}-p_{1, B} \\
\sigma & \geq \bar{\sigma} \equiv \frac{p_{1, A}-p_{1, B}}{2 \rho-1} . \tag{1.29}
\end{align*}
$$

We will show that in equilibrium it holds that $\bar{\sigma}<\sigma_{W}(A)$, which implies that consumers allocate as depicted in Figure 1.3 , consumers with $\sigma \in[0, \bar{\sigma})$ buy product $B$ in the advance selling period, consumers with $\sigma \in\left[\bar{\sigma}, \sigma_{W}(A)\right)$ buy product $A$ in the advance selling period, and consumers with $\sigma \in\left[\sigma_{W}(A), 1\right]$ buy their revealed preferred product in the consumption period.

The firms' profits are then given by

$$
\begin{align*}
& \Pi_{A}=p_{1, A}\left[\sigma_{W}(A)-\bar{\sigma}\right]+p_{2, A} \rho\left[1-\sigma_{W}(A)\right],  \tag{1.30}\\
& \Pi_{B}=p_{1, B} \bar{\sigma}+p_{2, B}(1-\rho)\left[1-\sigma_{W}(A)\right], \tag{1.31}
\end{align*}
$$

where $\sigma_{W}(A)$ and $\bar{\sigma}$ are given by (1.27) and (1.29) respectively. The corre-
sponding first-order conditions for firm $A$ are given by

$$
\begin{align*}
& 0=\sigma_{W}(A)-\bar{\sigma}-p_{1, A} \frac{\rho}{(1-\rho)(2 \rho-1)}+p_{2, A} \frac{\rho}{1-\rho},  \tag{1.32}\\
& 0=\rho\left(1-\sigma_{W}(A)\right)+\frac{\rho}{1-\rho}\left(p_{1, A}-\rho p_{2, A}\right) \tag{1.33}
\end{align*}
$$

and for firm $B$ are given by

$$
\begin{align*}
& 0=\bar{\sigma}-\frac{p_{1, B}}{2 \rho-1}  \tag{1.34}\\
& 0=(1-\rho)\left(1-\sigma_{W}(A)\right)-p_{2, B}(1-\rho) \tag{1.35}
\end{align*}
$$

Solving this system of equations leads to the unique equilibrium candidate $\sqrt[6]{6}$

$$
\begin{array}{ll}
p_{1, A}^{*}=\frac{2(2 \rho-1)}{3}, & p_{1, B}^{*}=\frac{2 \rho-1}{3} \\
p_{2, A}^{*}=\frac{3 \rho-1}{3 \rho}, & p_{2, B}^{*}=\frac{1}{3} \tag{1.37}
\end{array}
$$

Proposition 1.3. Suppose $\rho \in\left(\frac{1}{2}, 1\right)$. In a price discrimination equilibrium prices are given by (1.36) - 1.37) and the following holds:

1. Prices are lower than in the uniform pricing benchmark.
2. The prominent firm sets higher prices and offers a higher advance purchase discount in absolute terms, that is $p_{2, A}^{*}-p_{1, A}^{*}>p_{2, B}^{*}-p_{1, B}^{*}$. The less prominent firm offers a higher advance purchase discount in relative terms, that is $\frac{p_{2, B}^{*}-p_{1, B}^{*}}{p_{2, B}^{*}}>\frac{p_{2, A}^{*}-p_{1, A}^{*}}{p_{2, A}^{*}}$.

As in the cost asymmetry case, in the advance selling period the consumers' willingness to pay is lower due to uncertainty. This, together with the fact that firms use advance purchase discounts to "lock-in" less choosy consumers and the increase in competition, decrease prices relative to timeinvariant pricing.

[^5]As in the uniform pricing benchmark case, the prominent firm can afford to set higher prices. Firm $B$, on the other hand, sets lower prices to make up for its lack of prominence. In the advance selling period no consumers would buy from firm $B$ if prices would be the same or higher than those of firm $A$, because the probability of preferring product $B$ is lower than the one of preferring product $A$. The higher probability of preferring product $A$ is also the motive that incentives firm $B$ in shifting the competition into the advance selling stage when products are more homogeneous. Firm $B$ does so by offering a higher relative advance purchase discount.

We observe that, in contrast to the benchmark case, prices are increasing in prominence asymmetry. From the point of view of the consumers an increase in firm $A$ 's prominence corresponds to an increase in information in the advance selling period. Consider, for example, the limit case $\rho \rightarrow 1$. For $\rho \rightarrow 1$ consumers already know in the first period that with almost complete certainty product $A$ will be their favorite product, which increases their willingness to pay. An higher $\rho$ corresponds therefore to a higher ex-ante product heterogeneity, which for the firms translates into less competition and higher prices.

Corollary 1.3. Suppose $\rho \in\left(\frac{1}{2}, 1\right)$. Both firms sell the same amount of output in the advance selling period. The prominent firm sells a higher output in the consumption period and a higher total output. Market shares are unaffected by dynamic pricing.

Thanks to its higher relative discount the less prominent firm $B$ can make up for its lack of attractiveness in the advance selling period and sell the same output as firm $A$. Among those consumers who wait, the majority prefers product $A$, which is why firm $A$ sells a higher output in the consumption period resulting in a higher market share. Interestingly, advance selling only changes the consumers' individual allocation but it does not affect the firms' market shares with respect to the time-invariant pricing.

Corollary 1.4. Suppose $\rho \in\left(\frac{1}{2}, 1\right)$. With advance selling, the prominent firm A earns higher profits than its competitor. Price discrimination reduces the firms' profits, but more strongly for the prominent firm.

This result is an obvious consequence of Proposition 1.3 and Corollary 1.3. Firm $A$ can afford higher prices and sells a higher output, which lead to a higher profit. With respect to the time uniform pricing benchmark, the quantity sold is unaffected but prices are lower. Advance purchase discounts induce more competition which decreases the profits. The difference in the firms' profits, however, is less pronounced under advance selling. That is, the competition hits the prominent firm more strongly.

### 1.5 Discussion and Conclusion

In this chapter, we consider an oligopolistic market with individual demand uncertainty in which asymmetric firms offer advance purchase discounts. Firms either differ in their efficiency or in their prominence. Less choosy consumers buy in advance under uncertainty regarding their preferred product. For very low values of choosiness, they even buy their least favorite product if it is cheaper than their favorite one.

The effect of firms' heterogeneity on the advance purchase discounts and on advance sales depends on the type of asymmetry considered.

We find that when firms differ in their efficiency, the firm with lower marginal costs sets lower prices than its competitor. Prices are lower than in the uniform pricing benchmark and both firms' prices are affected in the same way by dynamic pricing, implying that firms offer the same advance purchase discount (in absolute value).

However, if we consider the consumers' expected discounts, consumers whose favorite product is the cheaper one face a higher discount, which incentivizes them to buy in the advance selling period. The more efficient firm profits more from advance selling in the sense that it attracts more consumers and it sells a higher output in the advance selling period than its competitor. The market share of the efficient firm is higher under price discrimination than with time-invariant pricing.

Price discrimination harms both firms since it increases competition and impacts both firms to the same extent.

When firms differ in their prominence, we show that prices are lower than in the uniform pricing benchmark, but firms are affected differently by dynamic pricing. The prominent firm sets higher prices, offers a higher advance purchase discount in absolute terms and a lower one in relative terms. Prices are lower than with uniform pricing, but increase in the prominence asymmetry. A higher prominence asymmetry reduces the consumers uncertainty in the advance selling period and makes the products appear more differentiated. This reduces the competition between the firms and leads to a price increase. In the limit, the prices in the price discrimination case converge to the benchmark equilibrium.

Thanks to its lower prices and more homogeneous products, the less prominent firm can make up for its lack of prominence and sell as much as the prominent firm in the advance selling period. In the consumption period, however, the prominent firm sells more since consumers who wait buy their preferred product. The prominent firm therefore sells also a higher total output. Market shares are unaffected by dynamic pricing.

Because of the higher market share and prices, the prominent firm earns a higher profit. The increase in competition due to advance purchase discounts reduces the profits with respect to the case with time-invariant pricing. However, the impact on the prominent firm is stronger resulting in a smaller profit difference under price discrimination.

This model highlights how dynamic pricing differently affects asymmetric firms depending on the source of heterogeneity. While differences in efficiency do not affect dynamic pricing other than the differences already present in the uniform pricing benchmark equilibrium, prominence asymmetry can explain differences in advance purchase discounts among firms (see Borenstein and Rose, 1994; Asplund et al., 2008; Gerardi and Shapiro, 2009). We observe the opposite with market shares: when firms differ in their prominence the total output sold is unaffected by advance selling, but it is affected when they differ in their efficiency.

This model can be adapted to study the effect of dynamic pricing on market entry. Evidence from the liberalization of the airline market shows an increase in intertemporal price discrimination practices, such as advance pur-
chase discounts, following market deregulation (Borenstein and Rose, 2014). This model can bring new insights on this topic by considering a prominent incumbent facing a more efficient entrant.

## Chapter 2

## Entry in Advance Purchase Markets

Evidence from the Italian Railway Market

### 2.1 Introduction

In the past years, there has been an increase in price dispersion after the deregulation in the airline industry. In the United States, following the 1978 Airline Deregulation Act, the amount of discounted fares, such as advance purchase discounts or minimum stay requirements, increased (Borenstein and Rose, 2014). Deregulation also promoted the entry of Low Cost Carriers (LCCs), which changed the airline market's structure and pricing systems. In particular, while Full Service Carriers (FSCs) used to price discriminate by offering seats with different quality (e.g., Economy vs. Business Class), LCCs rely on intertemporal price discrimination: they provide a basic quality flight, but lower fares if tickets are bought days or weeks before departure. Nowadays, advance purchase discounts are a widespread pricing strategy also among FSCs. ${ }^{1}$

[^6]Advance purchase discounts are a form of price discrimination that is also present in other markets. Well-known examples are early bird tickets for concerts and festivals sold at a reduced price several months before the event occurs. Some types of train tickets can also be purchased with a discount if booked in advance. Other examples are hotels offering refundable and non-refundable rooms or pre-orders of new and not yet launched products. What characterizes these types of markets is that buying in advance entails some degree of uncertainty. For example, at the time of an early purchase, buyers might not be sure that they will be able to attend at the date of the event. Thanks to this pricing strategy, firms can separate consumers who are more flexible and have a lower willingness to pay from less flexible consumers with a higher willingness to pay. The former prefer to buy in advance at a discounted rate, the latter are willing to pay a higher price in exchange for more flexibility.

Price discrimination is often under scrutiny when there is a monopolist or a prominent firm in the market. One of the worries is that price discrimination can be used as an anti-competition instrument to prevent other firms' entry into the market. The airline market example contradicts this worry since LCCs have been promoters of intertemporal price discrimination.

The main question that this chapter wants to answer is therefore: can price discrimination promote entry in markets with advance selling? We also address other issues such as what role discounts play in the competition between an entrant and an incumbent and the influence of different degrees of asymmetry between the firms. Additionally, we provide empirical evidence for our results.

To answer these questions, we adapt the model presented in Chapter 1 and allow the firms to differ in both marginal cost and prominence. More precisely, we assume that the entrant is more efficient than the incumbent and that the incumbent is more prominent than the entrant. We focus on the special case where the signal is uninformative as prominence asymmetry assures heterogeneity also in the advance selling period. To simplify the model, we equivalently assume that there is no signal.

In the theoretical analysis, we find that the entrant sets lower advance
selling and consumption period prices, a lower absolute advance purchase discount, and a higher relative advance purchase discount than the incumbent. This result follows mainly from the prominence asymmetry. Uncertainty in the first period diminishes the difference in the expected gross values of the two goods. The entrant then has an incentive to move the competition in the advance selling stage where its prominence disadvantage is less evident. Consequently, the entrant provides a low advance selling price. Its efficiency advantage also helps in using advance purchase discounts as a competition instrument.

Moreover, we find that intertemporal price discrimination increases the entrant's market share. Thanks to advance purchase discounts, the entrant sells a higher total output than with uniform pricing. For certain parameter values, the entrant even has a higher market share than the incumbent. The competition in the advance selling period, however, erodes profit margins, so the gain in output sold comes at the cost of lower short term profits.

Regulations on advance selling usually focus on consumer protection. For example, in the United States, passengers have the right to a refund for all airline tickets within 24 hours of booking 2 while in Israel, this right is valid for 14 days $3^{3}$ Although aimed at the consumers, these policies might affect the market structure. For example, if the entrant cannot set its price freely, it might achieve a lower market share than with advance selling. In that case a less efficient incumbent would continue serving the majority of the market.

Furthermore, we propose an empirical example using data on Italian railway ticket pricing. The railway companies Trenitalia and Italo match our set-up of incumbent and entrant particularly well. We analyze advance selling and consumption prices, as well as advance purchase discounts. We observe that the less prominent entrant Italo offers lower prices, lower absolute advance purchase discounts and higher relative discount than its prominent competitor Trenitalia. We therefore find promising evidence for our theoretical results.

This chapter is structured as follows: Section 2.2 introduces the model.

[^7]In Section 2.3 we solve a benchmark case of the model without price discrimination. In Section 2.4 we solve the model with price discrimination, and in Section 2.5 we discuss the effect on entry profitability. In Section 2.6 we present the empirical application. Section 2.7 concludes. Proofs can be found in Appendix B.

### 2.1.1 Related Literature

Whether price discrimination can act as an entry deterrent or enhances competition has been mostly analyzed for two forms of price discrimination.

The first group of literature considers the form of tying. In this set-up, there are two markets, each with a different product. The incumbent, active in both markets, can sell the two products as a bundle. This strategy can prevent the entry of new firms in one (Whinston, 1990) or both markets (Choi and Stefanadis, 2001; Carlton and Waldman, 2002).

The second form of price discrimination is of third-degree. In Armstrong and Vickers (1993), the incumbent operates in two (identical) markets and faces entry in one of the two. Price discrimination here is considered as the incumbent setting different prices in different markets. When this is not allowed, entry occurs more frequently. Cheung and Wang (1999), however, find that in this set-up whether price discrimination enhances or discourages entry depends on the relative elasticity of demand in the two markets.

Bouckaert et al. (2013) expand the model of Armstrong and Vickers (1993) by considering price discrimination not only across markets but also within markets. The two types of price discrimination have opposite effects on competition. The authors find that banning (any type of) price discrimination encourages entry only if the market in which the incumbent is a monopolist is large enough.

Another type of price discrimination is considered by Gehrig et al. (2011). In their set-up, the incumbent can implement switching costs. They find that the entrant's profit, and thus its decision to enter the market, is independent on the incumbent's choice regarding price discrimination.

