Essays in Heterogeneity in Macroeconomics

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Introduction

This thesis comprises three chapters, which study normative aspects in the field of monetary policy, under the lens of Heterogeneous Agents New Keynesian theoretical framework (HANK).

In the first chapter, optimal monetary and fiscal policy are jointly analyzed in a heterogeneous two-agents New Keynesian environment, where fiscal policy is modeled in the form of lump-sum transfers set by the government. The main result is that transfer policy does not serve as a substitute for forward guidance - as it entails consumption dispersion costs - and does not affect its optimal duration. Transfers indeed influence the length of stay at the zero lower bound through two offsetting channels: a *shortening* channel works through an initial increase in transfers that mitigates the recession (reducing the need for forward guidance), and a *lengthening* channel works through a later transfer cut that curbs the undesired expansion (making forward guidance desirable for a longer horizon). Imposing a homogeneous transfer policy across agents does not change the stabilization outcome or the effect on the duration of forward guidance, nor does so allowing for cyclical income differences.

The second chapter analyzes the monetary policy trade-off between defending purchasing power of consumers and keeping moderate debt cost to borrowers, in the framework of a heterogeneous agents New Keynesian open economy hit by a foreign energy price shock. Raising the interest rate indeed fights the real depreciation of domestic goods and wages due the energy shock, at the expense of an increase in debt costs of borrowers. The trade-off can be resolved by implementing a milder interest rate policy during the crisis in exchange for a prolonged contraction beyond the energy shock time span: this forward guidance approach allows to still enjoy a real appreciation today at the expense of a more smoothed effect on mortgage cost over time. This policy counterfactual is analyzed in a quantitative model of the UK economy under the 2022-2023 energy price hike, where the loss of consumers' purchasing power and the vulnerability of mortgagors to higher policy rates have been elements of paramount empirical relevance.

The third chapter proposes a new method to solve for optimal policy in heterogeneous agents New Keynesian models (HANK). It builds on the discretize-then-optimize method by Nuño, Gonzalez, Thaler, and Albrizio (2023), and reduces its computational complexity by leveraging the linearity of the first order conditions of the Ramsey planner in terms of the co-states of the problem. An application is carried out in the case of optimal management of energy shocks in HANK.

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Chapter 1

Optimal Monetary and Transfer Policy in a Liquidity Trap

1 Introduction

The challenges posed by a liquidity trap to stabilization policy relate to the impossibility to fully address a demand shock by conventional monetary tools, due to the zero lower bound on the nominal interest rate. Several authors have explored policy alternatives to overcome this émpasse, within the perimeter of monetary policy or fiscal policy. The range of considered policies include forward guidance on the nominal interest rate (according to the seminal paper by Eggertsson and Woodford (2003)), quantitative easing (Gertler and Karadi (2013)), distortionary taxation (Eggertsson and Woodford (2006)), helicopter money (Benigno and Nisticò (2020)) and lump sum transfers.

This paper studies lump sum transfers from the perspective of an optimal monetary and fiscal policy problem. The setting is one with heterogeneous agents, in which transfers have different effects on consumption due to a heterogeneity in marginal propensity to consume (MPC). In this perspective, the conventional view is that transfers can be used as a substitute of forward guidance over the recessionary phase of the liquidity trap, as they can produce an expansionary stimulus (see Eggertsson and Krugman (2012), Farhi and Werning (2016), Wolf (2021)). In this paper I challenge this view by showing first that, from an optimal policy perspective, transfers are not a substitute tool for forward guidance; secondly, that their optimal use over the liquidity trap does not even influence the optimal time of the liftoff of the interest rate from the zero lower bound.

The first result relies on the fact that transfer policies create consumption variations across households, that negatively affects welfare: therefore, a government seeking to use transfers to mitigate output fluctuations in the liquidity trap faces a trade-off between stabilization and consumption dispersion. The second result relies on the presence of a *shortening* and *lengthening* role of transfer policy with respect to the duration of forward guidance, which quantitatively offset each other. Over the early stages of the liquidity trap, the government wants to transfer resources to the high MPC agents (in my model, "hand to mouth" households) in order to mitigate the drop in output: this effect alone would reduce the room for forward guidance intervention, and shorten the optimal stay of the interest rate at the zero lower bound. When the shock is over and the interest rates are still kept at zero, then the government would like to cut the transfers to the hand to mouth in order to cool down the overheating of the economy - making a delay of the interest rate liftoff more desirable, as forward guidance becomes less costly in terms of output expansion. The shortening and lengthening channels lean against each other, giving rise to an analytically ambiguous effect on the duration of forward guidance. I calibrate the model using standard parameters assumed in the liquidity trap literature, finding that quantitatively these effects offset each other, leaving no significant influence on the optimal duration of stay at the zero lower bound.

Results are robust to two important extensions. First, I consider an environment in which the government is constrained to set the same transfer for all the households (as in Wolf (2021)). While in the baseline case the transfer to the hand to mouth could be used for stabilization, and the transfer to the other households could be set to satisfy public debt solvency, instead, with homogeneous transfer policy, the transfer is set equal for all the households: therefore it needs to satisfy both the goals, creating a trade-off between stabilization and public debt solvency. My finding points out that this additional trade-off is negligible, as any necessary public debt adjustment required by government policy can be smoothed out by long run movements in transfers - so well beyond the end of the liquidity trap.

Then, I study optimal transfer policy in a general environment where I allow for cyclical income differences between the hand to mouth and the other households (see Bilbiie (2018)), other than the ones implied by transfer policy. In this case, the extent to which the hand to mouth consumes more than the other households over the trap depends both on transfers and these additional sources of income differences. I show that the optimal transfer pattern is replaced by an optimal *augmented transfer* pattern, which incorporates both the transfer and the other cyclical income differences. By setting the transfer, the government can fully control the pattern of the augmented transfer, so it can still achieve the same stabilization results of the baseline setting. The results in terms of the role of transfers and their effect on forward guidance extend also to this more general framework.

This paper formulates an optimal fiscal-monetary policy problem in a liquidity trap, following in spirit Eggertsson and Woodford (2006). While they model fiscal policy as a distortionary VAT tax, I analyse lump sum transfers. The model builds on a literature exploring optimal policy in a TANK environment: Bilbiie, Monacelli, and Perotti (2020) analyse optimal monetary and transfer policy, where consumption dispersion arises from the tax scheme financing government spending; Hansen, Mano, and Lin (2020) treat instead optimal monetary policy alone in a two agents new keynesian environment. An analysis of optimal monetary policy in a TANK setting over the liquidity trap is carried out in Eggertsson and Krugman (2012) and in Benigno, Eggertsson, and Romei (2020): I contribute to these works by using the TANK framework to analyse a joint fiscal and monetary policy. The paper also contributes to the analysis of inequality effects of stabilization policies in TANK models, carried out in Debortoli and Galí (2017), Punzo and Rossi (2023) and Komatsu (2023): these papers allow for an effect of monetary policy on inequality through wealth effects on hand to mouth households (when characterized as *borrowers*) or through asymmetric profit redistribution. I abstract from these features, obtaining inequality-neutral monetary policy, and focus instead on consumption dispersion driven by transfer policy alone.

In the heterogeneous agents (HANK) literature, optimal monetary policy has been analysed in Acharya, Challe, and Dogra (2021), Nuño et al. (2023) and Ragot (2017) - the latter in a liquidity trap scenario; Le Grand, Martin-Baillon, and Ragot (2022) treats optimal fiscal policy, while Bhandari, Evans, Golosov, and Sargent (2021) and Wolf (2022) analyse the optimal fiscal-monetary mix. I contribute to these last two papers by studying optimal fiscal and monetary policy in a liquidity trap.

The effect of transfers on aggregate output , disentangled from an optimal policy perspective, is addressed in Farhi and Werning (2016), McKay and Reis (2013), Mehrotra (2018), Giambattista and Pennings (2017). Wolf (2021) shows an equivalence result in aggregate inflation-output stabilization between interest rate and stimulus check policies. I embed the results of this literature in my paper, by considering the role of transfers in achieving output and inflation stabilization.

Last, this paper relates also to the analysis of cyclical inequality in the liquidity trap (see Bilbiie (2021)), which I account for in the second extension of the model. The paper is organized as follows: section 2 reports the model's features: section 3 illustrates the main results in terms of transfer policy over the liquidity trap. Sections 4 and 5 develop the extensions with respect to the homogeneous transfer response and cyclical income difference.

2 Model

2.1 Households

An infinite-horizon economy features unit mass of households, with a fraction $1-\lambda$ of "ricardian" and λ of "hand-to-mouth" ("HtM"). The ricardian households can access to a financial market for short term bonds, in which they can save or borrow, whereas this possibility is instead precluded to the hand to mouth. The ricardian solves the following utility maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \xi_s \beta^t \left(1 - \exp(-zC_t) - \delta \exp(\eta L_t)\right)$$

s.t.

$$P_t C_t + \frac{B_t}{1+i_t} \leq W_t L_t + B_{t-1} + T_t + \frac{1-\chi}{1-\lambda} P_t \bar{D}_t \tag{2.1.1}$$

$$\lim_{s\to\infty} \beta^{s-t} \frac{B_s}{P_s} = 0 \tag{2.1.2}$$

where C_t is consumption, L_t is labor supply, B_t is bond holding, W_t is the nominal wage, P_t is the aggregate price index, T_t is a nominal transfer from the public sector, L_t is labor supply, β is the discount factor, ξ_t is an intertemporal preference shock[1], and z, δ, η are positive parameters. I assume that each period a fraction $1-\chi$ of the total amount of real firms' profits $\bar{D}_t$ is rebated evenly across the $1-\lambda$ ricardian households, and the rest to the hand to mouth. The last condition (2.1.2) is the transversality condition on bond holding. A particular remark relates to the adoption of exponential utility: it is suitable to maintain tractability in building an aggregate demand and supply for the economy in a heterogenous agents setting as the current one.

Consumption is specified by a Dixit-Stigliz aggregator of a unit mass of varieties:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{2.1.3}$$

where $C_t(j)$ is ricardian household's consumption of good of variety j and $\theta > 1$ is the elasticity of substitution between goods. First order conditions imply variety demand

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} C_t$$

whose sensitivity to the ratio between the variety price $P_t(j)$ and the price index P_t is measured by the elasticity θ - a standard result. The first order condition for labor supply implies:

$$\delta\eta \exp(\eta L_t) = z \exp(-zC_t) \frac{W_t}{P_t} \tag{2.1.4}$$

where the marginal disutility of labor is equated to the marginal utility of consumption multiplied

[1]Without loss of generality, I set $\xi_{-1} = 1$

by the real wage. The Euler equation is given by:

$$z\exp(-zC_t) = \xi_t\beta(1+i_t)E_t\left[z\exp(-zC_{t+1})\frac{P_t}{P_{t+1}}\right] \tag{2.1.5}$$

where the shock ξ_t affects the intertemporal consumption choice of the household: the higher is the realization of ξ_t, the more the household is propense to shift consumption from period t to $t+1$.
Let us now turn the attention to the hand to mouth problem. The latter writes similarly to the ricardian's one, with the notable differences that the household cannot trade in bonds. The hand to mouth receives a transfer T_t^* from the public sector- analogously to the ricardian household; moreover, a fraction χ of the total real dividend amount is rebated evenly across the λ hand to mouth households. The problem of the hand to mouth writes

$$\max E_0\sum_{t=0}^{\infty}\prod_{s=0}^{t-1}\xi_s\beta^t\left(1-\exp(-zC_t^*)-\delta\exp(\eta L_t^*)\right)$$

s.t.

$$P_tC_t^* \leq W_tL_t^* + \frac{\chi}{\lambda}P_t\bar{D}_t + T_t^* \tag{2.1.6}$$

where C_t^* and L_t^* are the consumption level and the labor supplied, respectively. I will assume for now $\chi = \lambda$, so that the share of profit levied to the hand to mouth is equal to the share of this type of household out of total population. This assumption implies that each period the hand to mouth receives the same dividend amount of the ricardian household.
Assuming C_t^* to have the same Dixit-Stigliz aggregator form of (2.1.3), the hand to mouth demand for the variety of good is specular to the ricardian household case:

$$C_t^*(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta}C_t^* \tag{2.1.7}$$

Taking the first order condition with respect to labor in the hand to mouth problem, we also obtain a labor supply condition analogous to the ricardian household:

$$\delta\eta\exp(\eta L_t^*) = -z\exp(-zC_t^*)\frac{W_t}{P_t} \tag{2.1.8}$$

Optimally, the budget constraint (2.1.6) hold with equality, pinning down the consumption of the hand to mouth for each period t:

$$P_t C_t^* = L_t^* W_t + T_t^* + P_t \bar{D}_t \tag{2.1.9}$$

Due to the lack of access to the bond market, the hand to mouth cannot save or borrow: therefore each period the whole sum of labor income and transfers is spent in consumption.

2.2 Firms

There is a unit mass of monopolistically competitive firms, each one producing a different variety j of good, with technology:

$$Y_t(j) = AL_t(j)$$

where $L_t(j)$ is labor demanded by firm j, and A is labor productivity. Each firm faces a probability α each period of not being able to reset its price; in that case, its price automatically increases by the steady state inflation Π. When a firm resets its price, it seeks to maximize its expected discounted sum of profits, adjusted for the probability of not being able to reoptimize in the future:

$$\begin{aligned} \max_{P_t(j)} E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \frac{1}{P_T} \Bigg[& \Pi^{T-t} P_t(j) \left(\frac{P_t(j)\Pi^{T-t}}{P_T} \right)^{-\theta} Y_T \\ & - \frac{W_T}{A}(1-\nu) \left(\frac{P_t(j)\Pi^{T-t}}{P_T} \right)^{-\theta} Y_T - \zeta_T \Bigg] \end{aligned} \tag{2.2.1}$$

where the term ν is a government subsidy on labor costs and ζ_t is a lump sum tax. Firms value future profits according to an average Λ_t of marginal utilities of the two households, weighted by the respective profit shares: $\Lambda_t = (1-\chi)z\exp(-zC_t) + \chi z \exp(-zC_t^*)$ (see Benigno et al. (2020)). The first order condition for the optimal pricing problem yields:

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta-1}(1-\nu) \frac{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \frac{W_T}{P_T} \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta} \frac{Y_T}{A}}{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \prod_{s=t}^{T-1} \xi_s \Lambda_T \left(\frac{P_T}{P_t} \frac{1}{\Pi^{T-t}} \right)^{\theta-1} Y_T} \tag{2.2.2}$$

where P_t^* is the optimal price set by the resetting firms at time t. Equation (2.2.2) shows how firms set current price price by taking into account future discounted flow of costs and revenues, weighted by the probability of not be able to reset the price in the future.

Calvo pricing implies the following standard motion for inflation:

$$P_t^{1-\theta} = (1-\alpha)P_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\Pi^{1-\theta} \tag{2.2.3}$$

Or equivalently:

$$1 = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1} \tag{2.2.4}$$

The optimal price setting condition (2.2.2) and the law of motion (2.2.4) give rise to the usual forward-looking expression for inflation in sticky price models (New Keynesian Phillips curve):

$$\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}} = \frac{F_t}{K_t} \tag{2.2.5}$$

where:

$$F_t = Y_t\Lambda_t + \alpha\beta\xi_t E_t\left\{F_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta-1}\right\} \tag{2.2.6}$$

$$K_t = \frac{\theta}{\theta-1}(1-\nu)\Lambda_t\frac{W_t}{P_t}\frac{Y_t}{A} + \alpha\beta\xi_t E_t\left\{K_{t+1}\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\theta}\right\} \tag{2.2.7}$$

2.3 Public sector

Public sector sets bond supply $\bar{B}_t$ and taxes T_t. It needs to satisfy the following flow constraint:

$$\bar{B}_t\frac{1}{1+i_t} = \bar{B}_{t-1} + (1-\lambda)T_t + \lambda T_t^* + V_tP_t - \zeta_tP_t \tag{2.3.1}$$

The resources gathered through the new debt issued $\bar{B}_t$ serves to repay the existing debt $\bar{B}_{t-1}$ and to finance the transfers to the agents T_t, T_t^*. The spending for subsidy $V_t = \nu(W_t/P_t)((1-\lambda)L_t + \lambda L_t^*)$ is exactly financed by the lump sum tax on firms ζ_t:

$$V_t = \zeta_t \tag{2.3.2}$$

Levying the lump sum fiscal burden of subsidies on firms allows to isolate the transfers T_t, T_t^* as the only lump sum fiscal instrument affecting the budget constraint of the household. I assume that transfers are set by the public sector in real terms. I will refer to these quantities by the following notation:

$$\tau_t \equiv \frac{T_t}{P_t} \quad , \quad \tau_t^* \equiv \frac{T_t^*}{P_t} \tag{2.3.3}$$

The public sector also sets the nominal interest rate i_t. The interest rate policy is constrained by a zero lower bound:

$$i_t \geq 0 \tag{2.3.4}$$

2.4 Equilibrium

The equilibrium is given by the households' optimality conditions (2.1.4),(2.1.5),(2.1.8),(2.1.9), the firms' optimality condition (2.2.5), the public sector budget constraint (2.3.1) together with the market clearing conditions:

$$Y_t(i) = (1-\lambda)C_t(i) + \lambda C_t^*(i) \quad \forall i \tag{2.4.1}$$

$$Y_t = (1-\lambda)C_t + \lambda C_t^* \tag{2.4.2}$$

$$\bar{B}_t = (1-\lambda)B_t \tag{2.4.3}$$

$$\frac{Y_t \Delta_t}{A} = (1-\lambda)L_t + \lambda L_t^* \equiv \bar{L}_t \tag{2.4.4}$$

The first two market clearing conditions above are the ones holding in the the goods market: for each variety and at the aggregate level, supply needs to be equal to the sum of the consumption levels of each household type, multiplied by the relative mass. The second condition equalizes aggregate bond supply to the aggregate demand for bonds of the ricardian households, which are the only ones who can hold them. The third condition is the market clearing condition in the labor market, displaying aggregate firms' labor demand on the left hand side - distorted by price dispersion[2] $\Delta_t = \int_0^1 (P_t(i)/P_t)^{-\theta}$ - and the aggregate labor supply of the households on the right hand side ($\bar{L}_t$).