We add to this literature by considering a second-degree type of price
discrimination in markets with individual demand uncertainty.
Our chapter contributes also to the literature on advance selling in the presence of individual demand uncertainty. Advance purchase discounts have been shown to be profitable for monopolies (Gale and Holmes, 1993; Nocke et al., 2011; Möller and Watanabe, 2010) and for perfectly competitive markets (Dana, Jr., 1998; Gale, 1993).

As we consider a market in which an entrant just joined the market, our set-up is the one of an oligopolistic market and therefore close to Möller and Watanabe (2016). In their model, two symmetric firms offer two differentiated products. Selling in advance at a lower price is profitable because it allows the firms to steal consumers from their competitor. Karle and Möller (2020) confirm the profitability of advance purchase discounts in a similar set-up with loss averse consumers. By considering a prominent incumbent versus a less prominent entrant, we expand this literature to asymmetric firms.

The few papers that allow for asymmetry mainly focus on price discrimination of the third-degree and its effect on consumer welfare. Chen (2008) studies the effect of history-based price discrimination over multiple time periods on the consumers' welfare. He shows that if the weaker firm stays in the market, consumers can benefit from price discrimination. Carroni (2016) considers customer poaching with asymmetric firms. The author shows that asymmetry has an important effect on pricing and, consequentially, on the consumers' behavior. Consumers who, in this case, are harmed by price discrimination. Both Chen (2008) and Carroni (2016) suggest that price discrimination might cause the smaller firm's exit. In our model, we find the opposite; the less prominent firm (the entrant) is the one that can have an advantage under the ability of price discriminating.

Also the empirical literature has focused on the correlation between asymmetry and price discrimination. However, results have, so far, been conflicting. Gerardi and Shapiro (2009) find that price discrimination increases with market share, while Borenstein and Rose (1994) and Asplund et al. (2008) find opposite evidence.

The difference between these papers is that Gerardi and Shapiro (2009)
also find a negative correlation between price discrimination and competition, implying that price discrimination results from the firms' exploitation of consumers' heterogeneous elasticities of demand. On the contrary, Borenstein and Rose (1994) and Asplund et al. (2008) find a positive correlation between price discrimination and competition, signaling that price discrimination is used as a competition instrument. In particular, Asplund et al. (2008) claim that small newspapers are more prone to offer discounts to new readers than their larger competitors. Our model is in line with this last interpretation since, in a duopoly, advance purchase discounts are used to steal consumers from the competitor Möller and Watanabe, 2016).

In our model we assume individual demand uncertainty, a feature that benefits the entrant more than the incumbent, as in the advance selling period the less prominent firm takes advantage of the fact that products look more similar than with perfect information. Therefore an important benchmark is the case without demand uncertainty. A comparable literature would be the one on competition with multidimensional screening. Rochet and Stole (2002) and Armstrong and Vickers (2001) consider symmetric duopolies in which consumers differ in their horizontal as well as vertical preferences. They show that if the two dimensions are independent and the market is covered, then firms do not price discriminate using different quality levels.

### 2.2 Model

We adopt the model presented in Chapter 1 and consider the following parametrization: $c_{A}=c, c_{B}=0, \rho>\frac{1}{2}$, and $\gamma=\left.\frac{1}{2}\right|^{4}$ For the sake of completeness, we include a description of the model in this section.

We consider a market in which two firms, an incumbent $I$ and an entrant $E$, sell two differentiated products $i \in\{I, E\}$. Consumers can purchase one of the products in two periods: the advance purchase period $(t=1)$ and the consumption period $(t=2)$. Both firms are active in the market in both

[^8]periods. Therefore we consider a situation in which the entrant just joined the market.

Prices are established in advance, that is, each firm $i \in\{I, E\}$ commits to a price schedule $\left(p_{i}, P_{i}\right) \in \mathbb{R}_{+}^{2}$ prior to the sales periods. $p_{i}$ is the price for good $i \in\{I, E\}$ in the advance purchase period and $P_{i}$ is the price for good $i \in\{I, E\}$ in the consumption period.

Firm $I$ has unit production costs equal to $c$, with $\frac{1}{2}>c>0.5$ and firm $E$ has unit production costs equal to 0 , meaning that we let the entrant be more efficient than the incumbent. To simplify our analysis we abstract from discounting of future payoffs.

We consider a unit mass of consumers with unit demand. Consumers are characterized by their choosiness $\sigma \in[0,1]$, which is private information of the consumer. The utility that consumer $\sigma$ obtains from his preferred product is given by $s+\frac{\sigma}{2}$, while the utility he obtains from his least preferred product is given by $s-\frac{\sigma}{2}$. Therefore, for more choosy consumers the difference in the products' characteristics weighs more heavily. We assume that $\sigma$ follows a uniform distribution over $[0,1]$. We let the incumbent be more prominent, in the sense that a fraction $\rho \in\left(\frac{1}{2}, 1\right)$ of consumers prefers, net of prices, product $I$ and a fraction $1-\rho$ prefers, net of prices, product $E$.

The key difference between the advance purchase period and the consumption period is the uncertainty in the consumers' preferences over the two products. Consumers are aware of their choosiness $\sigma$, but in the first period, they don't have any information about which one could be their preferred product other than their knowledge of the distribution $\rho$ of aggregate preferences. The preferred product is then revealed in the consumption period.

We further assume that the parameters are such that each consumer buys exactly one of the two products, and each consumer consumes the product he purchased, even if he bought his least preferred one. In the case of indifference between buying in the advance purchase period or the consumption period,

[^9]consumers purchase the product in the second one.

### 2.3 Benchmark Case: Uniform Pricing

We first look at the case in which price discrimination is ruled out, that is $p_{i}=P_{i}$, for $i \in\{I, E\}$. This later allows us to understand better the effect of advance purchase discounts on entry.

When price discrimination is not allowed, given that there is no discount and a firm's price is the same in both periods, consumers always wait and buy in the second period. Purchasing in advance might result in buying the least preferred product and has no benefit from a reduced price. The problem is similar to the standard Hotelling model, with the difference that firms differ in their cost efficiency and in their prominence.

We assume that the incumbent sets a higher price than the entrant $\left(P_{I}>\right.$ $\left.\left.P_{E}\right)\right]^{6}$ A consumer whose preferred product is $E$ always buys product $E$, since it is cheaper. A consumer with choosiness $\sigma$, whose preferred product is $I$, buys product $I$ if and only if

$$
\begin{align*}
s+\frac{\sigma}{2}-P_{I} & \geq s-\frac{\sigma}{2}-P_{E} \\
\sigma & \geq \hat{\sigma} \equiv P_{I}-P_{E} . \tag{2.1}
\end{align*}
$$

Therefore, the fraction of consumers who buys from the incumbent is given by $\rho(1-\hat{\sigma})$. The entrant sells to the consumers' fraction who prefers product $E$, which is given by $1-\rho$, and to those consumers who prefer product $I$ and have choosiness level lower than $\hat{\sigma}$, which are given by $\rho \hat{\sigma}$.

Thus, the firms' profits are given by

$$
\begin{align*}
\Pi_{I} & =\left(P_{I}-c\right) \rho(1-\hat{\sigma})  \tag{2.2}\\
\Pi_{E} & =P_{E}(1-\rho+\rho \hat{\sigma}) \tag{2.3}
\end{align*}
$$

[^10]Solving the system of equations generated by the first-order conditions yields the following egulibrium prices

$$
\begin{align*}
P_{I}^{U} & =\frac{2}{3} c+\frac{1+\rho}{3 \rho}  \tag{2.4}\\
P_{E}^{U} & =\frac{1}{3} c+\frac{2-\rho}{3 \rho} \tag{2.5}
\end{align*}
$$

and profits

$$
\begin{align*}
& \Pi_{I}^{U}=\frac{(1+(1-c) \rho)^{2}}{9 \rho}  \tag{2.6}\\
& \Pi_{E}^{U}=\frac{(2-(1-c) \rho)^{2}}{9 \rho} \tag{2.7}
\end{align*}
$$

Lemma 2.1. With uniform pricing the entrant sets lower prices than the incumbent. If $\rho>\frac{1}{2(1-c)}$, then the incumbent's market share and profit are higher than the entrant's. Higher cost asymmetry increases prices for both firms, increases the entrant's profit and decreases the incumbent's profit. Higher prominence asymmetry decreases prices and profits for both firms.

The price difference among the two firms is motivated by both types of asymmetry: Higher marginal costs obviously increase the incumbent's price more than the entrant's. The incumbent's prominence means that more consumers prefer product $I$ over $E$, hence the incumbent can afford to set a higher price, which comes at the cost of losing part of the demand, but with the benefit of higher gains from those consumers who still buy its product.

If the incumbent's prominence advantage is high enough compared to the cost asymmetry, then its market share is higher than the entrant's. Moreover, the difference in market shares increases with $\rho$.

A higher cost asymmetry $c$ means that the incumbent has to set a higher price to cover its production costs. The effect of a cost increase on the incumbent's profit is clearly negative. Given the incumbent's price increase, the entrant can then also afford to set a higher price and gain higher margins, which increases its profit.

An increase $\rho$ leads to a decrease in prices for both firms. We can think of $\rho$ as the size of the contested market: Because of its cost advantage, the
entrant not only serves its "home market" of size $1-\rho$ but also attracts customers from the incumbent's market. As the fraction of consumers who prefer product $I$ increases, the competition to attract them increases as well and leads firms to lower their prices. Higher competition reduces profits for both firms.

### 2.4 Advance Selling Equilibrium

In this section, we solve the firms' profit maximization problem to find the equilibrium prices. We later use our findings to discuss the profitability of advance purchase discounts for entry and to formulate the hypotheses to be tested empirically.

### 2.4.1 Analysis

Firm $i \in\{I, E\}$ sells product $i$ and chooses a price schedule $\left(p_{i}, P_{i}\right)$ that maximizes its profits. It must hold $P_{i} \geq p_{i}$ for $i \in\{I, E\}$, otherwise, given the uncertainty in utility in period 1 , all consumers would wait until the second period and buy their preferred product at a cheaper price. Firms could therefore increase $p_{i}$ until $p_{i}=P_{i}$ without influencing their profits.

To define the profits of the two firms and derive the optimal price schedules, we first need to establish which consumers are buying from which firm and in which period.

Consider a consumer thinking of purchasing product $I$ in the first period. The probability of preferring product $I$ is given by $\rho$, while the probability of preferring product $E$ is given by $1-\rho$. Purchasing product $I$ in advance gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma \mid 1, I)=s+\rho \frac{\sigma}{2}-(1-\rho) \frac{\sigma}{2}-p_{I} . \tag{2.8}
\end{equation*}
$$

We assume that if the consumer waits, he then buys his preferred product. Given that the price schedule is fixed, if he knows that he is going to buy a certain product no matter what his revealed preference is, he will buy it in the first period at a lower price. Our equilibrium values agree with
this assumption. Waiting for the consumption period gives the consumer an expected utility of

$$
\begin{equation*}
U(\sigma \mid 2)=s+\frac{\sigma}{2}-\rho P_{I}-(1-\rho) P_{E} . \tag{2.9}
\end{equation*}
$$

Therefore, a consumer with choosiness $\sigma$ prefers waiting over buying product $I$ in the first period if and only if

$$
\begin{align*}
U(\sigma \mid 2) & \geq U(\sigma \mid 1, I) \\
\sigma & \geq \sigma_{W}(I) \equiv P_{E}+\frac{\rho P_{I}-p_{I}}{1-\rho} \tag{2.10}
\end{align*}
$$

Analogously, the consumer prefers waiting instead of buying product $E$ in the first period if and only if

$$
\begin{align*}
U(\sigma \mid 2) & \geq U(\sigma \mid 1, E) \\
\sigma & \geq \sigma_{W}(E) \equiv P_{I}+\frac{(1-\rho) P_{E}-p_{E}}{\rho} . \tag{2.11}
\end{align*}
$$

The thresholds $\sigma_{W}(I)$ and $\sigma_{W}(E)$ define which consumers are buying in advance and which are waiting for the consumption period. More precisely, a consumer of type $\sigma$ buys in the first period if and only if $\sigma \in$ $\left[0, \max \left\{\sigma_{W}(I), \sigma_{W}(E)\right\}\right)$, if $\sigma \in\left[\max \left\{\sigma_{W}(I), \sigma_{W}(E)\right\}, 1\right]$ the consumer waits for the second period and buys the preferred product. We assume from now on that $\sigma_{W}(E)<\sigma_{W}(I)$. In equilibrium this condition is satisfied.

Next, we establish among the consumers who buy in the advance selling period, which product they purchase. Given the firms' heterogeneity in marginal costs and prominence, we expect products' prices to differ. We assume (and later confirm) that $p_{I}>p_{E} \square^{7} \mathrm{~A}$ consumer buying in the first period therefore faces a tradeoff between buying a product that will more likely be his preferred one (product $I$ ) and buying a cheaper product (product $E$ ). Buying product $I$ is preferable if the gain in consumption value is

[^11]

Figure 2.1: Consumers' purchase decisions for $p_{I}>p_{E}$.
greater than the price difference, that is if and only if

$$
\begin{align*}
s+\rho \frac{\sigma}{2}-(1-\rho) \frac{\sigma}{2}-\left[s+(1-\rho) \frac{\sigma}{2}-\rho \frac{\sigma}{2}\right] & \geq p_{I}-p_{E} \\
\sigma & \geq \bar{\sigma} \tag{2.12}
\end{align*}
$$

We focus on interior solutions, meaning that both firms sell strictly positive quantities in both period. For this reason we assume that $\bar{\sigma}<\sigma_{W}(I)$. This assumption is satisfied for appropriate parameter values, as we discuss in more detail in Section 2.4.4.

Therefore, for $p_{E}<p_{I}$ consumers allocate as follows: Consumers with $\sigma \in[0, \bar{\sigma})$ buy product $E$ in the advance selling period. Consumers with $\sigma \in\left[\bar{\sigma}, \sigma_{W}(I)\right)$ buy product $I$ in the advance selling period. Consumers with $\sigma \in\left[\sigma_{W}(I), 1\right]$ wait and buy the preferred product in the second period. This distribution is depicted in Figure 2.1.

The firms' profit are thus given by

$$
\begin{align*}
\Pi_{I} & =\left(p_{I}-c\right)\left(\sigma_{W}(I)-\bar{\sigma}\right)+\left(P_{I}-c\right) \rho\left(1-\sigma_{W}(I)\right)  \tag{2.13}\\
\Pi_{E} & =p_{E} \bar{\sigma}+P_{E}(1-\rho)\left(1-\sigma_{W}(I)\right) \tag{2.14}
\end{align*}
$$

We focus on price discrimination equilibria, that is profit-maximizing price systems $\left\{\left(p_{I}^{*}, P_{I}^{*}\right),\left(p_{E}^{*}, P_{E}^{*}\right)\right\}$ such that $P_{i}^{*}>p_{i}^{*}$, for $i \in\{I, E\}$, with both firms selling strictly positive quantities in both periods.

The maximization problem yields the following first-order conditions for firm $I$

$$
\begin{align*}
& 0=\sigma_{W}(I)-\bar{\sigma}-\left(p_{I}-c\right) \frac{\rho}{(1-\rho)(2 \rho-1)}+\left(P_{I}-c\right) \frac{\rho}{1-\rho},  \tag{2.15}\\
& 0=\rho\left(1-\sigma_{W}(I)\right)+\frac{\rho}{1-\rho}\left[p_{I}-c-\rho\left(P_{I}-c\right)\right], \tag{2.16}
\end{align*}
$$

and the following for firm $E$

$$
\begin{align*}
& 0=\bar{\sigma}-\frac{p_{E}}{2 \rho-1},  \tag{2.17}\\
& 0=(1-\rho)\left(1-\sigma_{W}(I)\right)-P_{E}(1-\rho), \tag{2.18}
\end{align*}
$$

where $\sigma_{W}(I)$ and $\bar{\sigma}$ are given by $(2.10)$ and (2.12) respectively.

### 2.4.2 Equilibrium

From the necessary conditions (2.15)-(2.18) we obtain the following unique closed-form solution for $\left\{\left(p_{I}^{*}, P_{I}^{*}\right),\left(p_{E}^{*}, P_{E}^{*}\right)\right\}$

$$
\begin{array}{ll}
p_{I}^{*}=P_{I}^{U}-\frac{(1-\rho)(1+4 \rho)}{3 \rho}, & P_{I}^{*}=P_{I}^{U}-\frac{2(1-\rho)}{3 \rho} \\
p_{E}^{*}=P_{E}^{U}-\frac{2\left(1-\rho^{2}\right)}{3 \rho}, & P_{E}^{*}=P_{E}^{U}-\frac{2(1-\rho)}{3 \rho} \tag{2.20}
\end{array}
$$

where $P_{I}^{U}$ and $P_{E}^{U}$ are the equilibrium prices in the benchmark case without price discrimination. Because of asymmetric prices, an equilibrium in which both firms sell in both periods only exists for certain parameters. Later we discuss these conditions in more detail. In the following figures, functions are depicted only for those parameter values that allow for a price discrimination equilibrium. Figure 2.2 shows the equilibrium prices as functions of $\rho$ for $c=0.05$.

We first notice that, as expected, both firms offer an advance purchase discount, that is advance selling prices are lower than consumption period prices, and that in both periods prices are lower than the benchmark's equilibrium prices. As in the advance selling period consumers face uncertainty, their willingness to pay is lower than with perfect information. Consequently, prices must be lower in the advance selling period than in the consumption period. Advance purchase discounts are also an instrument to "lock-in" consumers before true preferences are revealed. However, if the discount is too high, consumers with higher choosiness would also buy in the advance selling period. Firms therefore decrease second period prices to exploit consumers' higher willingness to pay for their preferred product.


Figure 2.2: Incumbent's (black) and entrant's (grey) equilibrium prices (solid) compared to the uniform pricing benchmark (dashed) for $c=0.05$.

Moreover, we observe that the effect of efficiency asymmetry on the equilibrium prices is the same with and without price discrimination. That is, prominence plays a key role for the firms' different strategies with time variant pricing.

In the Appendix we prove the following.
Proposition 2.1. In the price discrimination equilibrium the entrant sets a lower advance selling price ( $p_{i}^{*}$ ), a lower consumption period price ( $P_{i}^{*}$ ), a lower absolute advance purchase discount $\left(P_{i}^{*}-p_{i}^{*}\right)$, and a higher relative advance purchase discount $\left(\frac{P_{i}^{*}-p_{i}^{*}}{P_{i}^{*}}\right)$ than the incumbent.

To understand this result, we look at the effect of efficiency and prominence asymmetry separately. As with uniform pricing, the difference in marginal costs makes the entrant's advance selling price $p_{E}^{*}$ and the consumption period price $P_{E}^{*}$ smaller than those of the incumbent. Intuitively, the firm with lower marginal costs can afford lower prices and consequentially attract more consumers. However, the effect of marginal costs is merely a shift on the overall price level, and, for a given firm, the shift is the same in both periods. Hence, the advance purchase discount is not affected by the efficiency asymmetry in absolute terms, but it is in relative terms: given
that asymmetrical efficiency lowers the entrant's prices, it also increases the relative advance purchase discount.

Prominence increases the incumbent's advance selling and consumption period price: as in the case without price discrimination, the firm with a higher consumer base can afford to set higher prices, as the gain from nonmarginal consumers is higher than the loss from reduced demand. With price discrimination, both firms decrease their prices because of the increase in uncertainty and competition. The imperfect information in the first period, however, harms the incumbent more than the entrant: With uncertainty the two product are closer in their expected value (net of prices) and the incumbent's prominence advantage is reduced. Therefore the incumbent needs to decrease its price further than the entrant, with respect to the uniform pricing case. This results in the incumbent offering a higher advance purchase discount in absolute level. However, because of the entrant's lower consumption period price, the entrant's relative advance purchase discount is higher than the incumbent's.

Hence, both types of asymmetry work in the same direction, decreasing the entrant's advance selling and consumption period price and increasing its relative advance purchase discount. However, cost asymmetry affects prices in both periods equally, while the effect of prominence differs and generates different absolute advance purchase discounts for the two firms.

We observe how the firms differ in their trade-off between profit margins and quantity sold in the advance selling period. For several parameter constellations, the entrant gains lower profit margins and sells a higher quantity than the incumbent ${ }^{8}$ Figure 2.3 shows this for $c=0.05$. In the consumption period, the incumbent is advantaged in both senses. Compared to the entrant, the incumbent earns higher marginal profits and sells a higher output. In particular, the quantity sold by the entrant is minimal. There are two reasons for this: first, most consumers buy in the first period attracted by the lower prices. Second, as mentioned in the analysis, if a consumer waits,

[^12]he buys his preferred product, which, due to the incumbent's prominence, is with higher probability product $I$. Figure 2.4 depicts the profit margins and the quantity sold in the second period for $c=0.05$.

Regarding total quantity sold, the incumbent has a higher market share if $\rho>\frac{3 c+\sqrt{c(9 c+8)}+2}{4(c+1)}$. This threshold is higher than the correspondent one in the uniform pricing case (see Lemma 2.1), meaning that advance purchase discounts favor the entrant in terms of market share. We discuss this in further detail in Section 2.5.

### 2.4.3 Comparative Statics

We now turn to the effect of both types of asymmetry, marginal cost $c$ and prominence $\rho$, on the equilibrium prices and advance purchase discounts.

Advance selling and consumption period prices increase with efficiency asymmetry $c$. The effect for the incumbent is straightforward: As production costs increase, a higher price is required to cover them. This relieves competitive pressure on the entrant, who can in turn also set a higher price.

Relative discounts decrease with efficiency asymmetry $c$ for both firms, which follows from the increase in equilibrium prices. For a given firm, a higher $c$ increases advance selling prices and consumption period prices at the same rate, meaning that in absolute terms, the discount is not affected by $c$, however, because of the higher prices, the relative discount decreases.

A higher prominence asymmetry $\rho$ leads to an increase in advance selling prices for both firms. Intuitively, we can interpret an increase in $\rho$ as an increase in the consumers' level of information in the first period. When $\rho \rightarrow 1$, for example, consumers know already in the first period that, with almost complete certainty, $I$ is their preferred product. This increases their willingness to pay, and firms increase their advance selling prices.

Absolute and relative discounts are affected in a similar way. An increase in $\rho$ has indeed the following two effects: first, as we mentioned, advance selling prices increase with $\rho$. This directly decreases the discounts of both firms. Intuitively, if the consumers' uncertainty level drops, they also require a lower discount to be convinced of purchasing the product in the advance

Figure 2.4: Profit margin and quantity sold in the
consumption period for $c=0.05$.

Figure 2.3: Profit margin and quantity sold in the
advance purchase period for $c=0.05$.
selling period. The second effect is that a higher $\rho$ means that the firms are more asymmetric and their products are perceived as less homogeneous, which lowers the competitive pressure. The need for an advance purchase discount as a competition instrument consequentially weakens. Both effects work in the same direction, and therefore, absolute as well as relative discounts decrease with $\rho$.

The incumbent's consumption period price increases with $\rho$, while the entrant's price is unaffected. The incumbent's discount strategy can affect the consumers' purchase timing decision through the threshold $\sigma_{W}(I)$. If the competition in the first period decreases (because of more heterogeneous products), the incumbent can afford to set a higher consumption period price, without shifting the demand too much in the advance selling period. Hence, as $p_{I}^{*}$ increases, $P_{I}^{*}$ increases as well. The entrant pricing decisions in the first and second period are independent from each other (see equations (2.17) and (2.18). Hence, the decrease in competition in the advance selling period, does not affect its consumption period price.

It is worth noting that, as we mentioned, with $\rho \rightarrow 1$ consumers have almost perfect information about their preferred product already in the advance selling period. Therefore, the situation is similar to the uniform pricing benchmark in which all consumers wait and make an informed purchase in the consumption period. The price discrimination equilibrium is then similar to the benchmark one. In particular, advance selling prices $p_{I}^{*}$ and $p_{E}^{*}$ converge to $P_{I}^{U}$ and $P_{E}^{U}$. Advance purchase discounts are no longer used to lock-in consumers in the first period, so they converge to zero. Consumers who, in the advance selling period, buy product $E$, do so knowing that they are buying their least preferred product, so the threshold $\bar{\sigma}$ in the price discrimination equilibrium converge to the threshold $\hat{\sigma}$ in the benchmark equilibrium. It follows that the total quantities sold in the price discrimination equilibrium by the incumbent and the entrant also converge to the respective quantities in the benchmark case.

### 2.4.4 Equilibrium Existence

The firms' price differences and the advance purchase discounts influence the consumers' product choice as well as the time of purchase choice. It also might be the case that an interior solution does not exist because too many or too little consumers want to buy a certain product in a certain period. The next proposition pins down the necessary conditions for the existence of a price discrimination equilibrium. ${ }^{9}$

Proposition 2.2. A necessary condition for the existence of a price discrimination equilibrium is

$$
\rho>\frac{1}{2(1-c)} .
$$

The proof can be found in the Appendix. Suppose the prominence asymmetry is low compared to the cost asymmetry. In that case, no consumer is willing to buy the incumbent's product in the first period, that is, $\sigma_{W}(I)-\bar{\sigma}>0$ might be violated. Hence, this inequality determines the parameters' condition in Proposition 2.2. A lower $\rho$ means that the two products are perceived as more homogeneous in the advance selling period, so the price difference generated by the marginal costs makes the entrant's product more attractive. That is, the threshold $\bar{\sigma}$ tends to be higher. A higher cost asymmetry decreases the threshold $\sigma_{W}(I)$ : a higher $c$ increases the incumbent's prices more than the entrant's, so consumers are more likely to wait than to buy $I$ in the advance selling period. These two effects combined might result in the incumbent not selling in the first period, especially if $\rho$ is low compared to $c$.

### 2.5 Entry with Advance Selling

In this section we address the question whether advance purchase discounts can promote entry. Policymakers usually tend to forbid price discrimination as it would make it more difficult for new firms to join the market. We want to find whether with advance selling this is also the case.

[^13]

Figure 2.5: Entrant's equilibrium profit with and without advance selling for $c=0.05$. Price discrimination reduces the entrant's profits.

We focus on the entrant and look at the effect of advance selling on profits and market share. In the Appendix we prove the following.

Proposition 2.3. If an equilibrium in the market with advance purchase discounts exists, then price discrimination decreases the entrant's profit.

Figure 2.5 shows the negative impact of price discrimination on the entrant's profit. If we consider the profit of the entrant as the maximum willingness to pay to join the market, advance selling discourages entry and the effect is stronger for lower values of $\rho$.

The introduction of advance purchase discounts lowers the equilibrium prices and decreases the profit margins. The lower the prominence asymmetry, the more similar the two firms are perceived in terms of expected value (net of prices). Firms then conduct an aggressive advance selling strategy, which erodes their margins. Moreover, with advance purchase discounts, the majority of the consumers buy in the first period when prices are even lower.

For higher prominence asymmetry, the price difference with and without discrimination decreases, and so does the profit difference. First, an increase in $\rho$ leads to an increase in consumers' certainty in the advance selling period, which increases their willingness to pay. Second, as firms are perceived as less


Figure 2.6: Entrant's market share with and without advance selling for $c=0.05$. Price discrimination increases the output sold by the entrant.
homogeneous, the competition between the two in the advance selling period weakens, allowing for higher first-period prices. When price discrimination is not allowed, on the contrary, an increase in $\rho$ leads to a decrease in $P_{E}^{U}$. For $\rho \rightarrow 1$ profits with advance purchase discount and uniform pricing converge.

On the other hand advance selling has a positive effect on the entrant's total output sold as stated in the following proposition, that is proved in the Appendix. Figure 2.6 shows the entrant's market share with advance selling and with uniform pricing.

Proposition 2.4. If an equilibrium in the market with advance purchase discounts exists, then price discrimination increases the entrant's market share.

As previous work shows, competition with advance selling decreases the firms' prices and shifts sales to the first period (Möller and Watanabe, 2016). This goes to the benefit of the entrant, which wouldn't serve many consumers in the second period and, thanks to its efficiency advantage, can offer a low price and attract a high share of consumers in the first one. The advantage is such that the entrant can make up for its lack of prominence in terms of quantity sold and sell in total a higher output than with uniform pricing. For certain parameter values, it can even achieve a higher market share than
the incumbent.
Summarizing, even though advance selling decreases its profit, it increases the total quantity sold by the entrant compared to uniform prices. Hence, if the increase in market share is profitable for future profits, advance purchase discounts can be a valuable entry strategy.

### 2.6 Empirical Analysis

This section provides an example of data about ticket pricing of railway companies that matches our theoretical results. We consider the Italian railway companies Trenitalia and Italo as the incumbent and the entrant, and test the predictions of our theoretical model about the pricing and the advance purchase discounts.

The case of these two firms suits our purposes because of three reasons: (i) the firms operate on the same routes, i.e. they provide a similar good, (ii) one firm clearly has a larger market share, and (iii) advance purchase discounts are a wide-spread pricing strategy for railway companies.

### 2.6.1 The Italian Railway Market - The Dataset

The Italian railway market was liberalized in the early 2000s. Trenitalia was founded in 2000 following the liberalization of the market as a subsidiary of the Ferrovie dello Stato Italiane, a state-owned holding. Today, Trenitalia is still the leading train operator in Italy.