[2] Aggregate labor demand is indeed given by the sum of all the firm-specific demands for good variety, divided by labor productivity: $L^{demand} = \int_0^1 (Y_t(j)/A)dj = \int_0^1 (P_t(j)/P_t)^{-\theta}(Y_t/A)dj = Y_t\Delta_t/A$

2.5 Steady state

In steady state the firm's problem (2.2.1) boils down to a static problem yielding to the real wage ω determination.

$$\omega = (1-\nu)\frac{\theta-1}{\theta}A \tag{2.5.1}$$

The subsidy ν is set to eliminate the monopolistic distortions, yielding an undistorted steady state ($\omega = A$).

Following Benigno et al. (2020) and Wolf (2022), I also assume that the steady state distribution of transfers τ, τ^* is such that the consumption levels C and C^* are the solutions of a static Ramsey problem of the government seeking to maximise in steady state a welfare function given by weighted average of the flow utility of the two agents[3]. This, together with the optimal subsidy to firms, implies that in steady state the first best is achieved. This assumption is made to prevent any steady state suboptimality concern from interfering with the optimal policy formulation in the dynamics of the liquidity trap.

In what follows I will assume $C = C^* = Y$, so that the government's optimum is to let the two household consume the same amount of goods in steady state: this implies, by (2.1.4) and (2.1.8), also an equal labor supply between household types $L = L^* = \bar{L}$. This assumption, together with the equal dividend split, is necessary to rule out endogenous cyclical differences in income between ricardian and hand to mouth households: asymmetries in steady state labor supply yield indeed different labor - and then income - response over the liquidity trap. I will come back to this in Section 5, when both the steady state labor-consumption equalization and the equal dividend split assumptions will be lifted.

3 The stabilization role of transfer policy

In this section I will consider a government solving a dynamic Ramsey problem of maximization of the average utility of the two household types (weighting each type as in the steady state static Ramsey problem discussed in section 2.5). In order to set up the welfare objective function of the government it suffices to take a second order expansion of the weighted sum of the utility of the two types of households around the efficient steady state (details are reported in the online appendix),

[3]Details about the Ramsey problem in steady state are reported in the online appendix

yielding the following object that the government aims at minimizing:

$$\min \; E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2 + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{\sigma}{\phi}\lambda(1-\lambda)(\hat{c}_t - \hat{c}_t^*)^2 \right\} \tag{3.0.1}$$

where, denoting $U(C_t) \equiv 1 - \exp(-zC_t)$ and $V(L_t) \equiv \delta \exp(\eta L_t)$, we have $\phi \equiv (V''(\bar{L})/V'(\bar{L}))\bar{L}$ as the inverse Frisch elasticity of labor supply and $\sigma \equiv -(U''(Y)/U'(Y))Y$ as the relative risk aversion, when labor and consumption are equal to the aggregate steady state levels $\bar{L}$ and Y (which, in the current case, correspond to the equal steady state labor and consumption levels of the two household types). The coefficient κ is given by $\kappa = [(1-\alpha)(1-\alpha\beta)]/\alpha](\phi+\sigma)$. "Hat" variables are log-linear deviations around the steady state. Since households face a concave utility function, and their utility levels are weighted equally by the government, any departure from equalized consumption $\hat{c}_t = \hat{c}_t^*$ entails welfare costs, under the form of *consumption dispersion* $(\hat{c}_t - \hat{c}_t^*)^2$. This term shows up in the loss function together with the usual output gap and inflation costs.

The linearized budget constraint of the hand to mouth - see the online appendix for a detailed derivation - can be written as:

$$\hat{c}_t^* = \hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^* \tag{3.0.2}$$

where I define the linearized transfer $\hat{\tau}_t^*$ as $\hat{\tau}_t^* \equiv (\tau_t^* - \tau^*)/Y$. Using (3.0.2) together with the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^*$, we can express consumption dispersion as a function of the HtM transfer only (see derivation in the online appendix):

$$(\hat{c}_t - \hat{c}_t^*)^2 = \left(\frac{1}{1-\lambda}\right)^2 \left(\frac{\phi}{\phi+\sigma}\hat{\tau}_t^*\right)^2 \tag{3.0.3}$$

Assuming steady state consumption and labor equalization, and dividends rebated equally to each household type ($\chi = \lambda$) is key to obtain the above result of consumption dispersion as a function only of the HtM transfer; any cyclical income differences over the dynamics of the model, depending on steady state asymmetries or on uneven dividend distribution, are indeed ruled out. Therefore, the only way to have the hand to mouth consume more - or less - than the ricardian household is through a positive - or negative - change in HtM transfer $\hat{\tau}_t^*$.

Problem (3.0.1) is constrained by the *aggregate demand* equation of the economy, that is derived as follows. The linearized version of the ricardian household's Euler equation (2.1.5) writes:

$$\hat{c}_t - E_t\hat{c}_{t+1} = -\frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} + \hat{\xi}_t) \tag{3.0.4}$$

Using the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^*$ at time t and $t+1$, together with (3.0.2) and (3.0.4), we obtain the aggregate demand equation:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - E_t\hat{\pi}_{t+1} + \hat{\xi}_t) - \frac{\lambda}{1-\lambda}\frac{\phi}{\phi+\sigma}E_t\Delta\hat{\tau}^*_{t+1} \tag{3.0.5}$$

Output $\hat{y}_t$ changes over time according both on the evolution in the ricardian and hand to mouth consumption. The former is determined by the intertemporal incentives given by interest rate, inflation and preference shock; the latter is pinned down by the variation in the transfer $\Delta\hat{\tau}_t^*$. The evolution of HtM transfer affects aggregate output proportionally to the overall fraction of hand to mouth λ.

The second constraint of problem (3.0.1) is the aggregate supply equation, given by the log-linear counterpart of (2.2.5):

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{3.0.6}$$

As a temporary simplifying assumption, let us drop constraint (3.0.6) from the government problem by imposing $\Pi = \Pi = 1$ $\forall t$, implying $\hat{\pi}_t = 0$ $\forall t$ [4].

Let us consider an unexpected shock $\hat{\xi}_{t_0} > 0$ hitting the economy at t_0, which then reverts to $\hat{\xi}_t = 0$ $\forall t > t_0$. Let us also define the long run HtM transfer $\hat{\tau}^{*\prime} \equiv \lim_{t\to\infty} \hat{\tau}_t^*$, that is the value that the government chooses to let the transfer converge to in the limit, after that the economy is hit by the shock. In the online appendix, I show that $\lim_{t\to\infty} \hat{y}_t = 0$, so that long run output converges to the initial steady state level and is policy invariant[5]. By iterating (3.0.5) forward (and taking into account $\hat{\pi}_t = 0$ $\forall t$) we obtain:

$$y_{t_0} = -\frac{1}{\sigma}E_{t_0}\left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty}\hat{i}_t - \sigma\Theta\hat{\tau}^*_{t_0} + \sigma\Theta\hat{\tau}^{*\prime}\right] \tag{3.0.7}$$

where $\Theta = \lambda\phi/[(1-\lambda)(\phi+\sigma)]$. The sum of the prospective interest rates $\sum_{t=t_0}^{\infty}\hat{i}_t$ affects current output, by acting on the intertemporal consumption choices of the ricardian household. On the side of transfer policy, the effect on current output can be summarized exclusively by the difference between the current transfer $\hat{\tau}^*_{t_0}$ and the long run transfer $\hat{\tau}^{*\prime}$. Increasing the current transfer $\hat{\tau}^*_{t_0}$

[4]This condition can be retrieved by setting $\kappa = 0$, that in turn can be obtained by setting the fraction of non-resetting firms α to 1.

[5]This is not a trivial result: the government may indeed impose a nonzero long run transfer deviation $\hat{\tau}^{*\prime} \neq 0$: reallocating wealth between households then would affect labor supply and the output level in the limit. In the online appendix I show that this effect is negligible up to a first order approximation.

with respect to the long run transfer $\hat{\tau}_t^{*\prime}$ boosts current output relatively to the long run policy-invariant output level, by increasing current hand to mouth consumption. Since setting nonzero transfers $\hat{\tau}_{t_0}^*, \hat{\tau}^{*\prime}$ implies consumption dispersion (by (3.0.3)), and cutting future nominal rates $\{\hat{i}_t\}_{t=t_0+1}^{\infty}$ produces future undesired output expansions, the only term the government can use in equation (3.0.7) to neutralize the shock without incurring in welfare costs is the current nominal rate $\hat{i}_{t_0}$, namely by setting $\hat{i}_{t_0} = -\hat{\xi}_{t_0}$. However, for a realization of $\hat{\xi}_{t_0}$ high enough, this is not feasible because it would require $\hat{i}_{t_0}$ to go below the lower bound $\hat{i}^{ZLB}$ (i.e. the log-linearized counterpart of the zero lower bound condition (2.3.4)). The government is then willing to keep $\hat{i}_t$ at the lower bound up to some period $T > t_0$, in order improve the recession mitigation at time t_0 (see Eggertsson and Woodford (2003)). Figure 3.1 illustrates in gray the behavior of output

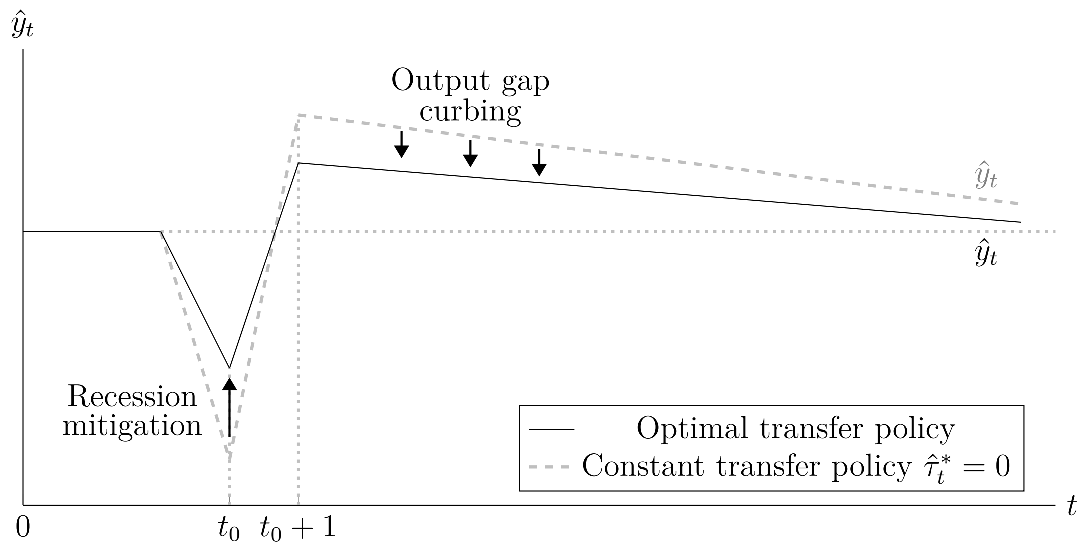


Figure 3.1: Output gap, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

when this forward guidance intervention on nominal rates is implemented, while keeping transfers $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$ at zero. Looking at the gray line first, output drops due to the shock at the onset of the trap, and then overshoots the steady state in the subsequent periods, when the nominal interest rates are still kept at the zero lower bound by forward guidance.

If the sequence of transfers $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$ is not kept at 0, but instead set optimally, the government can achieve a better stabilization of the output gap, by setting a positive transfer $\hat{\tau}_{t_0}^* > 0$ to mitigate even more the output drop at t_0, and by setting $\hat{\tau}_t^* < 0$ afterwards in order to curb the undesired output expansions arising from keeping the interest rates at 0 for $t > t_0$. Figure 3.2 and 3.1 report in black respectively the implied pattern of transfers and the response of output, under a policy

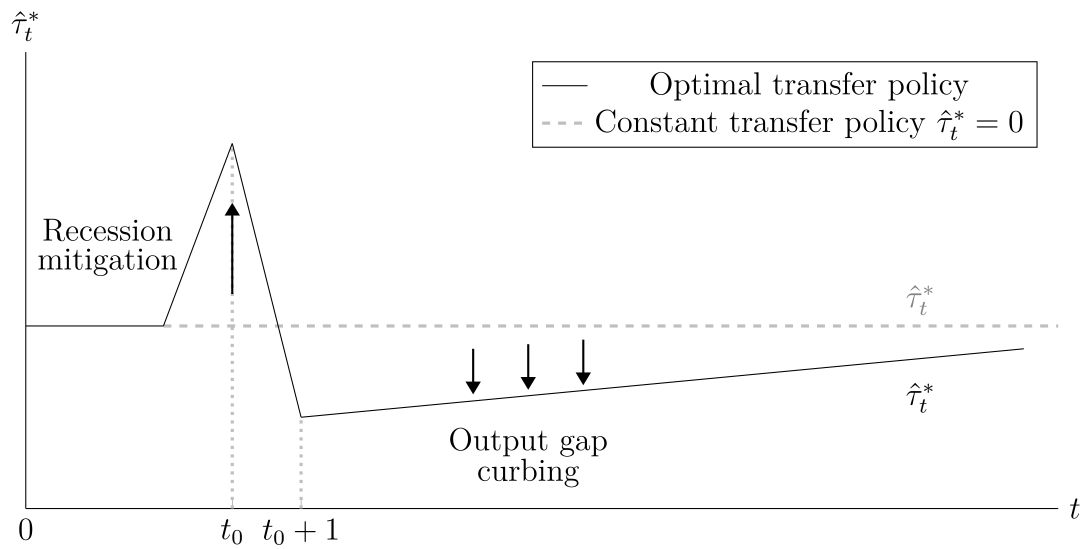


Figure 3.2: Hand to mouth transfer, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

setting transfers optimally and jointly with the interest rate. Notice that at the optimum the government does not want to completely stabilize output, because setting nonzero transfers entails consumption dispersion costs: transfers are not a substitute of the interest rate policy, which would instead be able to fully offset the shock by setting $\hat{i}_{t_0} = -\hat{\xi}_{t_0}$, absent the zero lower bound, and without yielding consumption dispersion. Moreover, transfer policy affects the optimal duration of

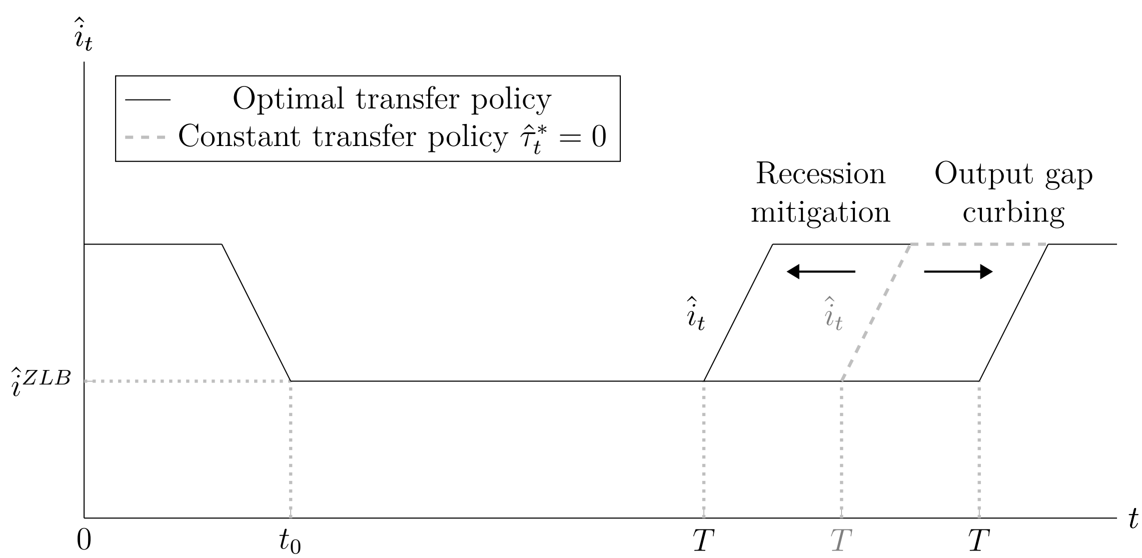


Figure 3.3: Interest rate, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

stay of the interest rate at the zero lower bound through two offsetting channels: at t_0 it entails a *shortening* effect on duration of the stay of the interest rate at the zero lower bound, as it exerts an additional expansionary effect on output, reducing the need for forward guidance. Afterwards, it counteracts the undesired output-boosting effect of monetary policy, so it makes the latter less costly in welfare terms: in this perspective, transfer policy plays a *lengthening* role with respect to forward guidance. The overall effect on the length of the stay of the interest rate at the zero lower bound remains ambiguous (see Figure 3.3).

3.1 An interpretation through the natural rate of interest

In what follows, I will define the effective *natural interest rate* as the natural interest rate that would be faced by a hypothetical representative agent with consumption levels aggregating the ones of the optimizer and the hand to mouth:

$$r_t^n = -\xi_t - \sigma\Theta E_t \Delta\hat{\tau}_{t+1}^* \tag{3.1.1}$$

Plugging indeed the term above into the AD equation (3.0.5), the latter becomes exactly alike the one that would be found in a representative agent framework (let us recall that $\hat{\pi}_t = 0\ \forall t$):

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - r_t^n) \tag{3.1.2}$$

Equation (3.1.1) shows that the natural rate can be manipulated through he HtM transfer variation $\Delta\hat{\tau}_{t+1}^*$: it is then endogenous to fiscal policy. Using again $\lim_{t\to\infty}\hat{y}_t = 0$, output gap y_{t_0} as from equation (3.0.5) (with the assumption $\hat{\pi}_t = 0\ \forall t$) can be rewritten by forward iteration as depending on the sum of the current and future deviations of the nominal from the natural interest rate, up to the liftoff period T, and from then onwards:

$$y_{t_0} = -(\hat{i}^{ZLB} - r_{t_0}^n) - \frac{1}{\sigma}E_{t_0}\sum_{t=t_0+1}^{T}(\hat{i}^{ZLB} - r_t^n) - \frac{1}{\sigma}E_{t_0}\sum_{t=T+1}^{\infty}(\hat{i}_t - r_t^n) \tag{3.1.3}$$

The shock term $\hat{\xi}_{t_0}$ is embedded into the natural rate $r_{t_0}^n$, that experiences a fall, involving a negative effect on current output. As discussed above, the government reacts by keeping the nominal rate $\hat{i}_t$ at the lower bound $\hat{i}^{ZLB}$ until time T. Therefore the future deviations up to the forward guidance horizon T, i.e. $\{\hat{i}^{ZLB} - r_t^n\}_{t=t_0+1}^T$, entail undesired future output expansions. The government can increase the current natural rate $r_{t_0}^n$ to strengthen the contemporaneous policy effect $(\hat{i}^{ZLB} - r_{t_0}^n)$ and cut future natural rates $\{r_t^n\}_{t=t_0}^T$ to curb the future expansionary effects $\sum_{t=t_0+1}^{T}(\hat{i}^{ZLB} - r_t^n)$.

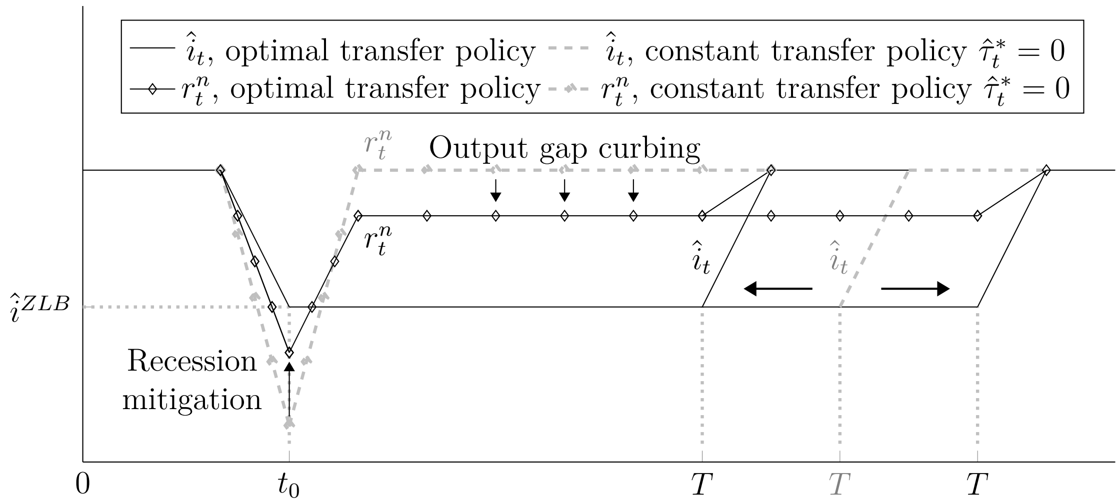


Figure 3.4: Natural and nominal interest rate, optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

In this way it can achieve a better output drop mitigation at t_0, at the expense of lower output expansions in the future periods. The optimal natural interest response is illustrated in Figure 3.4. Also under this interpretation, the two roles of transfer policy influence in an opposite way the duration of forward guidance.

According to (3.1.1), the optimal pattern of r_t^n is produced exactly by the optimal transfer policy illustrated in Figure 3.2: the fall of the transfer at $t_0 + 1$ after the initial peak creates an upward shift the natural rate at t_0, while then increasing the transfer back to steady state pushes the natural interest rate downwards.

3.2 Transfer financing

In the argument outlined so far I did not yet discuss how HtM transfers are financed. Let us take a log-linear approximation of the government's budget constraint (2.3.1) (after having substituted inside for bond market clearing (2.4.3)) in this simplified environment with $\hat{\pi}_t = 0 \ \forall t$:

$$\Gamma \hat{\tau}_t^* = \frac{1-\lambda}{\lambda} \left[\beta \hat{b}_t - \hat{b}_{t-1} - \beta \hat{i}_t - \Gamma \hat{\tau}_t \right] \tag{3.2.1}$$

where $\Gamma = Y\Pi/b = Y/b$, with $\hat{b}_t$ and b the deviation and the steady state level of ricardian bond holding B_t/P_t, respectively. We can see how at any time t the transfer $\hat{\tau}_t^*$, if positive, is financed by taking resources away from ricardian household, either through an increase in its public debt holding, or through an interest rate cut, or through a direct redistribution through the transfer

$\hat{\tau}_t$ (the opposite holds if $\hat{\tau}^*_t$ is negative). Taking the discounted sum with respect to the steady state discount factor β of both the right and the left hand side up to infinity, and imposing the transversality condition $\lim_{j\to\infty} \beta^j \hat{b}_{t_0+j} = 0$ and the predetermined condition $\hat{b}_{t_0-1} = 0$, we can write:

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}^*_{t_0+j} = -\frac{1-\lambda}{\lambda} \left[\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j} + \beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} \right] \tag{3.2.2}$$

The government can select any appropriate pattern of ricardian transfers $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ to satisfy the financing constraint above, without affecting the stabilization results (which depend uniquely on the aggregate demand determinants showing up in (3.0.7)). Of course, this will impact ricardian consumption. The latter is pinned down by the Euler equation and the ricardian intertemporal budget constraint (IBC)[6]:

$$\hat{c}_t - E_t \hat{c}_{t+1} = -\frac{1}{\sigma}(\hat{i}_t + \hat{\xi}_t) \tag{3.2.3}$$

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{c}_{t_0+j} = \Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{l}_{t_0+j} + \Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j} + \beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} \tag{3.2.4}$$

where in the IBC the real wage and dividend deviations do not show up, as they exactly offset each other (see the online appendix). Transfer financing through the term $\sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}$ has the effect of shifting up or down the whole discounted sum of ricardian consumption $\sum_{j=0}^{\infty} \beta^j \hat{c}_{t_0+j}$[7]. Therefore, while redistribution has a direct effect on current hand to mouth consumption, it can only affect ricardian consumption only with respect to its total discounted amount. The effectiveness of current HtM transfer movements in stabilizing output is not jeopardized by the financing scheme.

3.3 The general case

With the above considerations in mind, we can now consider the general case in which prices are not fully rigid and solve for the optimal policy problem of the government, which seeks to set

[6]The IBC of the ricardian household (3.2.4) is recovered by the infinite iteration forward of the log-linear version of (2.1.1) (the flow budget constraint), subject to the the transversality condition $\lim_{j\to\infty} \beta^j \hat{b}_{t_0+j} = 0$, the predetermined condition $\hat{b}_{t_0} = 0$, and the simplifying assumption $\hat{\pi}_t = 0 \ \forall t$

[7]I the current simplified case with rigid prices, it can be shown that the effect on ricardian consumption is null due to the adjustment in the labor sequence $\{\hat{l}_{t_0+j}\}_{j=0}^{\infty}$. This limit case is not further discussed here.

jointly the pattern of nominal interest rates and transfers. In this perspective I will lift the zero inflation assumption of the previous simplified setting; moreover, I will substitute the consumption dispersion term $(\hat{c}_t - \hat{c}_t^*)^2$ as a function of the transfer $\hat{\tau}_t^*$ using (3.0.3). The problem writes:

$$\min_{\{\hat{\pi}_t\}_{t_0}^{\infty},\{\hat{y}_t\}_{t_0}^{\infty},\{\hat{\tau}_t^*\}_{t_0}^{\infty},\{\hat{i}_t\}_{t_0}^{\infty}} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2+\frac{1}{2}\hat{y}_t^2+\frac{1}{2}\frac{\phi\sigma}{(\phi+\sigma)^2}\frac{\lambda}{1-\lambda}\hat{\tau}_t^{*2}\right\} \tag{3.3.1}$$

s.t

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_t) - \Theta E_t\Delta\hat{\tau}_{t+1}^* \tag{3.3.2}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{3.3.3}$$

$$\hat{i}_t \geq \hat{i}_{ZLB} \tag{3.3.4}$$

where (3.3.2) is the previously derived AD equation; constraints (3.3.3) and (3.3.4) are the log-linearized versions of the New Keynesian Phillips curve (2.2.5) and of the zero lower bound on the interest rate (2.3.4). The consumption deviations $\hat{c}_t$ and $\hat{c}_t^*$ can be determined residually by using the aggregate resource constraint $\hat{y}_t = (1-\lambda)\hat{c}_t + \lambda\hat{c}_t^*$ and the hand to mouth budget constraint (3.0.2).

The government needs to satisfy a solvency requirement (the generalized version of constraint (3.2.2)), obtained by iterating forward the log-linearized counterpart of the public sector budget constraint (2.3.1) and imposing the transversality condition $\lim_{j\to\infty}\beta^j\hat{b}_{t_0+j} = 0$:

$$\Gamma E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t_0+j}^* = -\frac{1-\lambda}{\lambda}\left[\Gamma E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t_0+j} + \beta E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{i}_{t_0+j} - E_{t_0}\sum_{j=0}^{\infty}\beta^j\hat{\pi}_{t_0+j}\right] \tag{3.3.5}$$

As discussed previously, the government can always choose one of the infinite possible appropriate sequences $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ to satisfy (3.3.5): as a consequence, condition (3.3.5) is not included among the constraints of the problem.

The system allows to analytically identify the trade-off between aggregate stabilization and consumption dispersion, with respect to transfer policy: taking the first order condition of problem (6.1.1)-(3.3.4) with respect to $\hat{\tau}_t^*$, we obtain:

$$\frac{\sigma}{\phi+\sigma}\hat{\tau}_t^* + \frac{1}{\beta}\nu_{t-1}^{AD} - \nu_t^{AD} = 0 \tag{3.3.6}$$

The zero consumption dispersion solution $\hat{\tau}_t^* = 0$ is not achieved over the dynamics, since $\hat{\tau}_t^*$ is valuable for aggregate stabilization: this value is embedded in the difference $(1/\beta)\nu_{t-1}^{AD} - \nu_t^{AD}$, where $\hat{\nu}_t^{AD}, \hat{\nu}_{t-1}^{AD}$, are the multipliers of the aggregate demand equation at time t and $t-1$, respectively. When the multiplier displays sizable variation over time, the further output is from steady state, and the more the government leans towards aggregate stabilization rather than consumption dispersion. In the simple case with $\hat{\pi}_t = 0$, condition (3.3.6) boils down to the following rule:

$$\hat{\tau}_t^* = -\frac{\phi + \sigma}{\sigma}\hat{y}_t \tag{3.3.7}$$

where I substituted for the multipliers using the first order condition on $\hat{y}_t$. In this case transfer reacts linearly to deviations in output. The higher is σ, the more concave is the utility function of the households, and the more relevant are consumption dispersion costs, calling for a weaker transfer reaction.

3.4 Simulation

In order to simulate the model under optimal policy, I adopt the following calibration: I set η and z such that $\phi \equiv (V''(\bar{L})/V'(\bar{L}))\bar{L} = 0.47$, and $\sigma \equiv -(U''(Y)/U'(Y))Y = 2$, where $\bar{L} = (1-\lambda)L + \lambda L^*$. In this way the impulse response functions of the model replicate exactly the ones that would be yielded by assuming a standard utility function of the type $U(C) = C^{1-\sigma}/(1-\sigma)$ and $V(L) = L^{1+\phi}/(1+\phi)$, with $\sigma = 2$ and $\phi = 0.47$ (see Eggertsson and Woodford (2006)). I set κ to 0.02 (see Benigno et al. (2020)). I assume $\lambda = 0.33$, according to the observation of Kaplan and Violante (2014) about hand-to-mouth the households in the Survey of Consumer Finance being approximately one third of the total amount of surveyed households. The discount factor $\beta = 0.9987$ and the steady state inflation rate $\Pi = 1.005$ implies a 2% inflation at the annualized level and a 2.5 % steady state nominal interest rate. I assume the ratio b/Y equal to 4 (translating a steady state debt-GDP annualized ratio of 1 in quarterly terms): this last calibrated value is not relevant for this setting but it will become so in the extension of the model developed in the next section.

Figure 3.5 reports the impulse response functions for the economy when hit by an unexpected shock at $t_0 = 1$, bringing $\hat{\xi}_t$ to 0.025 and lasting 12 periods. It compares optimal policy to an optimal policy when the HtM transfer does not vary. All the variables are in percentage deviation terms (let's recall here that transfer deviations are expressed in percentage of steady state output);

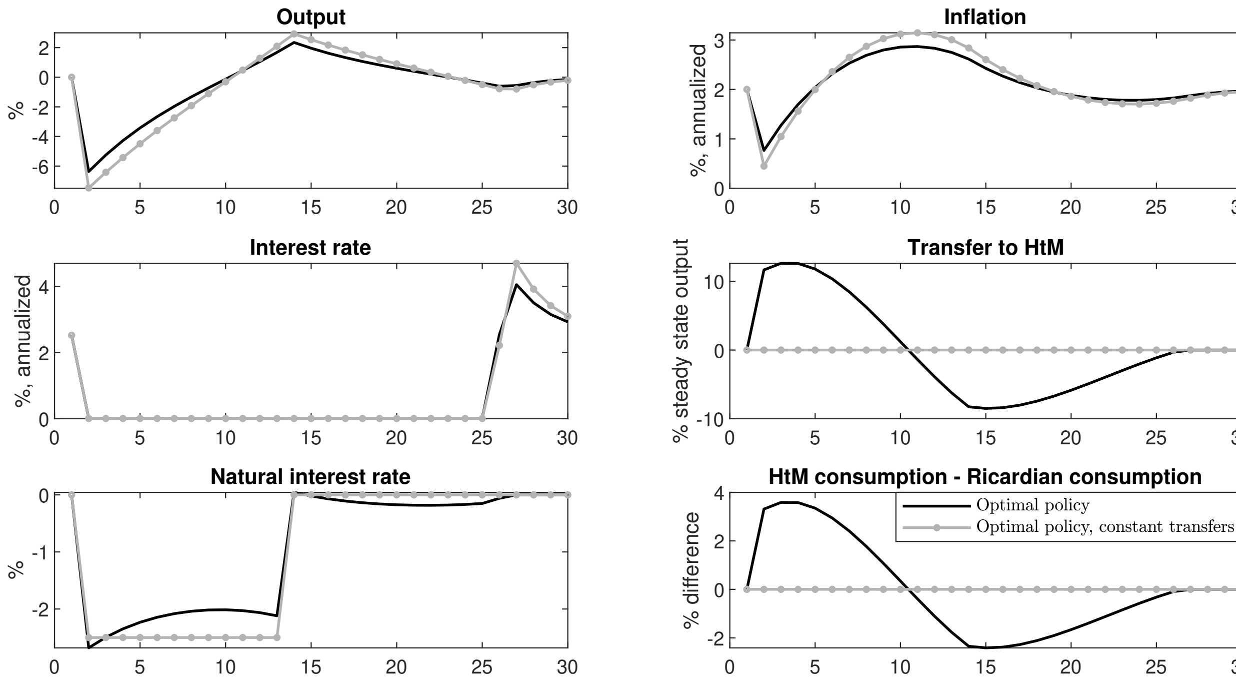


Figure 3.5: Optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$

inflation and nominal interest rate are annualized.

The shock is high enough to bring the interest rate to the zero lower bound. The duration of the stay at the zero lower bound is long and up to quarter 25, i.e. double the time span of the shock (which ends at quarter 13). The output and inflation drop in the early stages of the liquidity trap is mitigated at the expense of an output and inflation expansion later. At the onset of the shock, the government sets positive transfers for the hand to mouth to alleviate the output drop; as output increases, transfers are reduced, up until the former becomes positive; then, the government starts setting negative transfers to curb the expansion. The transfer deviation are sizable: the hand to mouth enjoys a rebate up to 12% of its steady state income over the recession and a down to -8% over the recovery. The pattern of transfers involve a manipulation of the natural interest rate: the transfer decreases gradually - after the initial peak - over the recession, implying an upward pressure on the natural rate, that has been dragged down by the shock. Then, over the recovery, the increasing pattern of transfers implies a downward shift in the natural rates, which allow to curb the expansion.

The introduction of the optimal response of transfer policy allows to achieve a better stabilization of output gap and inflation: the output trough of the constant transfer policy at the onset of the shock is reduced by nearly one fourth, and the same holds for the peak over the recovery. Also inflation fluctuates significantly less in the optimal policy with respect to the constant transfer policy scenario. This stabilization outcome is achieved at the expense of consumption dispersion costs: so transfers cannot perfectly substitute for the stabilization power of monetary policy that is foregone because of the zero lower bound. The HtM consumes more than the Ricardian when the transfer deviation is positive and less when it is negative. Remarkably, consumption difference between households moves less strongly than the the transfer: this is because agents can partially compensate the positive (or negative) transfer deviation by adjusting labor supply.

Notice that the impact of transfer policy on the duration of forward guidance is null: the nominal interest rate remains at zero until quarter 25 both in the constant transfer policy and in the optimal policy. This is due to the interaction of the two opposite roles of transfers with respect to monetary policy: on one side they call for a lower forward guidance horizon, when they mitigate the early recession; on the other side, they make more desirable an extension of the stay at the zero lower bound by counteracting the undesired output expansion of forward guidance. As discussed previously, these two effects act oppositely on the duration of stay of the rates at the zero lower bound. This can be seen in Figure 3.6, where the optimal transfer policy is decomposed into the optimal policy constrained by $\hat{\tau}_t^* \geq 0$ and the optimal policy constrained by $\hat{\tau}_t^* \leq 0$. In the first benchmark case, only positive transfer deviations are allowed, so only the recession mitigation and then the shortening role of transfers is active, and this implies a decrease in the horizon of forward guidance with respect to the optimal policy: the interest rate is kept at 0 for a quarter less. In the second case, only the later curbing of the expansion and then the lengthening role of transfers is in place - as only negative transfer deviations are allowed, and this drives the government to keep the interest rates at 0 for one quarter more.

The result of a null effect of transfer policy on the duration of forward guidance is robust to sensitivity analysis carried out on different parameters. Specifically, in the online appendix I consider lower and higher values - with respect to the current calibration - for λ, σ, ϕ, which are the parameters determine the effect of transfers on the economy (through stabilization via aggregate demand, or through the impact on consumption dispersion costs). I also consider lower and higher values for the shock ε_ξ and the parameter β, which determine the severity of the zero lower bound constraint[8]. Also in this case the alternative parametrizations lead still to the same finding.