Italo - Nuovo Trasporto Viaggiatori (NTV) (in short Italo) is a privateowned operator that focuses on high-speed connections, and it has been active on the Italian railway market since 2012. According to their estimations, Italo had a market share of $35 \%$ on the routes on which it operates in $2017 \sqrt{10}$

We analyze the two companies' pricing systems for high-speed connections on three routes: Milan-Bologna, Padua-Bologna, and Turin-Milan $\sqrt{11}$

[^14]Since all the destinations are located in Northern-Italy, we assume that the economic environment is similar on the three routes.

Data was collected through web scrapping and obtained by the booking site www.thetrainline.com. The data extraction took place once a week, always on Thursday, over three months, from November 2019 to February 2020. For each date, all daily connections were considered. For each train ride, the ticket price was collected one, eight, fifteen, and twenty-two days before departure. The collected prices are for non-refundable tickets.

|  | Trenitalia | Italo | Tot. |
| :--- | :---: | :---: | :---: |
| Milan-Bologna | 753 | 269 | 1022 |
|  | $(73.68)$ | $(26.32)$ |  |
|  | 326 | 150 | 476 |
|  | $(68.49)$ | $(31.51)$ |  |
| Turin-Milan | 443 | 194 | 637 |
|  | $(69.54)$ | $(30.46)$ |  |
| rush-hour | 600 | 234 | 834 |
|  | $(71.94)$ | $(28.06)$ |  |
| off-peak | 922 | 379 | 1301 |
|  | $(70.87)$ | $(29.13)$ |  |

Table 2.1: Descriptive statistics on the number of connections (row percentages in parenthesis).

Table 2.1 provides descriptive statistics about the total number of connections in our dataset provided by Trenitalia and Italo on the three different routes as well as the number of trains during rush- and off-peak-hours. The share of connections offered by Italo is very similar for Padua-Bologna and Turin-Milan, and it is a bit lower for Milan-Bologna. This is probably because Milan-Bologna is part of longer inter-city connections through all of Italy, and Trenitalia is active on some Southern-Italy routes where Italo was not at the time of data collection. In relation with our theoretical model, we can interpret this difference as Trenitalia having different levels of prominence over the three routes. In terms of connections during rush- and off-peak
hour, the two companies are similar. Based on the descriptives statistics, we are confident that the two companies fit into our theoretical framework and that the two firms can be compared across the three routes.

In what follows we will refer to Trenitalia as the incumbent and to Italo as the entrant.

### 2.6.2 Hypotheses

Based on our theoretical results of Proposition 2.1, we formulate the following hypotheses:

Hypothesis 1: The entrant charges lower prices than the incumbent, both in the advance selling and in the consumption period.

Hypothesis 2: The entrant offers a lower absolute advance purchase discount than the incumbent.

Hypothesis 3: The entrant offers a higher relative advance purchase discount than the incumbent.

### 2.6.3 Summary Statistics

In what follows we present a brief overview of the mean prices, absolute and relative advance purchase discounts.

|  | Milan-Bologna |  | Padua-Bologna |  | Turin-Milan |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Italo | Trenitalia | Italo | Trenitalia | Italo | Trenitalia |
| $p_{22}$ | 27.08 | 31.45 | 10.62 | 16.45 | 11.66 | 19.68 |
|  | $(7.00)$ | $(7.93)$ | $(1.98)$ | $(2.34)$ | $(3.66)$ | $(4.39)$ |
| $p_{15}$ | 27.80 | 32.34 | 10.59 | 17.01 | 12.43 | 20.24 |
|  | $(6.86)$ | $(7.70)$ | $(1.81)$ | $(2.26)$ | $(4.21)$ | $(4.10)$ |
| $p_{8}$ | 28.36 | 32.69 | 10.55 | 17.64 | 15.25 | 21.11 |
|  | $(6.28)$ | $(7.42)$ | $(1.69)$ | $(2.30)$ | $(3.04)$ | $(4.23)$ |
| $p_{1}$ | 42.24 | 47.89 | 19.03 | 26.38 | 30.15 | 36.59 |
|  | $(3.47)$ | $(3.63)$ | $(2.52)$ | $(2.92)$ | $(1.34)$ | $(2.23)$ |

Table 2.2: Summary statistics: mean price in euro (standard deviation).

|  | Milan-Bologna |  | Padua-Bologna |  | Turin-Milan |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Italo | Trenitalia | Italo | Trenitalia | Italo | Trenitalia |
| $\mathrm{a} \Delta p_{22}$ | 15.20 | 16.61 | 8.41 | 9.93 | 18.47 | 17.05 |
|  | $(6.15)$ | $(7.10)$ | $(3.00)$ | $(2.57)$ | $(3.73)$ | $(4.19)$ |
| $\mathrm{a} \Delta p_{15}$ | 14.48 | 15.73 | 8.45 | 9.37 | 17.72 | 16.50 |
|  | $(5.82)$ | $(6.72)$ | $(2.91)$ | $(2.33)$ | $(4.10)$ | $(3.96)$ |
| $\mathrm{a} \Delta p_{8}$ | 13.92 | 15.50 | 8.49 | 8.73 | 14.91 | 15.70 |
|  | $(5.06)$ | $(6.00)$ | $(2.85)$ | $(2.27)$ | $(3.19)$ | $(3.72)$ |

Table 2.3: Summary statistics: mean absolute advance purchase discount in euro (standard deviation).

|  | Milan-Bologna |  | Padua-Bologna |  | Turin-Milan |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Italo | Trenitalia | Italo | Trenitalia | Italo | Trenitalia |
| $\mathrm{r} \Delta p_{22}$ | .3616 | .3479 | .4278 | .3727 | .6125 | .4661 |
|  | $(.1454)$ | $(.1504)$ | $(.1481)$ | $(.0955)$ | $(.1202)$ | $(.1113)$ |
| $\mathrm{r} \Delta p_{15}$ | .3449 | .3298 | .4295 | .3515 | .5883 | .4508 |
|  | $(.1384)$ | $(.1434)$ | $(.1431)$ | $(.0914)$ | $(.1355)$ | $(.1035)$ |
| $\mathrm{r} \Delta p_{8}$ | .3316 | .3257 | .4315 | .3276 | .4941 | .4294 |
|  | $(.1188)$ | $(.1287)$ | $(.1398)$ | $(.0921)$ | $(.1012)$ | $(.0986)$ |

Table 2.4: Summary statistics: mean relative advance purchase discount (standard deviation).

The two companies' mean prices over the three routes are shown in Table 2.2, where $p_{t}$ corresponds to the price $t$-days before departure, for $t \in\{1,8,15,22\}$. Similarly Tables 2.3 and 2.4 show the mean absolute and relative advance purchase discounts for the two companies, where the subscript denotes the days before departure.

We observe that the incumbent (Trenitalia) sets higher prices than the entrant (Italo) not only in the consumption period $\left(p_{1}\right)$ but also in weeks before departure. This is true for all routes and all advance selling periods. This is in line with our first hypothesis.

The entrant's absolute advance purchase discount is in most cases lower than the incumbent's, with the exception for the route Turin-Milan 22- and 15 -days before departure. Most observations are therefore consistent with our second hypothesis.

As predicted by our model, the entrant offers a higher relative advance purchase discount than the incumbent. This is true for all routes and all advance selling periods.

### 2.6.4 Empirical Analysis

As stated earlier, we want to test whether being an entrant or an incumbent influences the charged prices and the offered advance purchase discounts. For each period $t$, where $t \in\{1,8,15,22\}$ denotes the days before departure, we estimate the price, the absolute, and the relative advance purchase discount as a function of the following variables: ROUTE, which takes the values 0 for Milan-Bologna, 1 for Padua-Bologna and 2 for Turin-Milan and the dummy variable ITALO. In a second estimation we also control for RUSHHOUR, a dummy which takes the value 1 if the departure time is between 6-9 AM or 4-7 PM. For example the model for $p_{22}$ is given by

$$
p_{22}=\beta_{0}+\beta I T A L O+\sum_{i} \beta_{i} R O U T E_{i}(+\gamma R U S H \text { HOU } R)+\varepsilon_{22} .
$$

Our first hypothesis would be confirmed for all three routes if $\beta$ is negative and significant.

Table 2.5 reports the coefficients of interest for the three hypotheses for each period $t \in\{1,8,15,22\}$. The results for the regression including the variable RUSHHOUR can be found in the Appendix.

First, we notice that the including the variable RUSHHOUR does not influence our results. We will therefore focus on Table 2.5.

Our first and third hypotheses are confirmed at the 1-\% significance level. In all advance selling periods the entrant (Italo) sets a lower price and a higher relative advance purchase discount than the incumbent (Trenitalia). Regarding our second hypothesis results are less significant but still go in the direction we would expect, that is, that the entrant offers a lower advance

|  | H1 |  |  |  | H2 |  |  | H3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $p_{22}$ | $p_{15}$ | $p_{8}$ | $p_{1}$ | $\mathrm{a} \Delta p_{22}$ | $\mathrm{a} \Delta p_{15}$ | $\mathrm{a} \Delta p_{8}$ | $\mathrm{r} \Delta p_{22}$ | $\mathrm{r} \Delta p_{15}$ | $\mathrm{r} \Delta p_{8}$ |
| Intercept | $\begin{gathered} 31.84^{* *} \\ (.20) \end{gathered}$ | $\begin{gathered} \hline 32.73^{* *} \\ (.19) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 32.98^{* *} \\ (.19) \end{gathered}$ | $\begin{gathered} \hline 48.06^{* *} \\ (.10) \end{gathered}$ | $\begin{gathered} 16.38^{* *} \\ (.19) \end{gathered}$ | $\begin{gathered} 15.50^{* *} \\ (.18) \end{gathered}$ | $\begin{gathered} \hline 15.35^{* *} \\ (.16) \end{gathered}$ | $\begin{aligned} & .3342^{* *} \\ & (.0046) \end{aligned}$ | $\begin{aligned} & .3157^{* *} \\ & (.0044) \end{aligned}$ | $\begin{aligned} & .3146^{* *} \\ & (.0040) \\ & \hline \end{aligned}$ |
| IT ALO | $\begin{gathered} \hline-5.84^{* *} \\ (.28) \\ \hline \end{gathered}$ | $\begin{gathered} \hline-6.00^{* *} \\ (.28) \\ \hline \end{gathered}$ | $\begin{gathered} -5.46^{* *} \\ (.26) \end{gathered}$ | $\begin{gathered} \hline-6.30^{* *} \\ (.15) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-.56^{*} \\ & (.26) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-.41 \\ & (.25) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline-1.02^{* *} \\ (.22) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline .0644^{* *} \\ & (.0065) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .0677^{* *} \\ & (.0062) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .0473^{* *} \\ & (.0056) \\ & \hline \end{aligned}$ |
| ROUTE $_{1}$ | $\begin{gathered} -15.39^{* *} \\ (.33) \end{gathered}$ | $\begin{gathered} -15.85^{* *} \\ (.32) \end{gathered}$ | $\begin{gathered} -15.85^{* *} \\ (.31) \end{gathered}$ | $\begin{gathered} -22.01^{* *} \\ (.17) \end{gathered}$ | $\begin{gathered} -6.75^{* *} \\ (.30) \end{gathered}$ | $\begin{gathered} -6.30^{* *} \\ (.29) \end{gathered}$ | $\begin{gathered} \hline-6.37^{* *} \\ (.26) \end{gathered}$ | $\begin{aligned} & .0355^{* *} \\ & (.0075) \end{aligned}$ | $\begin{aligned} & .0391^{* *} \\ & (.0072) \end{aligned}$ | $\begin{aligned} & .0308^{* *} \\ & (.0065) \end{aligned}$ |
| ROUTE $_{2}$ | $\begin{gathered} -12.83^{* *} \\ (.30) \end{gathered}$ | $\begin{gathered} -13.04^{* *} \\ (.29) \end{gathered}$ | $\begin{gathered} -12.00^{* *} \\ (.28) \end{gathered}$ | $\begin{gathered} -11.51^{* *} \\ (.15) \end{gathered}$ | $\begin{gathered} 1.28^{* *} \\ (.28) \end{gathered}$ | $\begin{gathered} 1.50^{* *} \\ (.26) \end{gathered}$ | $\begin{gathered} .42 \\ (.23) \end{gathered}$ | $\begin{aligned} & .1572^{* *} \\ & (.0069) \end{aligned}$ | $\begin{aligned} & .1567^{* *} \\ & (.0066) \end{aligned}$ | $\begin{aligned} & .1201^{* *} \\ & (.0060) \\ & \hline \end{aligned}$ |
| Obs. | 2135 | 2135 | 2134 | 2096 | 2096 | 2096 | 2095 | 2096 | 2096 | 2095 |
| $R^{2}$ | . 6231 | . 6469 | 0.6510 | 0.9066 | 0.2474 | 0.2489 | 0.2714 | 0.2370 | . 2520 | . 1922 |

purchase discount in absolute values. Hence, the empirical evidence supports our theoretical predictions.

### 2.7 Conclusion

This chapter considers an oligopolistic market with individual demand uncertainty in which asymmetric firms offer advance purchase discounts. We model the firms as a prominent incumbent and a more efficient entrant. Less choosy consumers buy in advance under uncertainty regarding their preferred product.

We find that the entrant offers a lower advance selling price, a lower consumption period price, a lower absolute advance purchase discount and a higher relative advance purchase discount than the incumbent. In particular, the entrant takes advantage of the fact that, in the advance selling period, uncertainty makes the expected gross value of the products more similar than what they actually are. Thanks to its higher efficiency, it can offer a low advance selling price and lock-in consumers before they learn their preferred product, which is most likely to be the incumbent's one due to the prominence asymmetry.

We show that price discrimination decreases the entrant's profit because the advance selling decreases profit margins. When the prominence asymmetry is high, however, the loss of profit is small. Moreover, with advance selling the entrant sells a higher total output than with uniform pricing. If, in a long-run perspective, the entrant is interested in market share, then advance purchase discounts are an optimal strategy to achieve this.

The Italian railway companies' price systems seem to confirm our result that the entrant sets a lower advance selling and consumption period prices, a lower absolute advance purchase discount, and a higher relative discount than the incumbent.

## Chapter 3

## Family Background and Labor Market Signaling

### 3.1 Introduction

Examples for jobs running in the family expand over time and professions. Back in the seventeenth and eighteenth century the Bernoullis contributed to several fields of mathematics and physics with fundamental principles that we still learn in school today. The Kennedys represent maybe the best-known political dynasty in the United States. The tradition started with P. J. Kennedy, elected to public office in 1884, continued through President John F. Kennedy, and recently came to an end with Joe Kennedy III, member of the U.S. House of Representatives until January 2021. Earlier this year the appearance of the 3rd generation soccer player Daniel Maldini to the field marked the 1000th game played by a Maldini family member for the Italian club AC Milan. ${ }^{1}$

These families are just some of many in which children follow the footsteps of their parents. According to a New York Times article, sons and fathers

[^15]are 2.7 times as likely as two unrelated people to have the same job. ${ }^{2}$ Some professions are more likely to be passed down, these include high income jobs such as legislator, lawyer and doctor, but also less remunerative professions such as farmer and fisher.