A key parameter to assess the stabilization power of transfer policy is the fraction of hand to mouth

[8]Consider that the steady state nominal interest rate is given by $i = \Pi/\beta - 1$. The higher is β, the closer is this value to 0, and the more binding will be the zero lower bound when the demand shock hits the economy

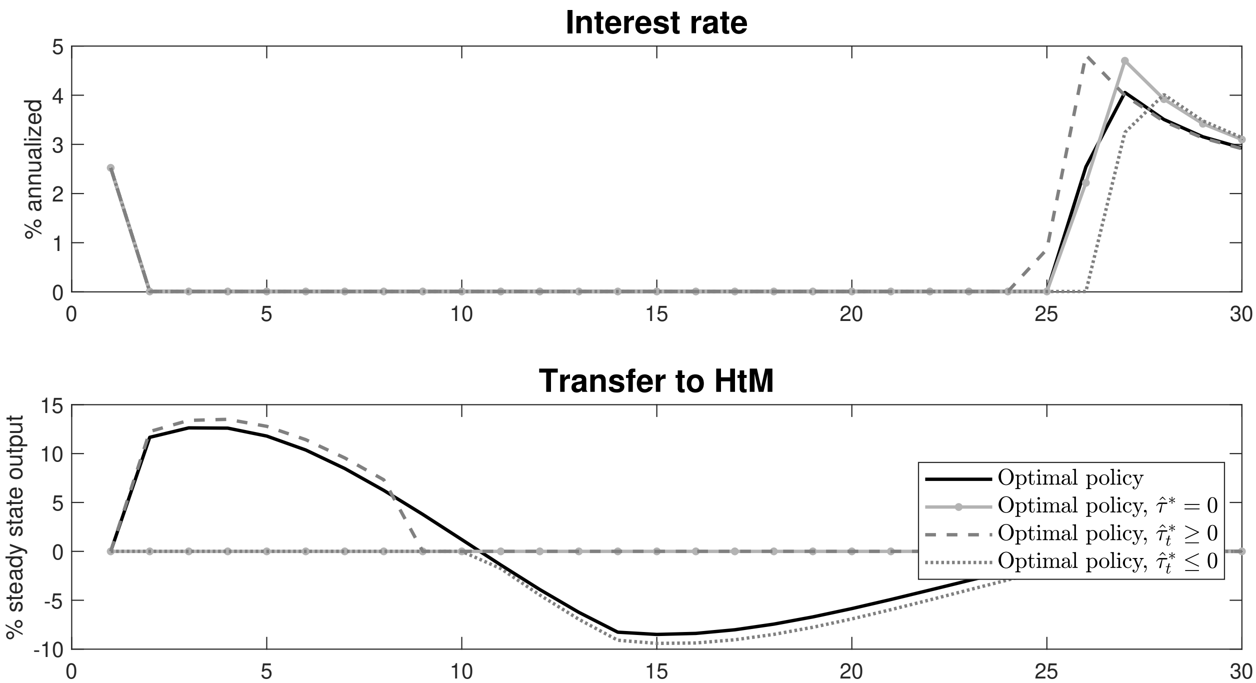


Figure 3.6: Optimal policy vs optimal policy with $\hat{\tau}_t^* \geq 0$ and $\hat{\tau}_t^* \leq 0$

households λ. As discussed previously, the higher is the fraction of hand to mouth in the economy, the stronger is the stabilizing effect on output of rebating them more resources, as λ enters in the AD curve (3.3.2) under the form of the coefficient $\Theta = \lambda/(1-\lambda)$. However, the same expression $\lambda/(1-\lambda)$ also appears in the coefficient of the consumption dispersion term in the government objective (6.1.1): this is because redistributing resources to or away from the hand to mouth impacts consumption dispersion more heavily the higher is their relative weight in the population. However, the effect of λ on output stabilization enters quadratically in the welfare objective (as the government draws disutility from output deviation squared), whereas it enters only linearly in the consumption dispersion coefficient. Therefore increasing λ entail better aggregate stabilization results at the expense of lower consumption dispersion: this can be seen by comparing Figure 3.5 (where λ is equal to 0.33) to Figure 3.7 - where we have instead $\lambda = 0.5$, implying half of the population being hand to mouth.

4 Optimal policy under a homogeneous tax response

So far I assumed that the government was able to freely differentiate lump sum taxation between Ricardians and hand to mouth. While using HtM tax $\hat{\tau}_t^*$ to stabilize output and inflation over

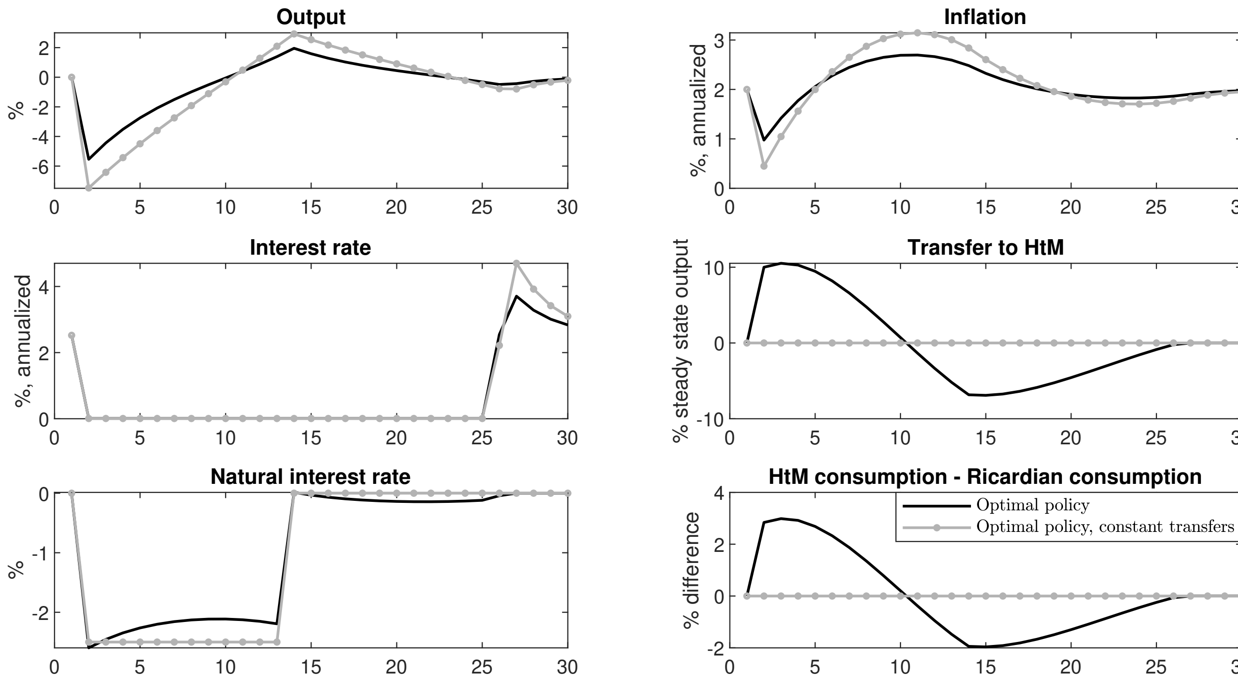


Figure 3.7: Optimal policy vs. optimal policy with $\hat{\tau}_t^* = 0$, $\lambda = 0.5$ case

the liquidity trap, the government could select any of the infinite possible sequences of ricardians' transfers $\{\hat{\tau}_{t_0+j}\}_{j=0}^{\infty}$ appropriate to guarantee the solvency constraint to hold (equation (3.3.5)). However, due to political constraints, a government could have hard time in implementing a heterogeneous tax response across households. In this section I explore to what extent the results in terms of the stabilizing effect of optimal transfers, as well as their imperfect substitutability with interest rate policy and their null effect on the duration of the stay at the zero lower bound, carry over to a case in which a unique stimulus check is rebated to all households in the economy (following in spirit Wolf (2021)). The additional constraint that I am setting is:

$$\tau_t - \tau = \tau_t^* - \tau^* \quad \forall t \tag{4.0.1}$$

Constraint (4.0.1) imposes that the the same increment of transfers with respect to steady state is set for the whole cross-section of households. This also implies that the transfer deviation terms - defined in output terms as previously - are equal:

$$\hat{\tau}_t = \frac{\tau_t - \tau}{Y} = \frac{\tau_t^* - \tau^*}{Y} = \hat{\tau}_t^* \tag{4.0.2}$$

I will thereafter call $\hat{\tau}_t^{**}$ the unique transfer deviation set on both ricardian and hand to mouth households.

Let us restate the solvency constraint (3.3.5), with only the unique transfer $\hat{\tau}_t^{**}$ available:

$$\Gamma E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^{**} = -(1-\lambda)\beta E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j} + (1-\lambda) E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\pi}_{t_0+j} \tag{4.0.3}$$

Now the transfer instrument used for aggregate stabilization $\hat{\tau}^{**}$ in (3.3.2) is the same that is used to guarantee solvency (4.0.3). It appears that the use of transfers now implies a trade-off not only between aggregate stabilization and consumption dispersion, but also with respect to public debt management.

Let us for now consider only the trade-off between aggregate stabilization and public debt management, leaving aside consumption dispersion concerns. Notice that when the prospective sequences of nominal interest rates and inflation change as a consequence of the preference shock hitting the economy, transfers have to adjust as well to guarantee solvency (4.0.3). However, since condition (4.0.3) is satisfied by setting an appropriate *sum* of discounted transfers, the government is free to smooth out the required fiscal response over time. In particular, given $\sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^{*}$ being the sum of discounted hand to mouth transfers generated by optimal policy in the benchmark heterogeneous transfer scheme of last section (either with or without the constant transfer constraint $\hat{\tau}_t^* = 0$), we can obtain the required total discounted transfer amount for public debt management in the current setting, $\sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^{**}$, by increasing every period $\hat{\tau}_t^*$ by a fixed amount Δ_{τ^*}:

$$E_{t_0} \sum_{j=0}^{\infty} \beta^j \hat{\tau}_{t_0+j}^{**} = E_{t_0} \sum_{j=0}^{\infty} \beta^j (\hat{\tau}_{t_0+j}^{*} + \Delta_{\tau^*}) \tag{4.0.4}$$

In this way we can obtain exactly the same aggregate output and inflation dynamics $\{\hat{y}_t\}_{t=t_0}^{\infty}$, $\{\hat{\pi}_t\}_{t=t_0}^{\infty}$ as in the benchmark case: the effect of the Δ_{τ^*} increase indeed cancels out in the output determination equation:

$$\begin{aligned} y_{t_0} &= -\frac{1}{\sigma} E_{t_0} \left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty} \hat{i}_t - \sum_{t=t_0+1}^{\infty} \hat{\pi}_t - \sigma\Theta(\hat{\tau}_{t_0}^{**} + \Delta_{\tau^*}) + \sigma\Theta(\hat{\tau}^{**\prime} + \Delta_{\tau^*}) \right] = \\ &= -\frac{1}{\sigma} E_{t_0} \left[\hat{\xi}_{t_0} + \sum_{t=t_0}^{\infty} \hat{i}_t - \sum_{t=t_0+1}^{\infty} \hat{\pi}_t - \sigma\Theta\hat{\tau}_{t_0}^{**} + \sigma\Theta\hat{\tau}^{**\prime} \right] \end{aligned} \tag{4.0.5}$$

The effect of the increase in current transfer $\hat{\tau}_{t_0}^{*}$ on output is indeed exactly offset by the increase in the limit transfer $\hat{\tau}^{*\prime}$. A rise in the latter implies indeed that resources are systematically rebated away from the ricardian budget constraint in the final steady state, making it poorer and forcing it to cut its current consumption.

From the argument developed so far, we can infer that there is no trade-off between aggregate stabilization and public debt management: this result follows from the fact that output at any time t is affected by the *difference* between current transfer $\hat{\tau}_t^{**}$ and long run transfer $\hat{\tau}^{**\prime}$; while public debt instead is determined by the *size* of transfers per se. Therefore the government is able to conduce a transfer policy that disentangles aggregate stabilization from public debt management. Intuition is that in the limit steady state, additional Δ_{τ^*} resources are rebated away from ricardian's consumption, which shrinks accordingly also its current consumption by an amount Δ_{τ^*}, offsetting the additional expansionary effect on output that passes through the current hand to mouth transfer $\hat{\tau}_{t_0}^{**} + \Delta_{\tau^*}$: output response remains therefore unchanged with respect to the baseline setting at time t_0.

Rearranging equation (4.0.4), we can back out Δ_{τ^*} as a function of the difference between the discounted sum of transfers in the homogeneous transfer scheme and the one in the heterogeneous transfer scheme:

$$\Delta_{\tau^*} = (1-\beta)E_{t_0}\left[\sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t_0+j}^{**} - \sum_{j=0}^{\infty}\beta^j\hat{\tau}_{t_0+j}^{*}\right] \tag{4.0.6}$$

The extra-fiscal deficit (or surplus) needed is multiplied by a coefficient $1-\beta$ to give rise to the required Δ_{τ^*}, that therefore turns out to be *small* - since β is close to 1. The government, by shifting the whole transfer sequence, can indeed smooth out the fiscal surplus/deficit over time.

As showed above, shifting all the transfers by a quantity Δ_{τ^*} does not interfere with aggregate stabilization; however it does affect consumption dispersion, as the latter is related with the squared size of the transfers:

$$(\hat{c}_t - \hat{c}_t^*)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}\hat{\tau}_t^{**}\right)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}(\hat{\tau}_t^{*}+\Delta_{\tau^*})\right)^2 \tag{4.0.7}$$

However, since the term Δ_{τ^*} is small in size, as showed above, the optimal solution of the government will not significantly deviate from the parallel shift of the whole HtM transfer sequence, nor it will display significant departures of output and inflation from the baseline heterogeneous transfer case. Figure 4.1 compares the impulse response functions of the economy in the case where the unique transfer $\hat{\tau}_t^{**}$ is set optimally to the case in which it is set constant to the level satisfying solvency (4.0.3), keeping the same calibration of parameters and specification of the shock as in section 3.

By equation (4.0.3) we can see that when debt dynamics are characterized by a low interest rate

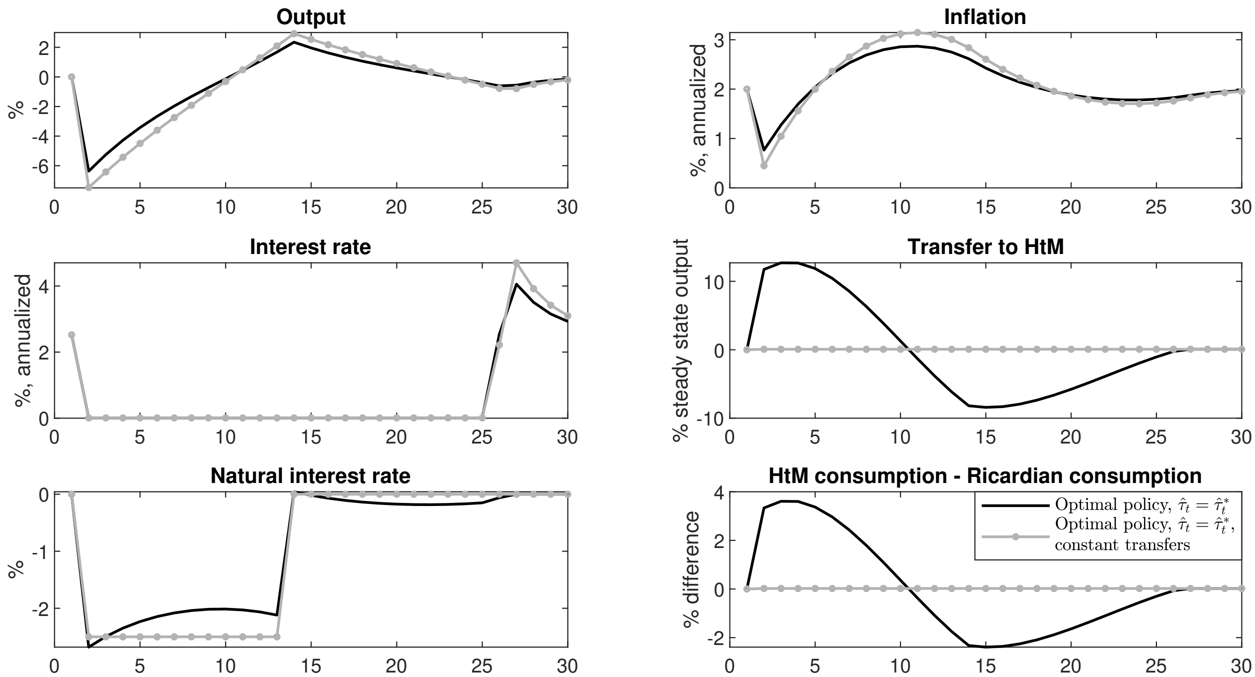


Figure 4.1: Optimal policy vs. optimal policy with constant transfers, $\tau = \tau^*$ case

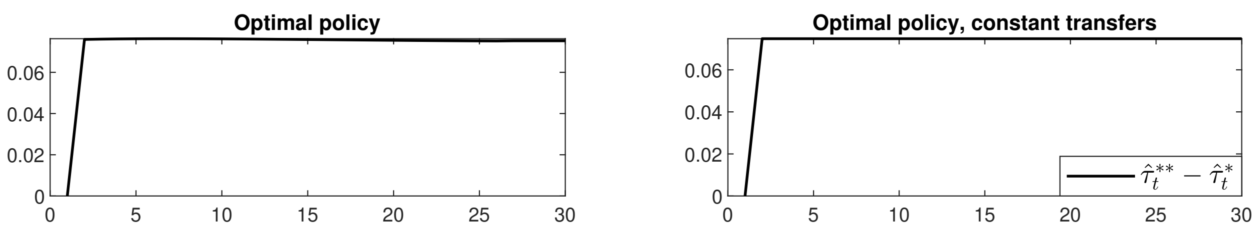


Figure 4.2: Optimal policy vs. optimal policy with homogeneous transfers: difference in HtM transfer

time span (a fall in the term $\beta \sum_{j=0}^{\infty} \beta^j \hat{i}_{t_0+j}$) which is a *wealth gain* for the government, that needs to be offset by an increase in transfers $\{\hat{\tau}_t^{**}\}_{t=t_0}^{\infty}$. According to the argument made above, the whole sequence of transfers need to be shifted (upward, in this case) with respect to the baseline path $\{\hat{\tau}_t\}_{t=t_0}^{\infty}$: in this way solvency is satisfied, and both optimal and constant transfer policy succeed in generating the same response of the economy as in the baseline setting (it can be indeed seen by comparing Figure 4.1 to Figure 3.5). The difference between the sequence of HtM transfers in the heterogeneous transfer scheme and in the homogeneous transfer response, reported in Figure 4.2,

is of a negligible degree of magnitude - as it is smoothed out in the long run.
Since all the variables' responses to the shock track closely the ones of the baseline setting, we can conclude that the results in terms transfers being imperfect substitutes for interest rate policy - as they generate consumption dispersion - and entailing a zero effect on the length of the stay at the zero lower bound, carry over to the homogeneous transfer response case.

5 Optimal policy under cyclical income differences

So far, cyclical income differences (unrelated to transfer policy) have been shut down through two assumptions: the equal dividend split $\chi = \lambda$ and the equalization of steady state consumption and labor levels $C = C^*$, $L = L^*$. In this section I relax these two assumptions and explore the implications for the formulation of an optimal monetary and transfer policy. The relevance of these forces in affecting hand to mouth consumption can be analytically identified by considering the HtM budget constraint, once these assumptions are lifted (see the online appendix for the derivation):

$$\frac{C^*}{Y}\hat{c}_t^* = \Phi\hat{y}_t + \hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^* \tag{5.0.1}$$

Hand to mouth consumption, standardized by the steady state consumption share C^*/Y, is determined by the same term of the baseline framework, $\hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^*$, plus an additional component $\Phi\hat{y}_t$, such that:

$$\Phi = \phi\left(\frac{L^*}{\bar{L}} - \frac{\chi}{\lambda}\right) \tag{5.0.2}$$

The sensitivity of HtM income to aggregate output $(1 + \Phi)$ depends on both the steady state heterogeneity - through the term $L^*/\bar{L}$ - and on the dividend split rule χ/λ. The higher is L^* with respect to aggregate labor supply $\bar{L}$, the stronger HtM labor supply varies with the aggregate output, making hand to mouth consumption more cyclical. The higher is the fraction of dividends $\frac{\chi}{\lambda}$ allocated to each hand to mouth household, the more countercyclical is HtM consumption instead. The latter feature is due to the unrealistic countercyclical nature of dividends in the New Keynesian models with stickiness in firms' price setting. I will nevertheless not take a stance on the sign and size of the whole cyclical coefficient Φ, but instead I will hereafter incorporate this quantity together with the transfer $\hat{\tau}_t^*$ into a single term $\tilde{\tau}_t^*$ - *the augmented transfer* - which consists in an endogenous cyclical component and a policy-driven component provided by the transfer:

$$\tilde{\tau}_t^* \equiv \frac{\phi+\sigma}{\phi}\Phi\hat{y}_t + \hat{\tau}_t^* \tag{5.0.3}$$

So that we can rewrite the hand to mouth budget constraint (5.0.1) as:

$$\tilde{c}_t^* = \hat{y}_t + \frac{\phi}{\phi+\sigma}\tilde{\tau}_t^* \tag{5.0.4}$$

Where $\tilde{c}_t^* \equiv (C^*/Y)\hat{c}_t$ is the hand to mouth consumption deviation standardized by its steady state consumption share. Notice that the government can always freely choose the augmented transfer level $\tilde{\tau}_t^*$, thanks to the degree of freedom provided by the transfer term $\hat{\tau}_t^*$ in equation (2.7.5). The augmented transfer affects the economy through the exact same channel of the transfer in the baseline model: by boosting hand to mouth consumption though its budget constraint. We can then reformulate the problem of the government in reaction to the shock ξ_{t_0} as an optimal policy setting jointly the interest rate and the augmented transfer sequence $\{\hat{i}_t, \tilde{\tau}_t^*\}_{t=t_0}^{\infty}$:

$$\min_{\{\hat{\pi}_t\}_{t_0}^{\infty},\{\hat{y}_t\}_{t_0}^{\infty},\{\tilde{\tau}_t^*\}_{t_0}^{\infty},\{\hat{i}_t\}_{t_0}^{\infty}} E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left\{\frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2 + \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\frac{\phi\sigma}{(\phi+\sigma)^2}\frac{\lambda}{1-\lambda}\tilde{\tau}_t^{*2}\right\} \tag{5.0.5}$$

s.t

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1} + \hat{\xi}_t) - \Theta E_t\Delta\tilde{\tau}_{t+1}^* \tag{5.0.6}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \tag{5.0.7}$$

$$\hat{i}_t \geq \hat{i}_{ZLB} \tag{5.0.8}$$

The problem is exactly equivalent to the baseline problem (6.1.1)-(3.3.4), with transfers $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$ replaced by augmented transfers $\{\tilde{\tau}_t^*\}_{t=t_0}^{\infty}$; therefore it gives rise to exactly the same optimal impulse response for the augmented transfer as for transfers in the setting without cyclical income differences; output, inflation, and consumption dispersion are also generated by the equivalent problem, so track exactly the ones produced in the case with no cyclical income difference. Consumption dispersion, in particular, is here given in terms of the consumption deviations of the households, standardized for the steady state consumption shares, that is expressed as a function of the augmented transfer:

$$(\tilde{c}_t - \tilde{c}_t^*)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}\tilde{\tau}_t^*\right)^2 \tag{5.0.9}$$

The sequence of transfers that the government needs to engineer to produce the desired sequence of the augmented transfer $\tilde{\tau}_t^*$ is now reliant on the sensitivity of HtM income to aggregate output (see (2.7.5)). If $\Phi > 0$, an endogenous cyclical component is introduced into hand to mouth consumption, dragging it downward over the recession and upwards during the boom. Optimal augmented transfer policy aims at achieving the opposite pattern (boosting HtM consumption initially and then curbing it), so transfers need to compensate for this effect: they will be raised more during the trough and cut more during the expansionary phase, with respect to the no-cyclical income difference scenario. By the same logic, transfers will display milder fluctuations with respect to the no-cyclical income difference case if $\Phi < 0$, i.e. if endogenous inequality boosts HtM consumption when output drops and curbs it when output expands. Figure 5.1 compares

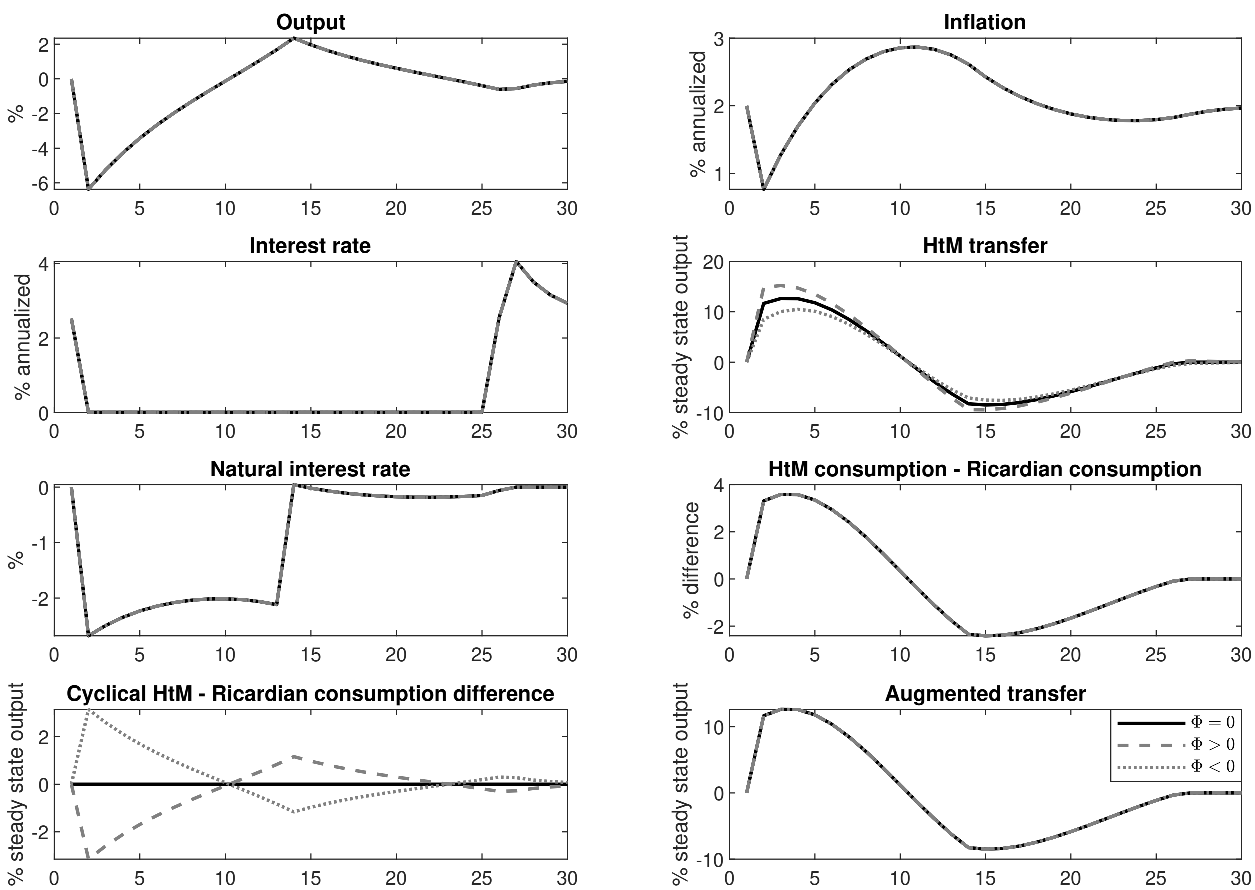


Figure 5.1: Optimal policy. $\Phi = 0$: no cyclical income difference; $\Phi > 0$: procyclical HtM income; $\Phi < 0$: countercyclical HtM income. The bottom left graph relates to cyclical income differences unrelated to transfer policy.

aggregate stabilization outcomes in the case $\Phi = 0$, with the case $\Phi = 0.094$ and $\Phi = -0.094$ - corresponding to a calibration where χ is kept equal to λ, and the steady state hand to mouth's hours worked are 20% more and less than the economywide labor supply $\bar{L}$, respectively. In the former case, the HtM supplies more labor to make partially up for a steady state consumption lower than average ($C^* < Y$), in the latter, it affords working less by a consumption advantage ($C^* > Y$). This consumption and labor steady state asymmetry arises from an uneven transfer distribution τ, τ^*, favouring the Ricardian in the former case and the hand to mouth in the latter. The shock process considered is the same as in section 3.4. In the case $\Phi = 0$, we are back to exactly the same impulse response as in Figure 3.5, since cyclical income difference is shut down. In the case $\Phi > 0$, the high steady state labor supply of the hand to mouth implies a higher cyclicality of its consumption, which calls for a more massive use of transfers over both the recession and the boom. In the case $\Phi < 0$ consumption of the hand to mouth is instead more countercyclical, and this feature substitute partially for the transfer intervention: the latter display then less sharp fluctuations over the trap. Overall, the government succeeds in making the augmented transfer term $\tilde{\tau}_t$ follow exactly the same optimal pattern in all the three cases, and that guarantees the same outcomes in terms of aggregate inflation-output stabilization and consumption dispersion.

The use of transfers affects consumption dispersion costs (together with cyclical income difference): then, in line with the results of in the baseline setting, the transfer instrument is not a substitute for monetary policy. Optimal transfers rise over the recession to mitigate the output drop, and fall over the later stages of the trap, to curb the expansion: these two forces once again imply countervailing effects on forward guidance duration, yielding an overall null impact on the duration of stay at the zero lower bound, as reported in Figure 5.2 and 5.3, and analogously to the baseline setting's results.

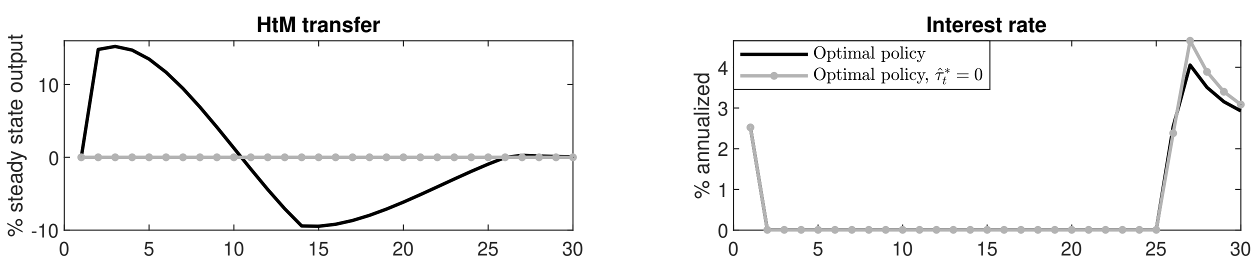


Figure 5.2: Optimal policy vs optimal policy with $\hat{\tau}_t^* = 0$, $\Phi < 0$ case

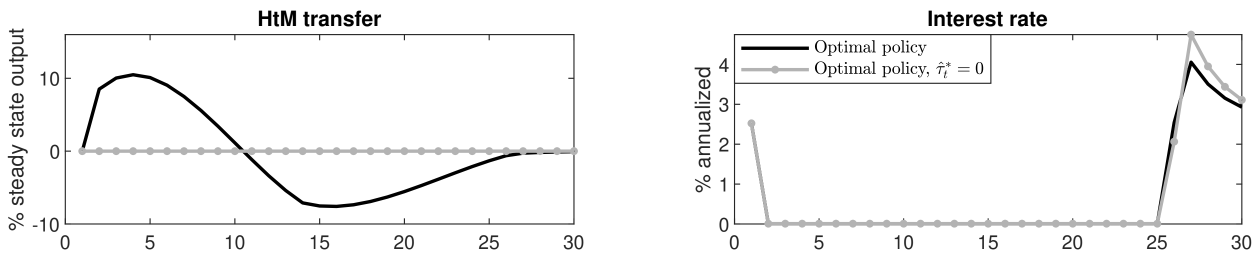


Figure 5.3: Optimal policy vs optimal policy with $\hat{\tau}_t^* = 0$, $\Phi > 0$ case

6 Conclusion

In this paper I formulate an optimal monetary and fiscal policy problem in a heterogeneous agents economy facing a shock that brings it to liquidity trap, where fiscal policy is modelled as transfer policy. Transfers are used by the government to manipulate the natural interest rate in the economy, at the expense of consumption dispersion, which prevent them from being an effective substitute of the foregone stabilization power of monetary policy. During the early stages of the liquidity trap, transfer policy is used jointly with monetary policy to mitigate the recession, while later it is used to curb the undesired output expansion implied by forward guidance. These two forces impact oppositely on the optimal duration of stay of nominal rates at the zero lower bound, with an overall impact that is negligible. The findings are robust to both restrictions imposing homogeneous transfer responses between household types, and to a broader framework allowing for cyclical income difference.

Remarkably, the optimal fiscal-monetary policy prescriptions of the paper do not call for a relaxation of treasury - central bank separation. Since the duration of forward guidance is not affected by transfers - when the latter are introduced in an optimal fashion - the optimal fiscal-monetary mix can be implemented with treasury observing the planned path of interest rates and setting transfers accordingly.

This paper opens up several avenues of extension: taking into account shocks triggering a liquidity trap through the hand to mouth side, as a deleveraging shock, would change the optimal transfer policy implication, introducing different trade-offs between aggregate stabilization and consumption dispersion. Also modeling the effect of dividends and stock market fluctuations on inequality can have relevant implications, as foreshadowed by the results of the last section. Finally, extending the model to a full heterogeneous agents New Keynesian environment (HANK) would allow to carry on further the analysis of the quantitative effects of transfer policy over the liquidity trap.

A Appendix

A.1 Derivation of the linearized labor supply conditions of the ricardian and hand to mouth

In the following, take into account the following definitions:

$$\sigma = -\frac{U''(Y)}{U'(Y)}Y = zY \tag{A.1.1}$$

$$\phi = \frac{V''(L)}{V'(L)}\bar{L} = \eta\bar{L} \tag{A.1.2}$$

Where $\bar{L} = (1-\lambda)L + \lambda L^*$. Taking a log-linear approximation of the Ricardian and HtM labor supply, we obtain:

$$\eta L\hat{l}_t = -zC\hat{c}_t + \hat{\omega}_t \tag{A.1.3}$$

$$\eta L^*\hat{l}_t^* = -zC^*\hat{c}_t^* + \hat{\omega}_t \tag{A.1.4}$$

Where $\hat{\omega}_t$ is real wage deviation. Aggregating up (A.1.3) and (A.1.4), and using $Y = A((1-\lambda)L + \lambda L^*) = \omega((1-\lambda)L + \lambda L^*)$, we obtain:

$$\eta\frac{Y}{\omega}\hat{y}_t = -zY\hat{y}_t + \hat{\omega}_t \tag{A.1.5}$$

Then:

$$\hat{\omega}_t = \left(\frac{\eta}{\omega}Y + zY\right)\hat{y}_t = (\phi+\sigma)\hat{y}_t \tag{A.1.6}$$

Therefore, plugging (A.1.1), (A.1.2) and (A.1.6), into (A.1.3) and (A.1.4), we get:

$$\frac{L}{\bar{L}}\hat{l}_t = -\frac{\sigma}{\phi}\frac{C}{Y}\hat{c}_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.7}$$

$$\frac{L^*}{\bar{L}}\hat{l}_t^* = -\frac{\sigma}{\phi}\frac{C^*}{Y}\hat{c}_t^* + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.8}$$

Which, in the baseline case with $C = C^* = Y$ and $L = L^* = \bar{L}$, boils down to:

$$\hat{l}_t = -\frac{\sigma}{\phi}\hat{c}_t + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.9}$$

$$\hat{l}_t^* = -\frac{\sigma}{\phi}\hat{c}_t^* + \frac{\phi+\sigma}{\phi}\hat{y}_t \tag{A.1.10}$$

A.2 Derivation of the linearized hand to mouth budget constraint

Let us take a log-linear approximation of the hand to mouth budget constraint (2.1.9) (in real terms) around the steady state:

$$C^*\hat{c}_t^* = L^*\omega(\hat{l}_t^* + \hat{\omega}_t) + Y\hat{\tau}_t^* + \frac{\chi}{\lambda}Y\hat{d}_t \tag{A.2.1}$$

Where I define the HtM transfer deviation $\hat{\tau}_t^* = (\tau_t - \tau^*)/Y$, and the dividend deviation $\hat{d}_t$ as $(\bar{D}_t - D)/Y$.