Generally, the family income level plays an essential role in a child's education and career path (Bowles et al., 2005, Björklund and Salvanes, 2011; Doepke and Zilibotti, 2019). The channels through which family background influences a child's future include natural factors, such as genetics, and environmental aspects, such as school choice, parenting style and opportunities (Björklund et al., 2006).

Education choice eventually has an impact on the future worker's income. Income increases a worker's well-being not only in absolute terms but also in relative ones (Clark and Oswald, 1996). In particular, Nikolaev and Burns (2014) show that the negative effect of downward mobility experienced by workers from high income households is stronger than the positive effect of upward mobility experienced by workers from low income households. This evidence suggests that (i) workers use the family income as a reference, and (ii) workers are subject to loss aversion. Therefore, when we study education choice (and eventually intergenerational mobility), the influence of the parents' income as a reference point is an important factor to consider.

Motivated by this evidence, this chapter introduces family background and loss aversion to a labor market signaling model. The goal is to link differences in intergenerational mobility to cultural and societal differences in the importance of family income or career as a reference for education choices. Understanding what drives intergenerational mobility is essential for the selection of optimal policy instruments that address this issue.

The model is based on Spence (1973). Workers differ in two dimensions: ability of the workers and income of their household of origin. They are employed by perfectly competitive firms that can observe a worker's education choice but not his type. Workers' utilities are reference-dependent, that is

[^16]the utility a worker derives from his wage depends on the wage itself and on the difference between his income and the income of his household of origin. Workers are characterized by loss aversion, in the sense that the negative effect of downward mobility experienced by workers from rich households is stronger than the positive effect of upwards mobility experienced by workers from poor households. A higher degree of loss aversion can be interpreted as a higher parental or cultural pressure to maintain a certain income. The education choice then depends on the income of the household of origin and the cost of acquiring education, which is lower for workers of higher ability.

The degree of loss aversion and the weight of the reference-based utility determine whether workers separate by ability or by the income of the household of origin. When workers are less prone to follow their parent's footsteps, workers split by ability, that is, workers of higher ability choose a higher level of education and earn a higher wage. In this case, high ability workers from poor households can achieve the higher wage and low ability workers from rich household earn the lower wage, resulting in higher intergenerational mobility. In a separation by income equilibrium, workers from poor households choose a lower level of education independently of their ability level, yielding lower intergenerational mobility.

We also show that wage inequality is lower when workers separate by income and we predict a positive correlation between intergenerational mobility and income inequality. Heterogeneity in the weight of family background in the education choice might explain differences in wage compression across countries.

A separating equilibrium reduces welfare because workers acquire (useless) education to distinguish themselves, but whether separation by ability or separation by income is more detrimental for welfare depends on two effects. A separation by income equilibrium requires a lower level of education than a separation by ability equilibrium, but the cost of education is not efficiently distributed as some of the low ability workers also acquire a higher level of education. A separation by ability equilibrium is more efficient when the difference in abilities is high enough.

After a review of the literature, the chapter is structured as follows: Sec-
tion 3.2 presents the model and shows how workers choose their education level. Section 3.3 describes the equilibria and Section 3.4 analyzes the welfare implications. Section 3.5 concludes. Proofs can be found in Appendix C.

### 3.1.1 Related Literature

This chapter contributes to the literature on behavioral labor economics (see Dohmen, 2014, for a review). In particular it combines labor market signaling to loss aversion.

Loss aversion has been famously introduced by Kahneman and Tversky (1979) and has since been applied to other models in labor economics.

Köszegi and Rabin (2006) develop a model including loss aversion in which the economic environment endogenously determines the reference point. They propose an application on the within-day labor supply decisions of taxi drivers and show that the expected wage and deviations from it play an important role for these decisions. Empirical evidence such as Fehr and Goette (2007) and Crawford and Meng (2011) backs Köszegi and Rabin's (2006) theory. Thiemann (2017) studies how peer performance influences an individual student's effort. She shows that in less competitive cultures, in which the degree of loss aversion and the importance of reference-dependent utility are lower, the students' average performance is higher when classrooms are sorted by ability than when classrooms include students with different abilities. The opposite is true for high competitive cultures. This literature focuses on the effect of loss aversion on workers' effort that maximize their own utility, while the model proposed here applies loss aversion in an asymmetric information framework.

So far, the impact of loss aversion on outcomes with asymmetric information has focused on screening and principal-agent models. For both set-ups loss aversion predicts less variation in the incentive schemes than standard assumptions on the agents would.

Herweg and Mierendorff (2013) and Hahn et al. (2018) show that firms serving loss averse consumers prefer to abandon price discriminating pricing schemes (e.g. two-part tariffs) in favor of a simpler one (e.g. a flat-rate).

Karle and Möller (2020) consider firms offering advance purchase discounts to loss averse consumers in the presence of individual demand uncertainty. Advance purchase discounts allow the firms to screen between flexible and "choosier" consumers. Loss aversion changes how advance sales react to a decrease in uncertainty and it is therefore an important factor to consider in assessing the impact of policies aimed to improve consumers' information.

De Meza and Webb (2007) consider a principal-agent model in which agents are characterized by loss aversion. They show that for certain intervals, the optimal wage scheme is unrelated to performance. In particular, if agents use the median wage as the reference point, the optimal wage is flat up to the median performance level and increasing thereafter.

The agency problem with loss averse agents is also considered by Herweg et al. (2010). In their model agents use their expected wage as the reference point. They show that since agents compare their wage with other possible wages, rewarding higher outcomes reduces the agents' expected utility because they anticipate the enhanced loss in case of a low wage. Thus, a richer wage scheme does not provide the intended incentives. They show that whereas standard preferences would imply a monotonic reward scheme, with loss averse agents a binary payment scheme is the optimal strategy.

Marchegiani et al. (2016) show that considering loss averse agents might explain the empirical evidence that supervisors have systematic leniency biases, that is they too often assess high performances. They claim that loss averse agents are more motivated when principals are biased towards high performances, than when they are towards low performances.

Our contribution to this literature is to consider the effect of loss aversion on a different model regarding asymmetric information, that is the signaling model.

This work, however, is not the first to analyze Spence's (1973) model from a behavioral point of view. Santos-Pinto (2012) introduces workers with biased self-confidence to the signaling model. In his model, workers acquire education according to their (biased) beliefs, meaning that in equilibrium one finds overconfident low ability workers among the high educated and underconfident high ability workers among the low educated. The presence
of biased workers induces a wage compression and might increase welfare compared to an equilibrium with rational workers.

### 3.2 Model

We introduce loss aversion to Spences (1973) signaling model. Workers differ in two dimensions: ability and income of the household of origin. Half are high ability workers, $\theta_{H}$, and half are of low ability, $\theta_{L}<\theta_{H}$. In terms of family background, half of the workers are from rich households with income $r_{R} \geq \theta_{H}$, and the other half from poor households with income $r_{P} \leq \theta_{L}$. The probability of being of high ability given that a worker comes from a rich household is given by $h>\frac{1}{2}$, while the corresponding probability from a poor household is given by $1-h$. Table 3.1 summarizes the workers' distribution.

Workers can acquire education $e$ at $\operatorname{cost} c(e, \theta)=\frac{e^{2}}{\theta}$. The cost function is increasing and convex in education and decreasing in ability. Note that $c_{e}\left(e, \theta_{L}\right)=\frac{2 e}{\theta_{L}}>\frac{2 e}{\theta_{H}}=c_{e}\left(e, \theta_{H}\right)$, that is a worker of higher ability has lower marginal costs and the Spence-Mirrlees single-crossing condition is satisfied. This condition assures the existence of a separating equilibrium by ability. The workers' utility function is modeled as in Köszegi and Rabin (2006), with the difference that the reference income is deterministic. That is, the utility of a worker of ability $\theta$ from a household of income $r$ with education level $e$ employed at wage $w$ is given by

$$
\begin{equation*}
U(w, e, \theta, r)=w+\mu(w-r)-c(e, \theta), \tag{3.1}
\end{equation*}
$$

where

$$
\mu(w-r)= \begin{cases}\eta(w-r) & \text { if } w \geq r  \tag{3.2}\\ -\eta \lambda(r-w) & \text { if } w<r\end{cases}
$$

with $\lambda>1$. The utility from not being employed is normalized to zero. The function $\mu(w-r)$ captures the reference-dependent preferences: The workers' utility depends on the difference between their own income $w$ and

|  | $r_{P}$ | $r_{R}$ |
| :---: | :---: | :---: |
|  |  | $\frac{h}{2}$ |
| $\theta_{L}$ | $\frac{1-h}{2}$ |  |
| $\theta_{H}$ | $\frac{1-h}{2}$ | $\frac{h}{2}$ |
|  |  |  |

Table 3.1: Workers' distribution across the two dimensions for $h>\frac{1}{2}$.
the income of the household of origin $r$. The parameter $\eta$ measures the weight that a worker attaches to the reference-dependent part of the utility. The assumption $\lambda>1$ captures loss aversion: in absolute values the disutility from a wage lower than the income of the household of origin is higher than the positive utility from a wage higher or equal to the income of the household of origin.

To abbreviate notation we define $\varphi \equiv \frac{1+\eta \lambda}{1+\eta}$ and interpret $\varphi$ as the propensity to follow the parents' footsteps. The parameter $\varphi$ is increasing in both $\lambda$ and $\eta$. Intuitively, this parameter can capture the link between cultural differences in loss aversion and the weight of family background and jobs' transmission within families. For example, in cultures in which social status is important, having a lower income or a less prestigious job than one's parent causes a higher disutility than in cultures with less pressure. We therefore expect to observe higher degrees of loss aversion (that is, a higher $\lambda)$ in these types of communities and a higher tendency to follow the parents' career path. Similarly, if the family income is an important reference point (higher $\eta$ ), workers should more often aim to achieve the same income level themselves.

The productivity of a worker of type $\theta$ with education level $e$ is given by $y(\theta, e)=\theta$, that is education is useless in terms of productivity and solely serves as a signal. A firm's profit from the employment of a worker of ability $\theta$ with education $e$ is then given by $\pi(w, e, \theta)=y(\theta, e)-w=\theta-w$, where $w$ is the wage paid to the worker. The workers' type is private information (in both dimensions) and consequently employers can only observe the education level. We assume that there are multiple profit maximizing employers who
are in perfect competition for workers.
The timing is as follows: i) Each worker chooses a level of education $e \geq 0$ based on his type and his beliefs. ii) Employers observe the worker's education and simultaneously make wage offers. iii) Workers decide if and which wage offer to accept.

### 3.2.1 Analysis

With four types of workers, separation can occur in several different ways. We focus on separating equilibria in which workers separate in two groups, either according to their ability or according to their family background, since these give rise to the most interesting results. Workers split by choosing either a low level of education $e_{L}$ or a high level of education $e_{H}$.

Wage offers. Let $\mu(e)$ be the employers' belief that a worker is of high ability after observing education level $e$. According to this belief the average productivity of workers with education level $e$ is given by $\mathbb{E}[y \mid e]=\mu(e) \theta_{H}+$ $(1-\mu(e)) \theta_{L}$. Because of risk-neutrality and perfect competition employers make wage offers

$$
\begin{cases}w_{L}=\mathbb{E}[y \mid e], & \text { if } e<e_{H},  \tag{3.3}\\ w_{H}=\mathbb{E}[y \mid e], & \text { if } e \geq e_{H}\end{cases}
$$

Education choice. If a worker chooses a low level of education it is then optimal to choose $e_{L}^{*}=0$. Since education does not influence productivity, an education level $e_{L}>0$ does not increase the wage as long as $e_{L}<e_{H}$, but it entails some costs. Therefore, for low educated workers a positive level of education only has costs and no benefits.

In a separating equilibrium workers must thus decide between acquiring no education $e_{L}^{*}=0$ and getting a wage $w_{L}$, and acquiring an education level $e^{H}>0$ and getting a wage $w_{H}$. The utility of a worker from a poor household with a higher level of education is given by

$$
\begin{equation*}
U\left(w_{H}, e_{H}, \theta, r_{P}\right)=w_{H}+\eta\left(w_{H}-r_{P}\right)-\frac{e_{H}^{2}}{\theta} \tag{3.4}
\end{equation*}
$$

While his utility with $e_{L}^{*}=0$ is given by

$$
\begin{equation*}
U\left(w_{L}, 0, \theta, r_{P}\right)=w_{L}+\eta\left(w_{L}-r_{P}\right) . \tag{3.5}
\end{equation*}
$$

A worker from a poor household then prefers a higher level of education if and only if

$$
\begin{align*}
U\left(w_{H}, e_{H}, \theta, r_{P}\right) & \geq U\left(w_{L}, 0, \theta, r_{P}\right) \\
e_{H} & \leq e_{H}\left(\theta, r_{P}\right) \equiv \sqrt{(1+\eta) \theta\left(w_{H}-w_{L}\right)} \tag{3.6}
\end{align*}
$$

Similarly, a worker from a rich household prefers a higher level of education if and only if

$$
\begin{align*}
U\left(w_{H}, e_{H}, \theta, r_{R}\right) & \geq U\left(w_{L}, 0, \theta, r_{R}\right) \\
e_{H} & \leq e_{H}\left(\theta, r_{R}\right) \equiv \sqrt{(1+\eta \lambda) \theta\left(w_{H}-w_{L}\right)} . \tag{3.7}
\end{align*}
$$

Note that $e_{H}\left(\theta_{H}, r\right)>e_{L}\left(\theta_{L}, r\right)$, that is, for the same wage difference and family background, a high ability worker is willing to acquire more education than a low ability worker. This follows from the fact that the marginal costs of education are lower for high ability workers. Moreover, $e_{H}\left(\theta, r_{R}\right)>$ $e_{H}\left(\theta, r_{P}\right)$, meaning that for the same wage difference and ability, a worker from a rich household is willing to acquire more education than one from a poor household. This follows from loss aversion: for the same cost of acquiring education, a rich worker is more motivated to achieve a higher income, since the difference from his reference income weighs more heavily than for a poor worker.

### 3.3 Intergenerational Mobility

In this section we discuss the effects of loss aversion on the separating equilibrium outcome. Using the thresholds (3.6) and (3.7) found in Section 3.2 we derive the following proposition. The proof can be found in the Appendix.