Aggregate real dividend $\bar{D}_t$ consists of aggregate output net of the labor cost, corrected for the subsidy and net of the lump sum tax ζ_t (which is given by $\zeta_t = ((1-\lambda)L_t + \lambda L_t^*)\omega_t\nu$):

$$\bar{D}_t = Y_t - ((1-\lambda)L_t + \lambda L_t^*)\omega_t(1-\nu) - ((1-\lambda)L_t + \lambda L_t^*)\omega_t\nu \tag{A.2.2}$$

In log linearized terms:

$$Y\hat{d}_t = Y\hat{y}_t - (1-\lambda)L\omega\hat{l}_t + \lambda L^*\omega\hat{l}_t^* - ((1-\lambda)L + \lambda L^*)\omega\hat{\omega}_t = -Y\hat{\omega}_t \tag{A.2.3}$$

Plugging (A.1.6),(A.1.8) and (A.2.3) into (A.2.1), we obtain the expression:

$$\frac{C^*}{Y}\hat{c}_t^* = \Phi\hat{y}_t + \hat{y}_t + \frac{\phi}{\phi+\sigma}\hat{\tau}_t^* \tag{A.2.4}$$

Where $\Phi = \phi\left(L^*/\bar{L} - \chi/\lambda\right)$. This is the general form for the hand to mouth budget constraint, reported in Section 5 (equation (5.0.1)). Setting $L = L^*$ and $\chi = \lambda$, we recover instead the budget constraint in the baseline case of Section 3 (equation (3.0.2)).

A.3 Proof of $\lim_{t\to\infty}\hat{y}_t = 0$

We do not restrict our analysis to optimal transfer policy, but account for any possible sequence of HtM transfer $\{\hat{\tau}_t^*\}_{t=t_0}^{\infty}$. Let us take a first order approximation of the aggregate resource constraint $(1-\lambda)C + \lambda C^* = Y_t$ around the initial steady state (namely, the state of the economy before the shock $\hat{\xi}_{t_0}$ hits). This approximation spans all the possible steady state to which the economy converges after the liquidity trap[9]:

$$(1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = (1-\lambda)L\hat{l} + \lambda L^*\hat{l}^* \tag{A.3.1}$$

Then, using (A.1.7) and (A.1.8), we get:

$$(1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = (1-\lambda)L\left(-\frac{zC}{\eta L}\hat{c} + \frac{1}{\eta L}\hat{\omega}\right) + \lambda L^*\left(-\frac{zC^*}{\eta L^*}\hat{c}^* + \frac{1}{\eta L^*}\hat{\omega}\right) \tag{A.3.2}$$

In steady state the real wage ω is fixed to the stationary level A (see section 2.5), then $\hat{\omega} = 0$. Then we can rearrange and simplify the equation above as:

$$(1-\lambda)C\hat{c} = -\lambda C^*\hat{c}^* \tag{A.3.3}$$

That implies

$$Y\hat{y}_t = (1-\lambda)C\hat{c} + \lambda C^*\hat{c}^* = 0 \tag{A.3.4}$$

Therefore steady state output is not affected by the crossectional distribution of consumption up to a first order approximation. So any long run HtM transfer $\hat{\tau}^{*\prime} \neq 0$ set by the government in the limit is not driving $\lim_{t\to\infty} \hat{y}_t$ away from 0.

A.4 Expressing consumption dispersion as a function of the the HtM transfer only

Using the aggregate resource constraint $Y\hat{y}_t = (1-\lambda)C\hat{c}_t + \lambda C^*\hat{c}^*_t$ and the hand to mouth budget constraint (A.2.4), we can derive:

$$Y\hat{y}_t = C(1-\lambda)\hat{c}_t + C^*\lambda\hat{c}^*_t = \tag{A.4.1}$$

$$= (1-\lambda)(C\hat{c}_t - C^*\hat{c}^*_t) + C^*\hat{c}^*_t = \tag{A.4.2}$$

$$= (1-\lambda)(C\hat{c}_t - C^*\hat{c}^*_t) + Y\Phi\hat{y}_t + Y\hat{y}_t + Y\frac{\phi}{\phi+\sigma}\hat{\tau}^*_t \tag{A.4.3}$$

Rearranging the equation above, we obtain:

$$\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}^*_t = -\frac{\phi}{\phi+\sigma}\frac{1}{1-\lambda}\tilde{\tau}^*_t \tag{A.4.4}$$

[9] The price dispersion term Δ_t is not considered up to a first order approximation

Where $\tilde{\tau}_t^* \equiv [(\phi+\sigma)/\phi]\Phi\hat{y}_t + \hat{\tau}_t^*$. And, squaring both sides, we obtain:

$$\left(\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}_t^*\right)^2 = \left(\frac{1}{1-\lambda}\right)^2\left(\frac{\phi}{\phi+\sigma}\tilde{\tau}_t^*\right)^2 \tag{A.4.5}$$

Notice that, if $L = L^*$ and $\chi = \lambda$, then $\Phi = 0$, $C = C^* = Y$, and $\hat{\tau}_t = \tilde{\tau}_t$, so we recover the formulation for consumption dispersion in the baseline setting without cyclical income difference (equation (3.0.3)).

A.5 The optimal steady state transfer problem

In what follows, we will approach the government Ramsey problem of optimal steady state transfer selection τ, τ^* in two steps: first, through a *social planner* problem, which selects the optimal steady state consumption and labor levels C, C^*, L, L^*; then, we will find the transfers τ, τ^* which implement this solution in the decentralized equilibrium. the formulation of a full social planner problem is possible as the steady state is not distorted thanks to the optimal labor cost subsidy ν, that guarantee the achievement of Pareto-efficiency.

The social planner problem is a utilitarian maximization of a linear combination of the utility of the households, according to some weights ψ, ψ^*, and subject to the aggregate resource constraint of the economy:

$$\max_{C,C^*,L,L^*} \ \psi(1-\lambda)\left(1-\exp(-zC)-\delta\exp(\eta L)\right) + \psi^*\lambda\left(1-\exp(-zC^*)-\delta\exp(\eta L^*)\right) \tag{A.5.1}$$
$$\text{s.t. } (1-\lambda)C + \lambda C^* = (1-\lambda)AL + \lambda AL^*$$

The first order conditions of the problem yield the following optimality conditions:

$$(1-\lambda)C + \lambda C^* = (1-\lambda)AL + \lambda AL^* \tag{A.5.2}$$

$$\delta\eta\exp(\eta L) = z\exp(-zC)A \tag{A.5.3}$$

$$\delta\eta\exp(\eta L^*) = z\exp(-zC^*)A \tag{A.5.4}$$

$$\psi z\exp(-zC) = \psi^* z\exp(-zC^*) \tag{A.5.5}$$

Optimally, the social planner equates the marginal disutility from labor to the marginal utility of consumption times the productivity, for each agent. Moreover, the household weighted more in the welfare function has lower marginal utility of consumption than the other one (and then , by

(A.5.3) and (A.5.4), lower marginal disutility of labor as well).

Turning the attention to the decentralized equilibrium, the consumption levels found through the social planner problem can be decentralized by setting appropriate transfers. The steady state budget constraints of the ricardian and the hand to mouth indeed write:

$$C = \omega L + \tau + \mathcal{B}\left(\frac{1}{\Pi} - \beta\right) \tag{A.5.6}$$

$$C^* = \omega L^* + \tau^* \tag{A.5.7}$$

Where $\mathcal{B}$ is steady state aggregate real bond quantity. Notice that aggregate dividends $\bar{D}$ are zero in steady state by (A.2.2), so they do not show up in the households' budget constraint. Equations (A.5.6) and (A.5.7) pin down the optimal steady state transfers τ, τ^*, given the optimal levels C, C^*, L, L^* and the aggregate bond real quantity[10].

A.6 Derivation of the welfare objective of the government (6.1.1)

Let us restate the flow welfare function of the government ((A.5.1)):

$$U_t = \psi(1-\lambda)\left(1 - \exp(-zC_t) - \delta\exp(\eta L_t)\right) + \psi^*\lambda\left(1 - \exp(-zC_t^*) - \delta\exp(\eta L_t^*)\right) \tag{A.6.1}$$

Taking a second order approximation of the expression above around the steady state U_{SS} yields:

$$\begin{aligned}
U_t \approx{} & U_{SS} + \psi(1-\lambda)z\exp(-zC)C\frac{C_t - C}{C} + \psi^*\lambda z\exp(-zC^*)C^*\frac{C_t^* - C^*}{C^*} + \\
& - \psi(1-\lambda)\delta\eta\exp(\eta L)L\frac{L_t - L}{L} - \psi^*\lambda\delta\eta\exp(\eta L^*)L^*\frac{L_t^* - L^*}{L^*} + \\
& - \frac{1}{2}\psi(1-\lambda)z^2\exp(-zC)C^2\left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\psi^*\lambda z^2\exp(-zC^*)C^{*2}\left(\frac{C_t^* - C^*}{C^*}\right)^2 \\
& - \frac{1}{2}\psi(1-\lambda)\delta\eta^2\exp(\eta L)L^2\left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\psi^*\lambda\delta\eta^2\exp(\eta L^*)L^{*2}\left(\frac{L_t^* - L^*}{L^*}\right)^2
\end{aligned} \tag{A.6.2}$$

[10] If I allowed the size of real debt to be chosen by the planner, that would have provided an additional and not necessary degree of freedom to implement the optimal allocation.

Using results (A.5.3)-(A.5.5), we can factor out some constant terms:

$$\begin{aligned}
U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[& (1-\lambda)\frac{1}{\omega} C \frac{C_t - C}{C} + \lambda \frac{1}{\omega} C^* \frac{C_t^* - C^*}{C^*} \\
& - (1-\lambda) L \frac{L_t - L}{L} - \lambda L^* \frac{L_t^* - L^*}{L^*} + \\
& - \frac{1}{2}(1-\lambda)\frac{1}{\omega} z C^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda \frac{1}{\omega} z C^{*2} \left(\frac{C_t^* - C^*}{C^*}\right)^2 \\
& - \frac{1}{2}(1-\lambda)\eta L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda \eta L^{*2} \left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg]
\end{aligned} \tag{A.6.3}$$

Using the aggregate resource constraint $Y_t = (1-\lambda)C_t + \lambda C_t^*$, the expression above becomes:

$$\begin{aligned}
U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[& \frac{Y}{\omega}\frac{Y_t - Y}{Y} - (1-\lambda) L \frac{L_t - L}{L} - \lambda L^* \frac{L_t^* - L^*}{L^*} + \\
& - \frac{1}{2}(1-\lambda)\frac{1}{\omega} z C^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda \frac{1}{\omega} z C^{*2} \left(\frac{C_t^* - C^*}{C^*}\right)^2 \\
& - \frac{1}{2}(1-\lambda)\eta L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda \eta L^{*2} \left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg]
\end{aligned} \tag{A.6.4}$$

A first order approximation of the market clearing condition (5.7.3) yields:

$$\frac{Y\Delta}{A}\frac{Y_t - Y}{Y} + \frac{Y\Delta}{A}\frac{\Delta_t - \Delta}{\Delta} = (1-\lambda) L \frac{L_t - L}{L} + \lambda L^* \frac{L_t^* - L^*}{L^*} \tag{A.6.5}$$

Where $\Delta = 1$. Recalling that $\omega = A$, and substituting for the above expression into (A.6.4) yields:

$$\begin{aligned}
U_t \approx U_{SS} + \psi z \exp(-zC)\omega \Bigg[& -\frac{Y}{\omega}\frac{\Delta_t - \Delta}{\Delta} - \frac{1}{2}(1-\lambda)\frac{1}{\omega} z C^2 \left(\frac{C_t - C}{C}\right)^2 - \frac{1}{2}\lambda \frac{1}{\omega} z C^{*2} \left(\frac{C_t^* - C^*}{C^*}\right)^2 + \\
& - \frac{1}{2}(1-\lambda)\eta L^2 \left(\frac{L_t - L}{L}\right)^2 - \frac{1}{2}\lambda \eta L^{*2} \left(\frac{L_t^* - L^*}{L^*}\right)^2 \Bigg]
\end{aligned} \tag{A.6.6}$$

Consider for any variable x_t the second order approximations $(x_t - x)/x \approx \hat{x}_t + (1/2)\hat{x}_t^2$ and $[(x_t - x)/x]^2 \approx \hat{x}_t^2$ where $\hat{x}_t$ is the log-deviation. Let us take also into account that $\hat{\Delta}_t^2 = 0$ up to a second order approximation. Then we can write the expression above as follows:

$$U_t \approx U_{SS} + \psi z \exp(-zC)\omega \left[-\frac{Y}{\omega}\hat{\Delta}_t - \frac{1}{2}(1-\lambda)\frac{1}{\omega} z C^2 \hat{c}_t^2 - \frac{1}{2}\lambda \frac{1}{\omega} z C^{*2} \hat{c}_t^{*2} - \frac{1}{2}(1-\lambda)\eta L^2 \hat{l}_t^2 - \frac{1}{2}\lambda \eta L^{*2} \hat{l}_t^{*2} \right] \tag{A.6.7}$$

By the aggregate resource constraint $Y\hat{y}_t = (1-\lambda)C\hat{c}_t + \lambda C^* \hat{c}_t^*$ and (A.6.5), notice the following first order equivalences (where the second order term $\hat{\Delta}_t$ does not show up):

$$C\hat{c}_t = Y\hat{y}_t + \lambda(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.8}$$

$$C^*\hat{c}_t^* = Y\hat{y}_t - (1-\lambda)(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.9}$$

$$L\hat{l}_t = \frac{Y}{\omega}\hat{y}_t + \lambda(L\hat{l}_t - L^*\hat{l}_t^*) \tag{A.6.10}$$

$$L^*\hat{l}_t^* = \frac{Y}{\omega}\hat{y}_t - (1-\lambda)(L\hat{l}_t - L^*\hat{l}_t^*) \tag{A.6.11}$$

Moreover, using (A.1.7) and (A.1.8) we can rewrite (A.6.10) and (A.6.11) as:

$$L\hat{l}_t = \frac{Y}{\omega}\hat{y}_t - \lambda\frac{z}{\eta}(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.12}$$

$$L^*\hat{l}_t^* = \frac{Y}{\omega}\hat{y}_t + (1-\lambda)\frac{z}{\eta}(C\hat{c}_t - C^*\hat{c}_t^*) \tag{A.6.13}$$

using (A.6.8), (A.6.9), (A.6.12) and (A.6.13), we can rewrite (A.6.7) as follows:

$$\begin{aligned}
U_t \approx\; & U_{SS} + \psi z \exp(-zC)\omega\Bigg[-\frac{Y}{\omega}\hat{\Delta}_t + \\
& -\frac{1}{2}(1-\lambda)\frac{z}{\omega}\left[Y^2\hat{y}_t^2 + \lambda^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 + 2\lambda Y\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}\lambda\frac{z}{\omega}\left[Y^2\hat{y}_t^2 + (1-\lambda)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 - 2(1-\lambda)Y\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}(1-\lambda)\eta\left[\left(\frac{Y}{\omega}\right)^2\hat{y}_t^2 + \lambda^2\left(\frac{z}{\eta}\right)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 - 2\lambda\frac{z}{\eta}\frac{Y}{\omega}\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right] + \\
& -\frac{1}{2}\lambda\eta\left[\left(\frac{Y}{\omega}\right)^2\hat{y}_t^2 + (1-\lambda)^2\left(\frac{z}{\eta}\right)^2(C\hat{c}_t - C^*\hat{c}_t^*)^2 + 2(1-\lambda)\frac{z}{\eta}\frac{Y}{\omega}\hat{y}_t(C\hat{c}_t - C^*\hat{c}_t^*)\right]\Bigg]
\end{aligned} \tag{A.6.14}$$

Rearranging the expression above, we get:

$$U_t \approx U_{SS} + \psi z \exp(-zC)Y\left[-\hat{\Delta}_t - \frac{1}{2}\left(z + \frac{\eta}{\omega}\right)Y\hat{y}_t^2 - \frac{1}{2}\lambda(1-\lambda)z\frac{\omega}{\eta}Y\left(z + \frac{\eta}{\omega}\right)\left(\frac{C}{Y}\hat{c}_t - \frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.15}$$

Consider a recursive formulation for price dispersion:

$$\Delta_t = \alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta}\Delta_{t-1} + (1-\alpha)\left(\frac{1-\alpha\left(\frac{\Pi_t}{\Pi}\right)^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}} \tag{A.6.16}$$

Taking a second order approximation of the equation above and summing through time yields

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{\Delta}_t=\frac{\alpha}{(1-\alpha)(1-\alpha\beta)}\theta\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\hat{\pi}_t^2 \tag{A.6.17}$$

As standard in the literature. Taking the infinite discounted sum of (A.6.15), we can substitute for the result above, obtaining the government's loss function:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\left(z+\frac{\eta}{\omega}\right)Y\frac{\theta}{\kappa}\hat{\pi}_t^2+\frac{1}{2}\left(z+\frac{\eta}{\omega}\right)Y\hat{y}_t^2+\frac{1}{2}z\frac{\omega}{\eta}\left(z+\frac{\eta}{\omega}\right)Y\lambda(1-\lambda)\left(\frac{C}{Y}\hat{c}_t-\frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.18}$$

where $\kappa=[(1-\alpha)(1-\alpha\beta)/\alpha](z+\frac{\eta}{\omega})Y$. Equivalently, using (A.1.1), (A.1.2) and $\bar{L}=Y/\omega$, we can write the loss function as:

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[\frac{1}{2}\frac{\theta}{\kappa}\hat{\pi}_t^2+\frac{1}{2}\hat{y}_t^2+\frac{1}{2}\frac{\sigma}{\phi}\lambda(1-\lambda)\left(\frac{C}{Y}\hat{c}_t-\frac{C^*}{Y}\hat{c}_t^*\right)^2\right] \tag{A.6.19}$$

Notice that, setting $C=C^*=Y$, the welfare objective is the one of section 3.3.

A.7 Sensitivity analysis

A key quantitative result of the paper is the mutual offsetting nature of the lengthening and shortening channels of optimal transfer policy with respect to the duration of forward guidance, which gives rise to a null effect on the optimal time of the liftoff of the nominal interest rate from the zero lower bound. In this section I perform numerical robustness analysis on this result, by considering a range of alternative parametrizations. I take into account the three parameters that show up in the aggregate demand equation (3.3.2) and in the coefficient of the consumption dispersion term showing up in the welfare objective of the government ((6.1.1)), i.e. the fraction of hand to mouth households λ, the relative risk aversion coefficient σ, and inverse Frisch elasticity of labor supply ϕ. These parameters determine the effect of transfers on the economy, either through the stabilization via aggregate demand, or through the impact on consumption dispersion. For each of these parameters, I select a couplet of alternative parametrizations, one higher and the other lower than the value used in the paper - see Table 1.