Proposition 3.1. With loss aversion two of the possible separating equilibria are

- Separation by ability: If $\varphi<\frac{\theta_{H}}{\theta_{L}}$ there exists an equilibrium in which workers separate by ability. Low ability workers choose education level $e_{L}^{*}=0$ and earn wage $w_{L}^{*}=\theta_{L}$. High ability workers choose education level $e_{H}^{*} \in\left[\sqrt{(1+\eta \lambda) \theta_{L}\left(\theta_{H}-\theta_{L}\right)}, \sqrt{(1+\eta) \theta_{H}\left(\theta_{H}-\theta_{L}\right)}\right]$ and earn wage $w_{H}^{*}=\theta_{H}$.
- Separation by income: If $\varphi>\frac{\theta_{H}}{\theta_{L}}$ there exists an equilibrium in which workers separate by the income of the household of origin. Workers from poor households choose education level $e_{L}^{*}=0$ and earn wage $w_{L}^{*}=h \theta_{L}+(1-h) \theta_{H}$. Workers from rich households choose education level $e_{H}^{*} \in\left[\sqrt{(2 h-1)(1+\eta) \theta_{H}\left(\theta_{H}-\theta_{L}\right)}, \sqrt{(2 h-1)(1+\eta \lambda) \theta_{L}\left(\theta_{H}-\theta_{L}\right)}\right]$ and earn wage $w_{H}^{*}=(1-h) \theta_{L}+h \theta_{H}$.

Whether workers separate by ability or by income depends on high ability workers from poor households (HP-type) and low ability workers from rich households ( $L R$-type). If loss aversion is low compared to the difference in productivity, the motivation of workers from rich households is low compared to the cost advantage of high ability workers. In this case a worker of type $H P$ is willing to get more education than a worker of type $L R$ and an equilibrium in which workers split according to their ability is possible. If, on the contrary, loss aversion is high compared to the productivity difference, then the motivation of $L R$-workers is high compared to the cost advantage of $H P$-workers and they are willing to get more education, in which case a separation by income occurs. Figure 3.1 shows which equilibrium occurs depending on the parameter values.

Proposition 3.1 links the cultural relevance of the parents' income a reference to intergenerational mobility. If the propensity to follow the parents' footsteps is low (low $\varphi$ ) then workers split by ability. A worker of high ability can achieve a high paying job, independently of the family background, that is, there is high intergenerational mobility. On the contrary, if the parents' career has a relevant influence (high $\varphi$ ) then workers split by the income of


Figure 3.1: Separating equilibria with loss aversion.
the household of origin. In this case workers from low income families get a low income independently of their ability, implying lower intergenerational mobility. This important insight shows that the level of mobility is not only determined by structural factors such as school system, school quality, financial support, and labor market structure, but also by cultural aspects. A policy meant to stimulate intergenerational mobility that only address structural issues might thus be less effective in countries in which social status is very important.

Note that without loss aversion (that is $\lambda=1$ ) a separation by income does not occur, since $\varphi=1<\frac{\theta_{H}}{\theta_{L}}$. Loss aversion is therefore essential here for a separation by income and might contribute to explaining why we may observe workers separating by family background.

The next corollary follows directly from Proposition 3.1.
Corollary 3.1. The wage difference between high and low educated work-
ers is smaller in an equilibrium in which workers separate by family income compared to one in which they separate by ability.

When workers separate by family income, the productivity of low educated workers is increased by high ability workers and the productivity of high educated workers is decreased by low ability workers, hence, the smaller wage gap in a separation by income equilibrium.

This result is in contrast with the evidence that income inequality and intergenerational mobility are negatively correlated (see, for example, OECD, 2018), a relationship that is also known as the "Great Gatsby Curve" (Krueger, 2012). However, new estimates for intergenerational mobility question this negative correlation (Jäntti and Jenkins, 2015) and evidence also exists for a positive correlation (Checchi et al., 1999). In countries in which maintaining a certain status is very important, the presence of low ability (and therefore less productive) workers among those with high education contributes in reducing income inequality.

Corollary 3.1 also relates to wage compression: When workers separate by income the difference in wages is smaller than the difference in productivity between high and low ability workers. Mourre (2005) presents evidence for variation in wage compression in the EU25 countries. For example, wage compression is present in the continental and southern EU15 countries, while none or contradictory evidence is found for the Anglo-Saxon and the northern countries. Cultural differences in the propensity of following the parents' footsteps could also contribute to explaining these cross-country differences.

### 3.4 Welfare

Higher intergenerational mobility is desirable in terms of equal opportunities. From a fairness point of view, high ability workers should be able to achieve a high income job, independently of whether they are born in a poor or in a rich family. A separation by ability equilibrium allows for such equality. The question we want address in this section is whether a separation by ability
equilibrium is also profitable in terms of overall welfare $3^{3}$
From a normative perspective it is unclear whether gain-loss utility should be considered in the welfare function (see the discussion in O'Donoghue and Sprenger, 2018). If we include loss aversion welfare is given by

$$
\begin{align*}
\mathcal{W}_{l}= & \frac{h}{2}\left[\theta_{L}+\eta\left(w_{L P}-r_{P}\right)-\frac{e_{L P}^{2}}{\theta_{L}}\right]+\frac{1-h}{2}\left[\theta_{L}-\eta \lambda\left(r_{R}-w_{L R}\right)-\frac{e_{L R}^{2}}{\theta_{L}}\right] \\
& +\frac{1-h}{2}\left[\theta_{H}+\eta\left(w_{H P}-r_{P}\right)-\frac{e_{H P}^{2}}{\theta_{H}}\right] \\
& +\frac{h}{2}\left[\theta_{H}-\eta \lambda\left(r_{R}-w_{H R}\right)-\frac{e_{H R}^{2}}{\theta_{H}}\right] \tag{3.8}
\end{align*}
$$

while if we do not include loss aversion it is given by

$$
\begin{align*}
\mathcal{W}_{0}= & \frac{h}{2}\left[\theta_{L}-\frac{e_{L P}^{2}}{\theta_{L}}\right]+\frac{1-h}{2}\left[\theta_{L}-\frac{e_{L R}^{2}}{\theta_{L}}\right] \\
& +\frac{1-h}{2}\left[\theta_{H}-\frac{e_{H P}^{2}}{\theta_{H}}\right]+\frac{h}{2}\left[\theta_{H}-\frac{e_{H R}^{2}}{\theta_{H}}\right], \tag{3.9}
\end{align*}
$$

where $w_{i j}$ and $e_{i j}$ are the wage and the education level of a worker with ability $\theta_{i}, i=\in\{H, L\}$, from a household with income $r_{j}, j=\{R, P\}$. We restrict our attention to the minimal level of education that allows for a separating equilibrium.

Lemma 3.1. Comparison among welfare levels of different equilibria is unaffected by the (non-)inclusion of gain-loss utility.

The proof is given in the Appendix. The lemma follows from the fact that without returns to education, average wages are the same in all equilibria. This result allows us to focus on a simpler welfare function that does not include gain-loss utility in the remainder of this section. In this case, welfare is higher if the total cost of acquiring education is lower.

The separation by ability equilibrium and the separation by income equilibrium do not coexists. Therefore, we cannot compare welfare levels for the same parameter values. To obviate this issue we set the parameter $\varphi$ at

[^17]$\varphi=\frac{\theta_{H}}{\theta_{L}}$ and compare welfares at this threshold value. In the Appendix we prove the following.

Proposition 3.2. For $\varphi=\frac{\theta_{H}}{\theta_{L}}$, a separation by ability equilibrium is more efficient than a separation by income equilibrium if and only if $\frac{\theta_{H}}{\theta_{L}}>\frac{2 h+1}{2 h-1}$.

Figure 3.2 illustrates Proposition 3.2. There are two effects at play and the stronger one determines which equilibrium leads to lower total costs for education and therefore a higher welfare. On the one hand, the minimal level of education that allows for separation is lower in the separation by income than in the separation by ability equilibrium for $\varphi=\frac{\theta_{H}}{\theta_{L}}$. On the other hand, with separation by income the cost for acquiring education is not efficiently distributed, since workers of type $L R$, who have higher cost of education than $H P$-workers, choose a positive level of education. When the difference in productivity, that is the difference in marginal costs, is high, then the increase in costs due to the low ability types acquiring education is higher than the decrease in costs due to the lower level of education, and therefore welfare is higher in a separation by ability equilibrium.

Result 3.1. In a separation by income equilibrium, welfare can be higher than in the standard equilibrium without loss aversion and gain-loss utility.

For appropriate parameter values the reduction in the cost for education is such that welfare in a separation by income equilibrium with $\varphi=\frac{\theta_{H}}{\theta_{L}}$ is higher than welfare in a separation equilibrium without gain-loss utility $(\eta=0) ._{4}^{4}$ In a separation by ability equilibrium with $\varphi<\frac{\theta_{H}}{\theta_{L}}$ this can never be the case because loss aversion implies higher levels of education with the same education distribution.

[^18]

Figure 3.2: Welfare comparison in the separating equilibria. Separation by ability has a higher welfare only if the productivity difference is high enough (gray area).

### 3.5 Conclusion and Discussion

Introducing family background and loss aversion to Spence's (1973) labor market signaling model allows to gain further insights about intergenerational mobility. It is not a novel insight that family background influences education choice and outcome: Literature shows significant effects for parental education, family structure, and parenting style (Björklund and Salvanes, 2011). This model sheds light on a new channel through which parental careers influence their children's education.

When workers use their family income as a reference, their education choice is not only based on their ability type but also on the wage difference with respect to their family.

We show that whether workers split according to their ability or to the income of the household of origin depends on the degree of loss aversion and
the weight of the reference-dependent utility. When these are low compared to the ability difference workers split by ability. A high ability worker from a poor household chooses the higher level of education, earns the higher wage and there is high intergenerational mobility. When these parameters are high with respect to the ability difference, then workers split by the income of the household of origin. High ability workers from poor households choose, despite their low costs, the lower level of education resulting in lower intergenerational mobility.

The presence of high ability workers among the low educated and of low ability workers among the high educated workers reduces the wage difference, which is lower than the difference in productivities. We therefore find a channel that reduces income inequality in equilibria with low intergenerational mobility.

In terms of welfare the two types of equilibria have different disadvantages. With separation by ability the level of education necessary for separation is higher than with separation by income. However, with separation by income the cost of acquiring education is not efficiently distributed as some low ability workers choose the higher level of education. When the cost advantage of high ability workers is high, then a separation by ability equilibrium is more efficient than one that separates by income. Otherwise, the opposite is true.

Further, for appropriate parameter values the low level of over-education in the separation by income equilibrium reduces the cost for education to the point that welfare is higher than in a separating equilibria without loss aversion.

This simple model shows how loss aversion contributes to explaining higher or lower levels of intergenerational mobility and the consequential effects on welfare. The following extensions, left for future research, might allow to gain further insights.

First, instead of a static model one could consider a dynamic one. For example, an overlapping generation model in which workers realize that their education choice not only influences their own income but also the education choice and correspondingly the income of their children. Such a model would allow to observe how loss aversion affects the evolution of intergenerational
mobility, wages and education levels over time.
Second, in this model workers from high income households can only be worse off than their parents, since it is assumed that they can achieve at most the same wage as them. Respectively, workers from low income households can only be better off than their parents. As a robustness check, the possibility that workers from high (respectively low) income households achieve a higher (respectively lower) income than their parents should be considered.

Lastly, it is assumed that education does not increase productivity. While there is evidence that education has a signal component (Murnane et al., 2000), it also increases productivity (Black and Lynch, 1996). Including education that is productive in this set-up might lead to different results, in particular in terms of welfare. If the marginal return to education is higher for high ability workers, then in the separation by income equilibrium there is a loss of productivity which lowers welfare. This might expand the set of parameter values for which a separation by ability equilibrium has a higher welfare than one with separation by income. Preliminary research shows that the introduction of productivity poses some challenges to the model's tractability. Alternatively, one could think of different job assignments as in Gibbons and Waldman (1999): Assume there are two type of job assignments, $H$ and $L$. High ability workers are more productive if they are assigned to job $H$ and low ability workers are more productive if assigned to job $L$. If the assignment is determined by the level of education, the loss of productivity in a separation by income should be qualitatively similar to the one in a model with returns to education.

## Appendix A

## Appendix to Chapter 1

## A. 1 Proofs

## A.1.1 Proof of Proposition 1.2

The prices (1.14) and 1.15 are the solutions to the necessary conditions (1.12)-(1.13). Because $\gamma<1$, it holds that $p_{1, i}^{*}<p_{2, i}^{*}$ for $i \in\{A, B\}$. To verify that they indeed constitute a price discrimination equilibrium, we need to check that both firms sell a positive output in both periods.

Replacing the equilibrium prices in (1.5), (1.6), and (1.7), we obtain the following equilibrium thresholds

$$
\begin{align*}
\bar{\sigma}^{*} & =\frac{c_{B}-c_{A}}{3(2 \gamma-1)},  \tag{A.1}\\
\sigma_{W}^{*}(A) & =\frac{2 \gamma}{-1+7 \gamma-4 \gamma^{2}}+\frac{c_{B}-c_{A}}{3},  \tag{A.2}\\
\sigma_{W}^{*}(B) & =\frac{2 \gamma}{-1+7 \gamma-4 \gamma^{2}}-\frac{c_{B}-c_{A}}{3} . \tag{A.3}
\end{align*}
$$

In the first period, $p_{1, A}^{*}<p_{1, B}^{*}$, hence firm $A$ sells a positive amount if and only if $\sigma_{W}^{*}(A)+\bar{\sigma}^{*}>0$, which is always true since both A.1) and A.2 are strictly positive given that $c_{B}>c_{A}$.

Firm $B$ sells a strictly positive amount in the first period if and only if
$\sigma_{W}^{*}(B)-\bar{\sigma}^{*}>0$, which is satisfied for

$$
\begin{equation*}
c_{B}-c_{A}<\frac{3(2 \gamma-1)}{-1+7 \gamma-4 \gamma^{2}} \tag{A.4}
\end{equation*}
$$

In the second period, both firms sell a strictly positive output, because $\sigma_{W}^{*}(B)<1$. To see why this is the case, we can rewrite the inequality as

$$
\begin{equation*}
\frac{2 \gamma}{-1+7 \gamma-4 \gamma^{2}}<1+\frac{c_{B}-c_{A}}{3} \tag{A.5}
\end{equation*}
$$

and note that the term on the left-hand side is always strictly smaller than 1 for $\gamma \in\left(\frac{1}{2}, 1\right)$.

Lastly, in order to focus on interior solutions, we also need $\sigma_{W}^{*}(A)<1$, which is true for

$$
\begin{equation*}
c_{B}-c_{A}<\frac{3\left(-1+5 \gamma-4 \gamma^{2}\right)}{-1+7 \gamma-4 \gamma^{2}} \tag{A.6}
\end{equation*}
$$

Condition (1.16) follows from equations A.4) and (A.6).