I also perform a sensitivity analysis with respect to the size of the shock ε_ξ and the discount factor β. Both these parameters indeed determine the extent to which the zero lower bound is binding during the liquidity trap (β in particular pins down the steady state value of the nominal interest

Table 1: Alternative parametrizations

Parameter	Low value	Paper	High value
λ	0.2	0.33	0.5
σ	1	2	3
ϕ	0.2	0.47	1
β	0.995	0.9987	0.999
ε_ξ	0.02	0.025	0.07

rate $i = \Pi/\beta - 1$, so the proximity of the latter to the zero lower bound). Also in this case I take into account a lower and a higher value with respect to the parametrization of the paper, which are reported as well in Table 1.

All these alternative simulations are carried out moving one parameter at a time. Results are summarized in Figure .1. In all the alternative configurations we can highlight the presence of the lengthening and shortening effects of transfer policy with respect to forward guidance; these effect offset each other, leaving the duration of the stay of the interest rate at the zero lower bound unchanged with respect to the baseline case with constant transfers.

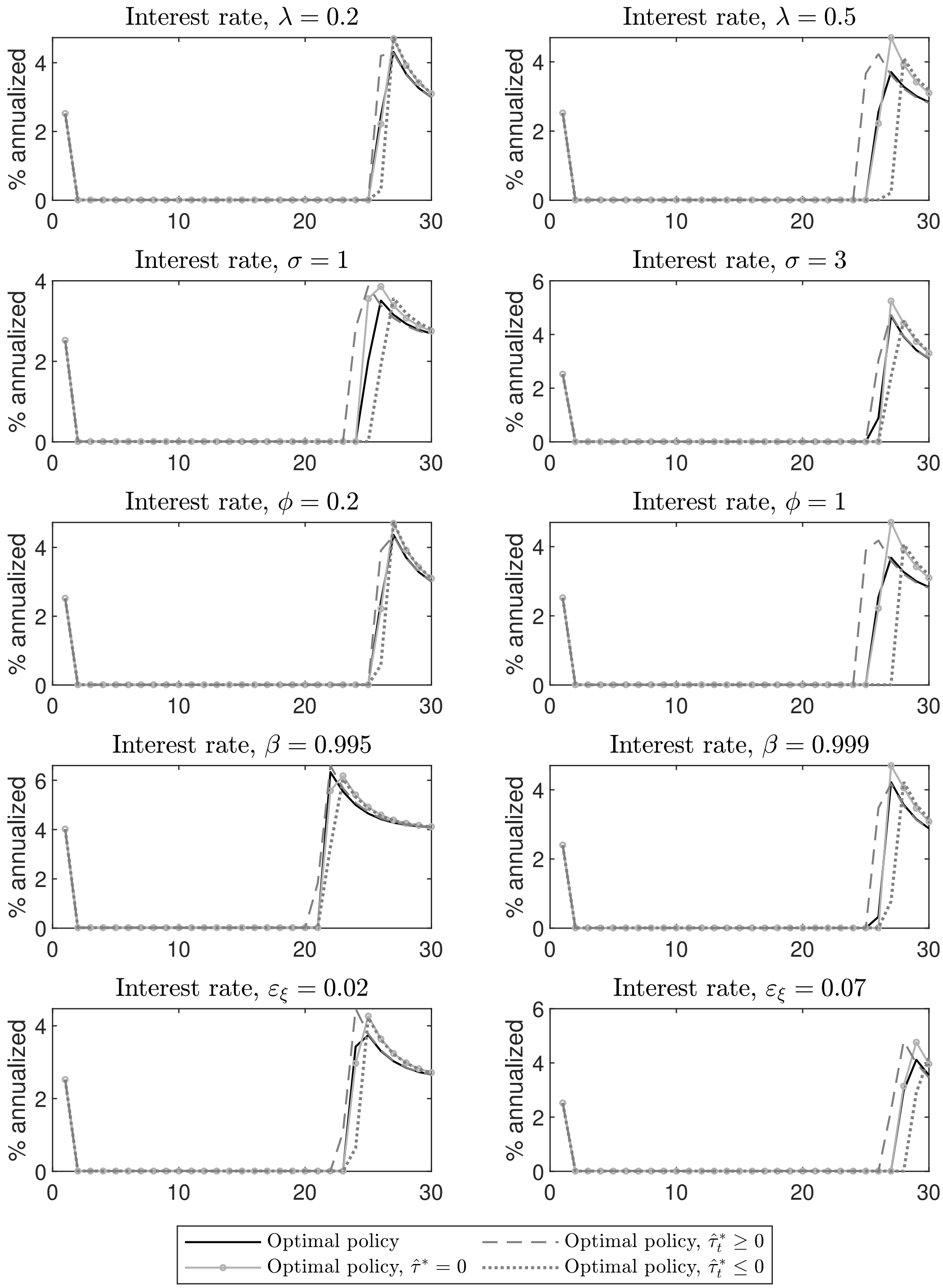


Figure A.1: Sensitivity analysis, nominal interest rate

Chapter 2

Interest Rate Smoothing in Face of Energy Shocks

1 Introduction

The years 2022 and 2023 witnessed a substantial rise in energy prices, exacerbating the inflationary pressures that had been steadily building since 2021; in response to the inflation surge, central banks in advanced economies have raised the policy rates, aiming to curb inflationary pressures and to safeguard the real income of consumers. Generally these interventions increased interest rates on variable-rate mortgages and fixed-rate mortgages due for renewal during the period of rate hikes. The case of the UK economy is particularly illustrative of this phenomenon: housing mortgages' cost are typically renegotiated every 5 years or less, making their interest rate particularly sensible to the movements in the policy rate set by the Bank of England (BoE).

The Central Banks' trade-off between shielding real income of consumers and maintaining moderate mortgage interest rates poses challenges for the formulation of a monetary policy reaction to an energy price shock. A contractionary interest rate policy effectively safeguards households' wages purchasing power by fostering a real exchange rate appreciation (by uncovered interest rate parity); on the other side, it increases the cost of mortgages.

The main theoretical result of the paper is that the trade-off between the protection of households' real income and preventing high interest rates for borrowers can be resolved once we account for monetary policy manipulating *the whole path of future interest rates.* If the central bank indeed commits to monetary tightening in the future, this implies a current real appreciation of domestic goods - through uncovered interest rate parity holding across the whole yield curve - that protects real wages' purchasing power ; therefore there is room to adopt a milder monetary policy at the onset of the shock, in order not to increase too much the financial burden on borrowers. The result of the paper echoes Silvana Tenreyro's argument in her final speech as Monetary Policy Committee member at the Bank of England, which stated that the monetary authority should commit in advance to a determined path of future interest rates, in order to partially offset the need to raise current rates in reaction to the surge in energy prices.

This paper analyses this trade-off in a small open economy new keynesian setting where agents are heterogeneous because of uninsurable idiosyncratic income risk. Agents trade in a liquid assets and are endowed with perpetual liabilities (mortgages) whose interest rate is in part fixed and in part variable, i.e. directly connected with the monetary policy rate. The presence of mortgages creates a quantitatively relevant adverse effect of contractionary monetary policy on the budget constraints of households. Agents' heterogeneity is a key assumption to make both the components of the trade-off (increases in temporary mortgage costs and falls in the real wage) quantitatively relevant from a welfare perspective: households indeed are unable to fully absorb income and mortgage cost shocks due to a precautionary saving motive, which especially holds true for the ones closer to the borrowing limit. Moreover, a full heterogeneous agents environment allows to have both a real wage fall and mortgage cost increases to be quantitatively relevant in affecting consumption over the whole crossection of agents (differently from a two-agents models, where these effects would only be numerically important for the borrowing constrained agents).

Once obtained the theoretical results in terms of benefit of interest rate smoothing, I proceed to a quantitative assessment of the implications of the model in the UK economy. The model is fed with the actual current and expected interest rate hike implemented by the BoE, as well as by the actual energy price data. The model is constructed and calibrated to match data both in an "aggregate" dimension (CPI inflation, real exchange rate, real wage, aggregate mortgage cost) and to align with the incidence of mortgages on the cross-sectional households' consumption patterns. The reference panel data for this analysis, "Understanding Society", reports nearly exclusively food expenditure among various expenditure items: therefore, I focus on comparing the model's outcomes to the data in terms of the effects of mortgage cost increases on food consumption.

The quantitative results of the paper point out that a *smoothed* interest rate policy - characterized by the interest rate peaking at 1 percentage point less than in BoE implemented policy, and requiring an additional three years to land on the new long-term level - is able to attain the same real exchange appreciation over the energy crisis, while reducing the food consumption difference between mortgagors and non-mortgagors by 4% over 2022, thanks to the reduced interest rate surge.

Contribution to the literature The model builds on the framework by Auclert, Rognlie, Souchier, and Straub (2023b), which study fiscal and monetary response to energy shocks in a HANK-type small open economy. Other recent literature studying the behavior of heterogeneous agents open economy in face of foreign shocks are Auclert, Rognlie, Souchier, and Straub (2023a) and Fukui, Nakamura, and Steinsson (2023) - for the case of depreciation shocks, and de Ferra, Mitman, and Romei (2020) - for sudden stops in capital inflows. This paper complements this strand of literature by analysing the trade-off - faced by a monetary policy reacting to the energy price shock - between fighting real wages deterioration and keeping moderate welfare costs for borrowers.

Pieroni (2023) studies the inflation - output gap trade-off faced by monetary policy during an energy supply shock in a closed economy HANK environment. Also in his framework the government's choice is characterized by a tension between raising interest rates to fight inflation, and the aim of not penalizing too much borrowers though the cost of debt channel. However, it restricts monetary policy to a Taylor-rule without room for monetary smoothing. The 2022-2023 energy crisis gives rise to other sources of welfare loss, which have been analyzed by recent literature: Olivi, Sterk, and Xhani (2023) study optimal monetary policy when consumption baskets vary across households: their model does not display neither an open economy dimension (so an appreciation channel of monetary policy) nor a debt cost channel of interest rate policy, which are the key factors of the trade-off examined in my work.

My paper, while assessing the trade-off between purchasing power defense and mortgage cost moderation, explicitly takes into account distributional effects of interest rate hikes, effects which are investigated empirically and theoretically in Del Negro, Dogra, Gundam, Lee, and Pacula (2024). Factoring inequality outcomes in the assessment of monetary policy performance is a robust implication of optimal policy analysis in heterogeneous agents' models such as in Bhandari et al. (2021), Wolf (2023), Ragot (2017), Acharya et al. (2021), Dávila and Schaab (2023) and Smirnov (2023). My paper naturally relates to this branch of literature by accounting for the asymmetric effect on monetary policy across the households' crossection in formulating an alternative monetary policy with respect to the benchmark one followed by the BoE over the energy crisis. In accordance with the findings from optimal policy literature, the proposed alternative suggests a "milder" contraction during the most severe stages of the economic cycle, to avoid excessively burdening borrowers.

The modelization of the heterogeneous agents' setting follow closely Nuño and Thomas (2022) and Achdou, Han, Lasry, Lions, and Moll (2021).

The paper is organized as follows: section 2 presents the model; section 3 analyzes the real appreciation - mortgage cost trade-off of the central bank, and provides the analytical result behind the interest rate smoothing policy prescription. Section 4 lays the ground for the quantitative application: it first presents the macro trends of the UK economy over the energy crisis and computes the empirical effect of mortgages on food consumption of households over the crossection; then proceeds to calibration and validation of the model. Section 5 explores the quantitative results of the model by comparing the benchmark BoE policy with a smoothed policy alternative. Section 6 concludes.

2 Model

The following general open economy framework builds on Auclert et al. (2023a) and Auclert et al. (2023b), while introducing two novel elements: long term bonds and mortgages (the latter modeled as perpetual debt, as in Burya and Davitaya (2022)), and food and non-food consumption (in order to construct a model-counterpart of food consumption variations analyzed in section 4).

2.1 Domestic households

A small open economy (the "domestic" economy) is populated by a unit mass of households, heterogeneous with respect to their wealth and their labor productivity. The discounted utility of a generic household i in economy j reads:

$$E_0 \int_0^\infty e^{\rho t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} \right] dt \tag{2.1.1}$$

where ρ is a subjective discount rate, σ is the coefficient of risk aversion, c_t is a Dixit-Stigliz consumption aggregator of a food c_t^f and non-food good c_t^{nf}, with elasticity ν and time-varying relative weight φ_t:

$$c_t = [\varphi_t^{\frac{1}{\nu}} c_{ft}{}^{\frac{\nu-1}{\nu}} + (1-\varphi_t)^{\frac{1}{\nu}} c_{nt}{}^{\frac{\nu-1}{\nu}}]^{\frac{\nu}{1-\nu}} \tag{2.1.2}$$

The Dixit-Stigliz formulation gives rise to the standard characterization of the price level as a harmonic average of the food and non-food goods:

$$p_t = [\varphi_t p_{ft}^{1-\nu} + (1-\varphi_t) p_{nt}^{1-\nu}]^{\frac{1}{1-\nu}} \tag{2.1.3}$$

Labor supply n_t is a bundle of a unit mass of labor varieties k supplied by the household:

$$n_t = \int_0^1 n_{kt} dk \tag{2.1.4}$$

where each variety's supply n_{kt} - equal across all household - is determined by a union, whose optimization problem will be discussed later.

I follow Nuño and Thomas (2022), in assuming that households trade in a nominal risk-free long-term bond a_t, among themselves and with foreign investors. A bond issued a time t promises a stream of nominal payments $\{\delta e^{-\delta(s-t)}\}_{s\in(t,\infty)}$ summing up to one unit of domestic currency over the infinite lifetime of the bond. A fraction ω of households is also endowed with *mortgage stock*, equal across all of them, that enter the budget constraint under the form of an nominal perpetual debt paid at interest rate i_t^d, and whose proceeds are rebated equally to each domestic household. Therefore the remaining fraction $1-\omega$ of households which are non-mortgagors (or "outright owners") still enjoy the stream of proceeds of mortgage revenues. The real levels of mortgage stocks $D_t^r \equiv D/p_t$ follows a law of motion which takes into account the effect of inflation $\pi_t \equiv \dot{p}_t/p_t$ on its denominator:

$$\dot{D}_t^r = -D_t^r \pi_t \tag{2.1.5}$$

The drift in the asset's dynamics is determined by the saving of the household, converted in asset units by division by the price X_t of the currently traded bond, net of the real reduction of asset amount by the amortization rate δ and inflation π_t:

$$\dot{a}_t = \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t \tag{2.1.6}$$

where $w_t \equiv W_t/p_t$ is the real wage, z_t is an idiosyncratic productivity shock that follows a diffusion process with parameters $\mu(z), \varsigma^2$; i_t^D is a household-specific interest rate on mortgages d_t and Π_t are dividends rebated to the household, generated respectively by the profits o firms and by the pooled economy-wide revenues from mortgages.

Each household's mortgage debt stock D is made up by a variable rate amount D^v and a fixed rate amount D^f, such that $D = D^v + D^f$. Both D^v and D^f have real value determined with the same process of (2.1.5): so the ratios D^v/D and D^f/D are constant over time. The variable rate mortgage yields interest rate i_t, anchored to the one provided by a security issued by the central bank (see section 2.5). The fixed rate mortgage consists instead in the sum of a continuum of mortgages of the same size D^f/S, indexed with subscript s and ranging from 0 to S:

$$D^f = \int\limits_0^S D^f(s)ds \tag{2.1.7}$$

Each $D^f(s)$ entails a household-specific interest rate $i_t^f(s)$: this implies $i_t^f = \frac{1}{S}\int\limits_0^S i_t^f(s)ds$. At each period t, only the mortgage $s(t)$ gets its interest rate updated, where $s(t)$ is the remainder of the division of t/S: this introduces a S-interval periodicity in the update of each mortgage s. When a mortgage $s(t)$ is renewed, it is paired with an interest rate $i_t^f(s) = i_{\tau\in[t,t+S)}^f(s)$, constant until next time of renewal $t+S$. I assume that this interest rate is set to the level that would guarantee to the foreign household the same total payment amount of domestic currency over the next S time interval that would be accrued if $D^f(s)$ were behaving as a variable rate mortgage (given the information set of the economy at time t). In other terms, the fixed interest rate is equal to the average of the variable rates over the time until the next mortgage rate renewal:

$$i_t^f(s) = i_{\tau\in[t,t+S)}^f(s) = \frac{1}{S}\int\limits_{[t,t+S)} i_\tau d\tau \tag{2.1.8}$$

It is here worth to highlight that the updating mechanism for i_t^d (2.1.8) is arbitrarily assumed in a stylized way to capture the forward-looking nature of the fixed rate of mortgage, and it will prove to be suitable to let the aggregate mortgage rate i_t^d track its empirical counterpart in section 4.3. Given the exogenous and non-tradable nature of the mortgage perpetuity D, the interest rate update rule for both fixed and variable mortgages is indeed detached from any market force in the model[11]. Let us define the aggregate interest rate on mortgages i_t^d

[11] The non-tradability of the perpetuity could be relaxed by assuming that the latter was sold only once in the life of the economy, by a private perfectly competitive intermediary with property rights equally split across all households, to only a subset of agents (the since then called "mortgagors"), up to a limit D, while no unexpected shock had yet hit the economy. At the trade time, the perpetuity D would be expected to yield the same interest rate $\bar{i}$ as the long-term debt a_t, for the whole infinite horizon on the economy. Every agent who could buy the perpetuity would do it up to the limit D, and invest the whole amount in the long term bond, in order to get its position in a_t as clear as possible from the borrowing limit $\bar{a}$. After that moment, mortgagors would be locked-in with their mortgage position D and converge to a steady state distributions of assets and states - that one that will be treated in section 2.8.

as the weighted average of the fixed and variable rate:

$$i_t^d = \frac{D^f}{D} i_t^f + \frac{D^v}{D} i_t \tag{2.1.9}$$

Households aim at maximizing lifetime utility (5.1.1) by choosing consumption, asset holding under constraints (5.1.7) and the borrowing limit. The intertemporal problem of the household can be formulated recursively under the form of a Hamiltonian-Bellman-Jacobi equation for household with productivity realization z, asset holding a:

$$\rho V_t(a,z) = \max_{a_t, c_t} \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\phi}}{1+\phi} + s_t(a,z) \frac{\partial V_t}{\partial a} \right] + \mu(z) \frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2} \frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t} \tag{2.1.10}$$

where

$$s_t(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if mortgagor} \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t + \Pi_t}{X_t} - (\delta + \pi_t) a_t & \text{if non-mortgagor} \end{cases} \tag{2.1.11}$$

We can define the joint density of wealth and productivity $f_t(a,z)$. Its dynamics over time are governed by a Kolmogorov-forward equation:

$$\frac{\partial f_t(a,z)}{\partial t} = -\frac{\partial}{\partial a} [s_t(a,z) f_t(a,z)] - \mu(z) \frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2} \frac{\partial^2 V_t}{\partial z^2} \tag{2.1.12}$$

I will assume that the process for z is normalized such that the idiosyncratic productivity realizations aggregate to one:

$$\int_0^1 z f_t(a,z) dz = 1 \tag{2.1.13}$$

Lastly, let us define C_t as aggregate consumption in the domestic economy - the integral of $c_t(a,z)$ over all states a, z.