## A.1.2 Proof of Corollary 1.1

Using the equilibrium thresholds defined by A.1)-A.3 we obtain the following equilibrium quantities

$$
\begin{align*}
q_{1, A}^{*} & =\frac{\gamma}{-1+7 \gamma-4 \gamma^{2}}+\frac{2 \gamma\left(c_{B}-c_{A}\right)}{6(2 \gamma-1)}  \tag{A.7}\\
q_{2, A}^{*} & =\frac{-1+5 \gamma-4 \gamma^{2}}{2\left(-1+7 \gamma-4 \gamma^{2}\right)}-\frac{(2 \gamma-1)\left(c_{B}-c_{A}\right)}{6}  \tag{A.8}\\
q_{1, B}^{*} & =\frac{\gamma}{-1+7 \gamma-4 \gamma^{2}}-\frac{2 \gamma\left(c_{B}-c_{A}\right)}{6(2 \gamma-1)}  \tag{A.9}\\
q_{2, B}^{*} & =\frac{-1+5 \gamma-4 \gamma^{2}}{2\left(-1+7 \gamma-4 \gamma^{2}\right)}+\frac{(2 \gamma-1)\left(c_{B}-c_{A}\right)}{6} \tag{A.10}
\end{align*}
$$

It this straightforward to see that $q_{1, A}^{*}>q_{1, B}^{*}$ and $q_{2, A}^{*}<q_{2, B}^{*}$. The market shares are given by

$$
\begin{align*}
q_{A}^{*} & =\frac{1}{2}+\frac{\left(-1+6 \gamma-4 \gamma^{2}\right)\left(c_{B}-c_{A}\right)}{6(2 \gamma-1)}  \tag{A.11}\\
q_{B}^{*} & =\frac{1}{2}-\frac{\left(-1+6 \gamma-4 \gamma^{2}\right)\left(c_{B}-c_{A}\right)}{6(2 \gamma-1)} \tag{A.12}
\end{align*}
$$

It then holds that $q_{A}^{*}>q_{B}^{*}$.
With uniform pricing the market share of the efficient firm is

$$
\begin{equation*}
q_{A}^{U}=\frac{1}{2}+\frac{c_{B}-c_{A}}{6} . \tag{A.13}
\end{equation*}
$$

Given $\gamma \in\left(\frac{1}{2}, 1\right)$ it then follows

$$
\begin{equation*}
q_{A}^{*}>q_{A}^{U} \Leftrightarrow \frac{-1+6 \gamma-4 \gamma^{2}}{2 \gamma-1}>1 \Leftrightarrow 4 \gamma(1-\gamma)>0 \tag{A.14}
\end{equation*}
$$

## A.1.3 Proof of Proposition 1.3

Given the equilibrium prices $(1.36)$ and $(1.37)$ the equilibrium thresholds are given by

$$
\begin{align*}
\sigma_{W}^{*}(A) & =\frac{2}{3}  \tag{A.15}\\
\sigma_{W}^{*}(B) & =\frac{1}{3 \rho},  \tag{A.16}\\
\bar{\sigma}^{*} & =\frac{1}{3} . \tag{A.17}
\end{align*}
$$

It is clear that all the relevant thresholds are in $(0,1)$ and that $\sigma_{W}^{*}(A)>$ $\sigma_{W}^{*}(B)$, which allows for the profits to be defined as in (1.30) and (1.31) and for the existence of an interior solution.

The price differences between the uniform pricing equilibrium prices and the advance selling equilibrium are given by

$$
\begin{array}{ll}
p_{A}^{U}-p_{1, A}^{*}=\frac{1}{3}\left(3-4 \rho+\frac{1}{\rho}\right), & p_{B}^{U}-p_{1, B}^{*}=\frac{2\left(1-\rho^{2}\right)}{3 \rho}, \\
p_{A}^{U}-p_{2, A}^{*}=\frac{2(1-\rho)}{3 \rho}, & p_{B}^{U}-p_{2, B}^{*}=\frac{2(1-\rho)}{3 \rho}, \tag{A.19}
\end{array}
$$

which are all positive for $\rho \in\left(\frac{1}{2}, 1\right)$, proving the first claim.
We now prove the second claim. In the advance selling period the price difference between the two firms is given by

$$
\begin{equation*}
p_{1, A}^{*}-p_{1, B}^{*}=\frac{2 \rho-1}{3}, \tag{A.20}
\end{equation*}
$$

and in the consumption period is given by

$$
\begin{equation*}
p_{2, A}^{*}-p_{2, B}^{*}=\frac{2 \rho-1}{3 \rho} . \tag{A.21}
\end{equation*}
$$

Both are clearly positive for $\rho \in\left(\frac{1}{2}, 1\right)$.
The absolute advance purchase discount difference is given by

$$
\begin{equation*}
\left(p_{2, A}^{*}-p_{1, A}^{*}\right)-\left(p_{2, B}^{*}-p_{1, B}^{*}\right)=\frac{(1-\rho)(2 \rho-1)}{3 \rho}, \tag{A.22}
\end{equation*}
$$

which is positive for all $\rho \in\left(\frac{1}{2}, 1\right)$.
The difference in the relative advance purchase discounts is given by

$$
\begin{equation*}
\frac{p_{2, A}^{*}-p_{1, A}^{*}}{p_{1, A}^{*}}-\frac{p_{2, B}^{*}-p_{1, B}^{*}}{p_{2, B}^{*}}=-\frac{(1-\rho)(2 \rho-1)}{(3 \rho-1)} \tag{A.23}
\end{equation*}
$$

which is negative for all $\rho \in\left(\frac{1}{2}, 1\right)$.

## A.1.4 Proof of Corollary 1.3

Given the equilibrium thresholds (A.15) and A.17, the output sold by the two firms in the first period are given by

$$
\begin{equation*}
q_{1, A}^{*}=\sigma_{W}^{*}(A)-\bar{\sigma}^{*}=\frac{1}{3}=\bar{\sigma}^{*}=q_{1, B}^{*} . \tag{A.24}
\end{equation*}
$$

The output sold in the second period are given by

$$
\begin{equation*}
q_{2, A}^{*}=\rho\left(1-\sigma_{W}^{*}(A)\right)=\rho \frac{1}{3}>(1-\rho) \frac{1}{3}=(1-\rho)\left(1-\sigma_{W}^{*}(A)\right)=q_{2, B}^{*} . \tag{A.25}
\end{equation*}
$$

It is then straightforward that $q_{1, A}^{*}+q_{2, A}^{*}>q_{1, B}^{*}+q_{2, B}^{*}$.
With uniform pricing the equilibrium outputs are given by

$$
\begin{align*}
q_{A}^{U} & =\frac{1+\rho}{3}  \tag{A.26}\\
q_{B}^{U} & =\frac{2-\rho}{3} \tag{A.27}
\end{align*}
$$

It then follows that $q_{1, A}^{*}+q_{2, A}^{*}=q_{A}^{U}$ and $q_{1, B}^{*}+q_{2, B}^{*}=q_{B}^{U}$.

## A.1.5 Proof of Corollary 1.4

We show that the profits difference between the firms is smaller under advance selling than in the uniform pricing benchmark. The rest of the Corollary follows from Proposition 1.3 and Corollary 1.3 .

The difference in profits in the benchmark equilibrium is given by

$$
\begin{equation*}
\Pi_{A}^{U}-\Pi_{B}^{U}=\frac{2}{3}-\frac{1}{3 \rho}, \tag{A.28}
\end{equation*}
$$

while the one in the advance selling equilibrium is given by

$$
\begin{equation*}
\Pi_{A}^{*}-\Pi_{B}^{*}=\frac{2 \rho-1}{3} \tag{A.29}
\end{equation*}
$$

It then holds

$$
\begin{equation*}
\Pi_{A}^{U}-\Pi_{B}^{U}>\Pi_{A}^{*}-\Pi_{B}^{*} \Leftrightarrow \rho \in\left(\frac{1}{2}, 1\right) . \tag{A.30}
\end{equation*}
$$

## Appendix B

## Appendix to Chapter 2

## B. 1 Proofs

## B.1.1 Proof of Proposition 2.1

Let $\left\{\left(p_{I}^{*}, P_{I}^{*}\right),\left(p_{E}^{*}, P_{E}^{*}\right)\right\}$ be the equilibrium prices that solve the system of equations generated by the first order conditions (2.15)-(2.18).

In the advance selling period the price difference between the two firms is given by

$$
\begin{equation*}
p_{I}^{*}-p_{E}^{*}=\frac{c}{3}+\frac{(2 \rho-1)}{3}, \tag{B.1}
\end{equation*}
$$

and in the consumption period is given by

$$
\begin{equation*}
P_{I}^{*}-P_{E}^{*}=\frac{c}{3}+\frac{(2 \rho-1)}{3 \rho} . \tag{B.2}
\end{equation*}
$$

Both are clearly positive for all $\rho \in\left(\frac{1}{2}, 1\right)$ and $c>0$.
The absolute advance purchase discount difference is given by

$$
\begin{equation*}
\left(P_{I}^{*}-p_{I}^{*}\right)-\left(P_{E}^{*}-p_{E}^{*}\right)=\frac{(1-\rho)(2 \rho-1)}{3 \rho}, \tag{B.3}
\end{equation*}
$$

which is positive for all $\rho \in\left(\frac{1}{2}, 1\right)$.
The difference in the relative advance purchase discounts is given by

$$
\begin{equation*}
\frac{P_{E}^{*}-p_{E}^{*}}{P_{E}^{*}}-\frac{P_{I}^{*}-p_{I}^{*}}{P_{I}^{*}}=\frac{(1-\rho)(c+2 \rho-1)}{(c+1)(2 c \rho+3 \rho-1)}, \tag{B.4}
\end{equation*}
$$

which is also positive for all $\rho \in\left(\frac{1}{2}, 1\right)$ and $c>0$.

## B.1.2 Proof of Proposition 2.2

Replacing the equilibrium prices in (2.10)-(2.12), we obtain the following equilibrium thresholds:

$$
\begin{align*}
\sigma_{W}^{*}(I) & =\frac{2-c}{3}  \tag{B.5}\\
\sigma_{W}^{*}(E) & =\frac{c \rho+1}{3 \rho}  \tag{B.6}\\
\bar{\sigma}^{*} & =\frac{c+2 \rho-1}{3(2 \rho-1)} . \tag{B.7}
\end{align*}
$$

It holds that

$$
\begin{equation*}
\sigma_{W}^{*}(I)-\bar{\sigma}^{*}=\frac{2(1-c) \rho-1}{3(2 \rho-1)}>0 \Leftrightarrow \rho>\frac{1}{2(1-c)} \tag{B.8}
\end{equation*}
$$

We show that if (B.8) is satisfied, then all other relevant thresholds are in the interval $(0,1)$ and $0<\bar{\sigma}^{*}<\sigma_{W}^{*}(E)<\sigma_{W}^{*}(I)<1$. Note that, since $\rho \in\left(\frac{1}{2}, 1\right)$, equation (B.8) implies that $c \in\left(0, \frac{1}{2}\right)$.

The equations (B.5), B.7) and B.8 together with $\rho \in\left(\frac{1}{2}, 1\right)$ and $c \in$ ( $0, \frac{1}{2}$ ) imply that $0<\bar{\sigma}^{*}<\sigma_{W}^{*}(I)<1$. Moreover, it holds

$$
\begin{array}{r}
\sigma_{W}^{*}(I)-\sigma_{W}^{*}(E)=\frac{2(1-c) \rho-1}{3 \rho}, \\
\sigma_{W}^{*}(E)-\bar{\sigma}^{*}=\frac{(1-\rho)[2(1-c) \rho-1]}{3 \rho(2 \rho-1)}, \tag{B.10}
\end{array}
$$

which are positive if (B.8) is satisfied.

## B.1.3 Proof of Proposition 2.3

The difference in the entrant's profits is given by

$$
\begin{equation*}
\Pi_{E}^{U}-\Pi_{E}^{*}=\frac{4(1-\rho)\left(2 \rho-1-c^{2} \rho^{2}\right)}{9 \rho(2 \rho-1)} \tag{B.11}
\end{equation*}
$$

which, given $\rho \in\left(\frac{1}{2}, 1\right)$, is positive if and only if

$$
\begin{equation*}
f(\rho, c) \equiv 2 \rho-1-c^{2} \rho^{2}>0 \tag{B.12}
\end{equation*}
$$

Note that the equilibrium existence condition (B.8) is equivalent to

$$
\begin{equation*}
c<1-\frac{1}{2 \rho} . \tag{B.13}
\end{equation*}
$$

It therefore holds

$$
\begin{align*}
f(\rho, c) & =2 \rho-1-c^{2} \rho^{2}  \tag{B.14}\\
& >2 \rho-1-c \rho^{2}  \tag{B.15}\\
& >2 \rho-1-\left(1-\frac{1}{2 \rho}\right) \rho^{2}  \tag{B.16}\\
& =\frac{5}{2} \rho-1-\rho^{2}>0 \quad \forall \rho \in\left(\frac{1}{2}, 1\right), \tag{B.17}
\end{align*}
$$

which proves B.12).

## B.1.4 Proof of Proposition 2.4

The difference in the entrant's market share is given by

$$
\begin{equation*}
q_{E}^{*}-q_{E}^{U}=\frac{4 c(1-\rho) \rho}{3(2 \rho-1)} \tag{B.18}
\end{equation*}
$$

which is positive for $\rho \in\left(\frac{1}{2}, 1\right)$ and $c>0$.

|  | H1 |  |  |  | H2 |  |  | H3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $p_{22}$ | $p_{15}$ | $p_{8}$ | $p_{1}$ | $\mathrm{a} \Delta p_{22}$ | $\mathrm{a} \Delta p_{15}$ | a $\Delta p_{8}$ | $\mathrm{r} \Delta p_{22}$ | $\mathrm{r} \Delta p_{15}$ | $\mathrm{r} \Delta p_{8}$ |
| Intercept | $\begin{gathered} \hline 31.18^{* *} \\ (.22) \end{gathered}$ | $\begin{gathered} 32.08^{* *} \\ (.22) \end{gathered}$ | $\begin{gathered} 32.32^{* *} \\ (.20) \end{gathered}$ | 47.78** <br> (.11) | $\begin{gathered} \hline 16.69^{* *} \\ (.21) \end{gathered}$ | $\begin{gathered} 15.79^{* *} \\ (.20) \end{gathered}$ | $\begin{gathered} 15.61^{* *} \\ (.17) \end{gathered}$ | $\begin{aligned} & .3438^{* *} \\ & (.0051) \end{aligned}$ | $\begin{aligned} & .3250^{* *} \\ & (.0049) \end{aligned}$ | $\begin{aligned} & .3232^{* *} \\ & (.0044) \end{aligned}$ |
| IT ALO | $\begin{gathered} -5.82^{* *} \\ (.28) \end{gathered}$ | $\begin{gathered} -5.98^{* *} \\ (.27) \end{gathered}$ | $\begin{gathered} -5.43^{* *} \\ (.26) \end{gathered}$ | $\begin{gathered} -6.29^{* *} \\ (.14) \\ \hline \end{gathered}$ | $\begin{aligned} & -.57^{*} \\ & (.26) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-.42 \\ & (.25) \\ & \hline \end{aligned}$ | $\begin{gathered} -1.02^{* *} \\ (.22) \end{gathered}$ | $\begin{aligned} & .0642^{* *} \\ & (.0064) \end{aligned}$ | $\begin{aligned} & .0676^{* *} \\ & (.0062) \end{aligned}$ | $\begin{aligned} & .0472^{* *} \\ & (.0056) \end{aligned}$ |
| ROUTE $_{1}$ | $\begin{gathered} \hline-15.35^{* *} \\ (.33) \\ \hline \end{gathered}$ | $\begin{gathered} \hline-15.81^{* *} \\ (.32) \\ \hline \end{gathered}$ | $\begin{gathered} -15.81^{* *} \\ (.30) \\ \hline \end{gathered}$ | $\begin{gathered} -22.00^{* *} \\ (.17) \\ \hline \end{gathered}$ | $\begin{gathered} \hline-6.77^{* *} \\ (.30) \\ \hline \end{gathered}$ | $\begin{gathered} \hline-6.31^{* *} \\ (.29) \\ \hline \end{gathered}$ | $\begin{gathered} \hline-6.38^{* *} \\ (.25) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline .0351^{* *} \\ & (.0075) \\ & \hline \end{aligned}$ | $\begin{aligned} & .0387^{* *} \\ & (.0072) \end{aligned}$ | $\begin{aligned} & \hline .0304^{* *} \\ & (.0065) \\ & \hline \end{aligned}$ |
| ROUTE $_{2}$ | $\begin{gathered} \hline-12.95^{* *} \\ (.30) \end{gathered}$ | $\begin{gathered} \hline-13.16^{* *} \\ (.29) \end{gathered}$ | $\begin{gathered} -12.12^{* *} \\ (.27) \\ \hline \end{gathered}$ | $\begin{gathered} -11.56^{* *} \\ (.15) \\ \hline \end{gathered}$ | $\begin{gathered} 1.33^{* *} \\ (.28) \end{gathered}$ | $\begin{gathered} 1.56^{* *} \\ (.26) \end{gathered}$ | $\begin{aligned} & .47^{*} \\ & (.23) \end{aligned}$ | $\begin{aligned} & .1589^{* *} \\ & (.0069) \end{aligned}$ | $\begin{aligned} & .1584^{* *} \\ & (.0066) \end{aligned}$ | $\begin{aligned} & .1217^{* *} \\ & (.0059) \end{aligned}$ |
| RUSHHOU R | $\begin{gathered} 1.75^{* *} \\ (.26) \end{gathered}$ | $\begin{aligned} & 1.71^{* *} \\ & (.25) \end{aligned}$ | $\begin{gathered} 1.76^{* *} \\ (.24) \end{gathered}$ | $\begin{aligned} & .76^{* *} \\ & (.14) \end{aligned}$ | $\begin{gathered} -.82^{* *} \\ (.24) \end{gathered}$ | $\begin{gathered} -.77^{* *} \\ (.23) \end{gathered}$ | $\begin{gathered} -.70^{* *} \\ (.21) \end{gathered}$ | $\begin{gathered} -.0257^{* *} \\ (.0060) \end{gathered}$ | $\begin{aligned} & -.0251^{* *} \\ & (.0058) \\ & \hline \end{aligned}$ | $\begin{aligned} & -.0231^{* *} \\ & (.0052) \end{aligned}$ |
| Obs. $R^{2}$ | 2135 .6308 | 2135 0.6542 | $\begin{gathered} 2134 \\ 0.6595 \end{gathered}$ | 2096 0.9080 | 2096 0.2516 | 2096 0.2528 | 2095 0.2754 | 2096 0.2436 | 2096 0.2586 | 2095 0.1997 |

Table B.1: Regression including $R U S H H O U R$ (standard deviation), ${ }^{* *}=1-\%$ significance, ${ }^{*}=5-\%$ significance.

## Appendix C

## Appendix to Chapter 3

## C. 1 Proofs

## C.1.1 Proof of Proposition 3.1

Separation by ability Let $\varphi<\frac{\theta_{H}}{\theta_{L}}$. For wages $w_{L}$ and $w_{H}$ it holds that $e_{H}\left(\theta_{L}, r_{R}\right)=\sqrt{(1+\eta \lambda) \theta_{L}\left(w_{H}-w_{L}\right)}<\sqrt{(1+\eta) \theta_{H}\left(w_{H}-w_{L}\right)}=e_{H}\left(\theta_{H}, r_{P}\right)$
where $e_{H}\left(\theta_{H}, r_{P}\right)$ and $e_{H}\left(\theta_{L}, r_{R}\right)$ follow from the equations (3.6) and (3.7). That is a high ability worker from a poor household is willing to get a higher level of education for a wage $w_{H}$ than a low ability worker from a rich household. $e_{H}\left(\theta_{L}, r_{R}\right)$ also defines the minimal level of education such that only high ability workers acquire education. The equilibrium wages are then given by the expected productivity of each group, that is $w_{H}^{*}=\theta_{H}$ and $w_{L}^{*}=\theta_{L}$. It follows that $e_{H}\left(\theta_{L}, r_{R}\right)=\sqrt{(1+\eta \lambda) \theta_{L}\left(\theta_{H}-\theta_{L}\right)}$. The upper bound for $e_{H}$ is defined by the maximum level of education a high ability worker from a poor household is willing to get, that is $e_{H}\left(\theta_{H}, r_{P}\right)=\sqrt{(1+\eta) \theta_{H}\left(\theta_{H}-\theta_{L}\right)}$.

Separation by income Let $\varphi>\frac{\theta_{H}}{\theta_{L}}$. For wages $w_{L}$ and $w_{H}$ it holds that

$$
\begin{equation*}
e_{H}\left(\theta_{H}, r_{P}\right)=\sqrt{(1+\eta) \theta_{H}\left(w_{H}-w_{L}\right)}<\sqrt{(1+\eta \lambda) \theta_{L}\left(w_{H}-w_{L}\right)}=e_{H}\left(\theta_{L}, r_{R}\right) \tag{C.2}
\end{equation*}
$$

where $e_{H}\left(\theta_{H}, r_{P}\right)$ and $e_{H}\left(\theta_{L}, r_{R}\right)$ follow from equation (3.6) and (3.7). That is a low ability worker from a rich household is willing to get a higher level of education for a wage $w_{H}$ than a high ability worker from a poor household. $e_{H}\left(\theta_{H}, r_{P}\right)$ also defines the minimal level of education such that only workers from rich households acquire education. The equilibrium wages are then given by the expected productivity of each group, that is $w_{H}^{*}=$ $h \theta_{H}+(1-h) \theta_{L}$ and $w_{L}^{*}=h \theta_{L}+(1-h) \theta_{H}$. Note that since $h>\frac{1}{2}$, $w_{H}^{*}>w_{L}^{*}$. It follows that $e_{H}\left(\theta_{H}, r_{P}\right)=\sqrt{(2 h-1)(1+\eta) \theta_{H}\left(\theta_{H}-\theta_{L}\right)}$. The upper bound for $e_{H}$ is defined by the maximum level of education a low ability worker from a rich household is willing to get, that is $e_{H}\left(\theta_{L}, r_{R}\right)=$ $\sqrt{(2 h-1)(1+\eta \lambda) \theta_{L}\left(\theta_{H}-\theta_{L}\right)}$.

## C.1.2 Proof of Lemma 3.1

The difference between welfare including gain-loss utility and not is given by

$$
\begin{equation*}
\mathcal{W}_{l}-\mathcal{W}_{0}=\frac{\eta}{2}\left[\bar{w}_{P}-r_{P}-\lambda\left(\bar{w}_{R}-r_{R}\right)\right], \tag{C.3}
\end{equation*}
$$

where $\bar{w}_{P}$ and $\bar{w}_{R}$ are the average wages of workers from poor or rich households, respectively. Since education does not increase productivity and firms are in competition for workers, the average wage is the same in all types of equilibria. This means that when comparing welfares of different equilibria, it does not play a role whether one includes gain-loss utility in the welfare function or not.

## C.1.3 Proof of Proposition 3.2

Let $\varphi=\frac{\theta_{H}}{\theta_{L}}$. With the equilibrium values from Proposition 3.1 the welfare in a separation by ability equilibrium is given by

$$
\begin{equation*}
\mathcal{W}_{0}^{A}=-\frac{1}{2} \eta\left(\theta_{H}-\theta_{L}\right)+\theta_{L} \tag{C.4}
\end{equation*}
$$

and the welfare in a separation by income equilibrium is given by

$$
\begin{align*}
\mathcal{W}_{0}^{I}= & \frac{1}{2 \theta_{L}}\left\{-(2 h-1)(1-h)(1+\eta) \theta_{H}^{2}+\left[1-(2 h-1)^{2}(1+\eta)\right] \theta_{H} \theta_{L}\right. \\
& \left.+[(1+\eta)(2 h-1) h+1] \theta_{L}^{2}\right\} . \tag{C.5}
\end{align*}
$$

It then holds

$$
\mathcal{W}_{0}^{A}>\mathcal{W}_{0}^{I} \Leftrightarrow \frac{\theta_{H}}{\theta_{L}}>\frac{2 h+1}{2 h-1} .
$$

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# Statement of Authorship 

## Selbständigkeitserklärung

Ich erkläre hiermit, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen benutzt habe. Alle Koautorenschaften sowie alle Stellen, die wörtlich oder sinngemäss aus Quellen entnommen wurden, habe ich als solche gekennzeichnet. Mir ist bekannt, dass andernfalls der Senat gemäss Artikel 36 Absatz 1 Buchstabe r des Gesetzes vom 5. September 1996 über die Universität zum Entzug des aufgrund dieser Arbeit verliehenen Titels berechtigt ist.

Bern, 21.08.2021

Nadia Jlenia Ceschi


[^0]:    ${ }^{1}$ Proposition 1.2 pins down the necessary parameter conditions for equilibrium existence.

[^1]:    ${ }^{2} \mathrm{~A}$ sufficient condition for the existence of a price discrimination equilibrium is that heterogeneity is sufficiently small. Möller and Watanabe (2016) show that such a price discrimination equilibrium exists for homogeneous firms. By continuity of firms' profit functions, this result extends to heterogeneous firms, as long as the asymmetry is not too high.

[^2]:    ${ }^{3}$ The expected discount of a consumer whose favorite product is $S=A$ is given by $\mathbb{E}[\Delta p \mid S=A]=\gamma p_{2, A}^{*}+(1-\gamma) p_{2, B}^{*}-p_{1, A}^{*}=\Delta p_{A}+(1-\gamma)\left(p_{2, B}^{*}-p_{2, A}^{*}\right)-p_{1, A}^{*}$. Anlogously, the one of a consumer with $S=B$ is given by $\mathbb{E}[\Delta p \mid S=B]=\Delta p_{B}+(1-\gamma)\left(p_{2, A}^{*}-p_{2, B}^{*}\right)-p_{1, B}^{*}$. From Proposition 1.1 it follows that $\mathbb{E}[\Delta p \mid S=A]>\mathbb{E}[\Delta p \mid S=B]$.

[^3]:    ${ }^{4}$ If we assume that the more prominent firm sets a lower price, we find a contradiction in the prices that solve the first-order conditions.

[^4]:    ${ }^{5}$ As in the case with asymmetric efficiency, if $p_{1, i} \geq p_{2, i}$ all consumers postpone their purchase to the second period and firms could therefore decrease $p_{1, i}$ until $p_{1, i}=p_{2, i}$ without reducing their profits.

[^5]:    ${ }^{6}$ Equilibrium existence follows from Möller and Watanabe (2016) for sufficiently small heterogeneity, like in the case of asymmetric costs.

[^6]:    ${ }^{1}$ Evidence of LCCs intertemporal pricing and its effect on FSCs can be found, for example, in Mason (2006), Alderighi et al. (2011), and Alderighi et al. (2012).

[^7]:    ${ }^{2}$ https://www.transportation.gov/individuals/aviation-consumer-protection/refunds $\sqrt[3]{\text { https://www.united.com/ual/en/us/fly/travel/israel-consumer-protection-law.html }}$

[^8]:    ${ }^{4}$ In Chapter 1 we assume $c_{A} \leq c_{B}$ and $\rho \geq \frac{1}{2}$. Since here we consider a prominent incumbent facing an efficient entrant, we set $c_{A}>c_{B}$. The results in Chapter 1 are still valid after the appropriate adjustments.

[^9]:    ${ }^{5}$ The upperbound on $c$ is determined by the existence of a price discrimination equilibrium, which we discuss in Section 2.4.4.

[^10]:    ${ }^{6}$ If we assume that the incumbent sets a lower price, we find a contradiction in the prices that solve the first-order conditions.

[^11]:    ${ }^{7}$ If we assume the opposite, we then find a contradiction to the assumption in the price-scheme that satisfies the necessary conditions.

[^12]:    ${ }^{8}$ If the cost asymmetry is high enough compared to the prominence asymmetry, then the entrant has higher profit margins and sells a higher quantity than the incumbent.

[^13]:    ${ }^{9}$ Regarding the sufficient conditions, see the Footnotes 2 and 6 in Chapter 1

[^14]:    ${ }^{10}$ https://italospa.italotreno.it/en/investor-relations/the-high-speed-rail-transportmarket.html
    ${ }^{11}$ Data on these routes was readily available from Etzensperger $(2020)$.

[^15]:    ${ }^{1}$ https://www.gazzetta.it/Calcio/Serie-A/Milan/07-01-2021/milan-famiglia-maldini-quota-1000-presenze-serie-3902299936691.shtml

[^16]:    $2^{2}$ https://www.nytimes.com/interactive/2017/11/22/upshot/the-jobs-youre-most-likely-to-inherit-from-your-mother-and-father.html

[^17]:    ${ }^{3}$ Conclusions on welfare should be considered carefully. As in Spences (1973) model we do not consider returns to education. If education increases a worker's productivity we might obtain different results.

[^18]:    ${ }^{4}$ More precisely, the difference workers' abilities must be low enough such that $\frac{\theta_{H}}{\theta_{L}}<\frac{-h(2 h-1)(1+\eta)+\sqrt{h^{2}(2 h-1)^{2}(1+\eta)^{2}+4(2 h-1)(1-h)(1+\eta)}}{2(2 h-1)(1-h)(1+\eta)}$.