2.2 Final good producers

A mass of perfectly competitive firms produce either the food or non-food good, according to a CES production function in energy input y_{Et} (supplied by the foreign economy) and non-energy domestic input y_{Dt} (supplied by domestic producers):

$$y_{jt} = [(1-\alpha_E)^{\frac{1}{\epsilon}} y_{Dt}^{\frac{\epsilon-1}{\epsilon}} + \alpha_E^{\frac{1}{\epsilon}} y_{Et}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{1-\epsilon}} \qquad j = f, n \tag{2.2.1}$$

where ϵ is the elasticity of substitution between energy and non-energy goods. Notice that, being the production function for the food and non-food good exactly equal, the marginal cost mc_t^f, mc_t^n for both goods is the same, and it immediately follows that $mc_t^f = mc_t^n = p_{ft} = p_{nt} = p_t$ by perfect competition. The CES production function gives rise to the following formulation for the latter nominal marginal cost (equal to the final consumer's price p_t):

$$mc_t^f = mc_t^{nf} = p_{ft} = p_{nt} = p_t = [(1-\alpha_E)p_{Dt}^{1-\epsilon} + \alpha_E p_{Et}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{2.2.2}$$

where p_{Dt} and p_{Et} are respectively the prices of the non-energy and energy inputs.
The non-energy input y_{Dt} is in turn itself a CES aggregator of a home-produced good y_{Ht} and foreign-produced good y_{Ft}:

$$y_{Dt} = [(1-\alpha)^{\frac{1}{\eta}} y_{Ht}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} y_{Ft}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.2.3}$$

Where η is the elasticity of substitution between the domestic and foreign good. The price of the non-energy good can be derived as:

$$p_{Dt} = [(1-\alpha)p_{Ht}^{1-\eta} + \alpha p_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{2.2.4}$$

Final producers and producers of the domestic non-energy good solve an optimal variety expenditure problem, which delivers a standard Dixit-Stigliz demand formulation for energy, domestic and foreign goods:

$$y_{Et} = \alpha_E \left(\frac{p_{Et}}{p_t}\right)^{-\epsilon} y_{jt} \tag{2.2.5}$$

$$y_{Ht} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} (1-\alpha) \left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta} y_{jt} \tag{2.2.6}$$

$$y_{Ft} = (1-\alpha_E) \left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon} \alpha \left(\frac{p_{Ft}}{p_{Dt}}\right)^{-\eta} y_{jt} \tag{2.2.7}$$

2.3 Intermediate good producers

The intermediate domestic good y_{Ht} is produced by a competitive mass of firms[12] which operate under a technology linear in aggregate labor N_t and aggregate productivity A:

$$Y_{Ht} = AN_t \tag{2.3.1}$$

This implies that dividends are zero ($d_t = 0$). Aggregate labor N_t is a Dixit-Stigliz aggregator of labor varieties:

$$N_t = \left(\int N_{kt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3.2}$$

where N_{kt} is the aggregate labor demand for variety k. The zero profit condition equates the real wage per unit of output to the price of the domestic good:

$$w_t \frac{1}{A} = \frac{p_{Ht}}{p_t} \tag{2.3.3}$$

Firms also face an optimal choice of the labor variety mix, leading to the standard optimal labor variety demand:

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t \tag{2.3.4}$$

where W_{kt} is the nominal wage in labor market k.

2.4 Unions

Each union k determines the labor supply of variety k, i.e. n_{kt} - equal across all households - standing ready to satisfy labor demand:

$$n_{kt} = N_{kt} \tag{2.4.1}$$

Following Wolf (2021), the union chooses the nominal wage W_{kt} at which it supplies labor in order to maximize the utility of the *average* agent; this utility is considered net of a nominal adjustment cost and a real wage stabilization motive (the latter being introduced

[12] Auclert et al. (2023a) instead assumes a monopolistically competitive sector with flexible prices

as in Auclert et al. (2023b):

$$\max \int_{\tau \geq 0} \exp \left[-\rho\tau \left(\{ u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right) \} - \frac{\psi}{2} \pi_t^{W2} N_{t+\tau} - \frac{\zeta}{2} \frac{(\varepsilon - 1)\tilde{N} u' \tilde{C}}{\tilde{w}} \left(w_{k,t+\tau} - \tilde{w}\right)^2 \right) \right] \tag{2.4.2}$$

where $\tilde{N}$, $\tilde{C}$ and $\tilde{w}$ are respectively the final steady state levels of labor, aggregate consumption and the real wage and ζ is a parameter measuring the extent of the real wage stabilization motive. The latter is an important element to produce a positive pattern of inflation even in the tail of the energy shock, when energy price inflation would turn negative. As shown in the appendix, I solve the maximization problem subject to constraint (2.3.4) and the real labor earnings specification derived from the household block, obtaining the New Keynesian Phillips curve for inflation in the labor market:

$$\pi_t^W = \frac{1}{\rho - \dot{N}_t / N_t} \left[\kappa \left(\chi N_t^{\phi} - \frac{\varepsilon - 1}{\varepsilon} w_t C_t^{-\sigma} - \zeta \frac{\varepsilon - 1}{\varepsilon} \frac{\tilde{N}}{N_t} \tilde{C}^{-\sigma} (w_t - \tilde{w}) \frac{w_t}{\tilde{w}} \right) + \dot{\pi}_t^W \right] \tag{2.4.3}$$

where the slope κ is given by $\frac{\varepsilon}{\psi}$.

2.5 Central bank

The central bank trades a short term (instantaneous) risk-free asset with foreign households - as in Nuño and Thomas (2022). and sets its nominal return i_t. I assume the central bank not to follow any rule, but instead to set the prospective i_t for $[t, \infty)$ according to a fully arbitrary path contingent to the information set of the policy-maker at time t. Given the perfect foresight nature of the model, the planned path for i_t updates only if an unexpected ("MIT") shock hits the model a time t. This modelization choice allows to replicate a close fit of actual interest rate policy data, as showed in section 4.2. Equilibrium implication of this unconventional assumptions for monetary policy will be discussed in section 5.7.

2.6 Foreign economy

The rest of the world displays a representative household with constant consumption C^* of a non-energy good ($C^* = y^*$). The good is produced by a foreign representative firm, with technology symmetric to the final producers in the domestic economy ((5.1.4)):

$$y^* = [\alpha^{\frac{1}{\eta}} {y_{Ht}}^{*\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} {y_{Ft}}^{*\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{2.6.1}$$

where y^*_{Ht} and y^*_{Ft} are respectively the quantities of domestic and foreign input used by the foreign representative firm; note that the coefficient $(1-\alpha)$ is paired with y^*_{Ft}, due to home bias, mirroring expression (5.1.4).

Exported domestic goods are priced in foreign currency. Therefore, the foreign firms features the following Dixit-Stigliz demand for the domestic good:

$$y^*_{Ht} = \alpha \left(\frac{p^*_{Ht}}{p^*_t} \right)^{-\eta} y^*_{HF} \tag{2.6.2}$$

Where p^*_{Ht} and p^*_t are the home good price and the foreign price level in foreign currency, respectively. The foreign price index p^*_t is given by the standard CES formulation, symmetric to (5.1.3):

$$p^*_t = [(1-\alpha)p^{*1-\eta}_{Ft} + \alpha p^{*1-\eta}_{Ht}]^{\frac{1}{1-\eta}} \tag{2.6.3}$$

with p^*_{Ft} being the price of the foreign good in foreign currency; I assume p^*_{Ft} to be itself a Dixit-Stigliz aggregator of a mass of varieties N^*, i.e. $p^*_{Ft} = \left(\int_0^{N^*} \tilde{p}^{*1-\eta}_{Ft}(n)dn \right)^{\frac{1}{1-\eta}}$. For $N^* \to \infty$, imposing symmetry across the foreign varieties' prices $\tilde{p}^*_{Ft}(n)$ implies $p^*_{Ft} \to p^*_t$ - namely, the foreign economy is "big" with respect to the domestic one, so its price index is not affected by domestic economy's price fluctuations.

Monetary policy in the foreign economy ensures full price stability:

$$p^*_t = p^*_{Ft} = 1 \tag{2.6.4}$$

where I normalize p^* to 1. I assume the law of one price to hold, hence I obtain:

$$p^*_{Ht} = p_{Ht}\mathcal{S}_t \tag{2.6.5}$$

$$p_{Ft} = p^*_{Ft}/\mathcal{S}_t = 1/\mathcal{S}_t \tag{2.6.6}$$

where $\mathcal{S}_t$ is the nominal exchange rate. Defining the real exchange rate as $Q_t = \mathcal{S}_t \frac{p_t}{p_t^*} = p_t \mathcal{S}_t$, and substituting y^* by C^* by foreign economy's good market clearing, we can rewrite foreign demand (5.4.2) as:

$$y_{Ht}^* = \alpha \left(\frac{p_{Ht}}{p_t} Q_t \right)^{-\eta} C^* \tag{2.6.7}$$

From the equation above, it can be noticed how a real appreciation (i.e. an increase in Q_t), leads foreign consumers to express a lower demand for the domestic good, which becomes relatively less convenient.

In the light of the foreign price stability and law of one price assumptions, and using the definition $Q_t = p_t \mathcal{S}_t$ we can also rearrange the domestic price index (5.1.3) formulation to obtain the real price of energy and the domestic and foreign goods as a functions of real exchange rate Q_t and energy price in foreign currency p_{Et}^*, that I assume to be exogenous :

$$\frac{p_{Et}}{p_t} = p_{Et}^*/Q_t \equiv p_E(Q_t, p_{Et}^*) \tag{2.6.8}$$

$$\frac{p_{Dt}}{p_t} = \left(\frac{1 - \alpha_E p_E(Q_t, p_{Et}^*)^{1-\epsilon}}{1 - \alpha_E} \right)^{\frac{1}{1-\epsilon}} \equiv p_D(Q_t, p_{Et}^*) \tag{2.6.9}$$

$$\frac{p_{Ht}}{p_t} = \left[\frac{1}{1-\alpha} \left(\frac{1 - \alpha_E p_E(Q_t, p_{Et}^*)^{1-\epsilon}}{1 - \alpha_E} \right)^{\frac{1-\eta}{1-\epsilon}} - \frac{\alpha}{1-\alpha} p_F(Q_t)^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv p_H(Q_t, p_{Et}^*) \tag{2.6.10}$$

$$\frac{p_{Ft}}{p_t} = 1/Q_t \equiv p_F(Q_t) \tag{2.6.11}$$

The real price of energy p_{Et}/p_t depends positively on the foreign nominal price of energy p_{Et}^*, and negatively on the real exchange rate Q_t: domestic goods' appreciation indeed makes imported energy relatively cheaper. Conversely, the price of the non$_energygood$p$_{Dt}$ is negatively related to the price of energy, so it is decreasing in p_{Et}^* and increasing in Q_t. p_{Ft}/p_t depends negatively on the real exchange rate: real appreciations indeed reduce the price of the foreign good relatively to the domestic one. The real price of the domestic good, p_{Ht}/p_t, depends negatively on both the real price of energy and the real price of foreign goods: therefore, a real appreciation (i.e. and increase in Q_t) boosts the real price of domestic goods by making energy and foreign goods relatively cheaper. An increase in energy price p_{Et}^* instead lowers p_{Ht}/p_t by reducing the relative price of domestic goods with respect to energy.

I assume that the foreign household can invest both in an short term foreign asset yielding nominal return $i^* + \xi_t$ (with ξ_t being a time varying component), and in the domestic central bank's security mentioned in section 2.5: to rule out arbitrage opportunities, the return from the two assets needs therefore to be equal (uncovered interest parity, "UIP"):

$$i_t = i^* - \frac{d\mathcal{S}_t}{\mathcal{S}_t} + \xi_t \tag{2.6.12}$$

The condition can also be expressed in real terms:

$$i_t - \pi_t = i^* - \pi^* - \frac{dQ_t}{Q_t} + \xi_t \tag{2.6.13}$$

where $\pi^* = 0$ due price stability in the foreign economy. The foreign households, being able to invest also in domestic long term bonds, discounts the coupon payments of the latter at the central bank's security short term interest rate, allowing us to pin down the price of bonds at time t:

$$X_t = \int_t^\infty \delta e^{-\left[\int_t^s i_s + \delta(s-t)\right]} ds \tag{2.6.14}$$

2.7 Equilibrium

Given a path for the interest rates i_t and energy prices p^*_{Et}, an initial distribution of wealth and productivity $f_0(a,z)$, and foreign consumption C^*, a competitive equilibrium is defined as a path for households' choices (a_t,c_{ft},c_{nt},c_t), firms' choices (N_t, y_{ft}, y_{nt},y_{Ht},y_{Et}), unions' choices (n_t,π^W_t), prices ($\mathrm{p}_H(Q_t, p^*_{Et})$,$p_E(Q_t, p^*_{Et})$,$p_F(Q_t)$,$w_t$,$Q_t$,$X_t$), aggregate quantities (Y_{ft},Y_{nt},Y_{Ht},C_t) and distributions ($f_t(a,z)$, consistent with the Kolmogorov forward dynamics (5.1.14)) such that households and firms optimize, and the following market clearing conditions in the goods and labor market are satisfied, as well as the uniform rebating rule for mortgage payment revenues:

$$
\begin{aligned}
Y_{Ht} =&(1-\alpha_E)\left(\frac{p_{Dt}}{p_t}\right)^{-\epsilon}(1-\alpha)\left(\frac{p_{Ht}}{p_{Dt}}\right)^{-\eta}(Y_{ft}+Y_{nt})+\alpha\left(\frac{p^*_{Ht}}{p^*_t}\right)^{-\eta}C^* = \\
=&(1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t,p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha(p_F(Q_t)/p_D(Q_t,p^*_{Et}))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft}+Y_{nt}) \\
&+\alpha\left(p_H(Q_t,p^*_{Et})Q_t\right)^{-\eta}C^* \qquad (2.7.1) \\
C_t =&Y_{ft}+Y_{nt} \qquad (2.7.2) \\
Y_{Ht} =&AN_t \qquad (2.7.3) \\
N_t =&n_t \qquad (2.7.4) \\
\Pi_t =&\omega D^r_t i^d_t \qquad (2.7.5)
\end{aligned}
$$

where (5.7.1) is market clearing in the domestic good's market[13], (2.7.2) is market clearing in the final goods' market[14], (5.7.2) is market clearing the labor market, and (5.7.3) stands for the assumptions of households complying with the unions' choices in setting their labor supply (by symmetry among unions, $\int_0^1 n_{kt}dk \equiv n_t\ \forall k$). The goods market clearing condition (5.7.1) in particular is given by the sum of domestic demand (the first term on the right hand side) and foreign demand (the second term on the right hand side).

It is here worth stressing that the variables that we need to take as exogenous in order to compute the equilibrium are not only the energy shocks P^*_{Et}, but also the interest rates i_t, unlike standard modelization choices which would introduce policy rules to endogenize monetary policy (as a Taylor rule). Assuming an arbitrary path for interest rate opens up the possibility for inflation and output indeterminacy due to mean-zero sunspot shocks (see Cochrane (2011); I acknowledge here the limitation of this approach and decide to focus only on purely deterministic equilibria.

[13]Condition (5.7.1) is retrieved by substituting for p_{Dt}/p_t and p_{Ht}/p_{Dt} by using the price indexes (5.1.3) and (5.1.5) and results (5.4.8)-(5.4.10).

[14]Since $p_{ft}=p_{nt}=p_t$, the aggregate demands for the food and non-food goods write $C_{ft}=\varphi_t C_t$ and $C_{nt}=(1-\varphi_t)C_t$. By market clearing in the two markets, we have $Y_{ft}=C_{ft}$ and $Y_{nt}=C_{nt}$, hence, since $\varphi_t\in(0,1)$, we obtain $C_t=Y_{ft}+Y_{nt}$

2.8 Steady state

In order to obtain a stationary value for D_t^r, I need calibrate the model to obtain price stability ($\pi = 0$): this is achieved by imposing the stationary interest rate $\bar{i}$ equal to $i^* + \xi$ in the steady state version of UIP ((5.5.3)), with ξ being a stationary value for ξ_t. The model exhibits an infinite number of steady states, each one indexed by a value for the stationary real stock of mortgage $\bar{D}^r$. This is due to the fact that any nonzero inflation path $\pi_t \in [0, \infty)$, for given initial stock D, determines a different limit value of D_t^r (for $t \to \infty$) - determined by the extent to which the inflation path reduces the real mortgage stock over time. The following discussion will characterize a steady state for *given* $\bar{D}^r$.

The real domestic price $p_H(Q)$ is determined uniquely by the steady state $\bar{Q}$, and so is $\bar{w}$, by (5.2.3). Therefore, by (5.1.7), each household's consumption in home and foreign good $c(a, z)$ is determined uniquely by $\bar{Q}$, the steady state interest rate $\bar{i}$ (which also pins down the mortgage rate $i^d = i$), labor $\bar{N}$ and the states a, z (provided that I already substitute for the mortgage proceeds' rebating rule (2.7.5) and the labor supply compliance (5.7.3)). This implies that the drift function $s(a, z)$ depends only on $\bar{i}$, $\bar{Q}$ and $\bar{N}$ and the states a, z. Then, by setting to 0 the left hand side of (5.1.14), we can obtain the whole steady state distribution $f(a, z)$ as a function of $\bar{i}$, $\bar{Q}$ and $\bar{N}$.

Aggregate consumption $\bar{C}$ is defined as the integral over steady state consumption for each combination of states, given the stationary distribution $\bar{f}(a, z)$: $\bar{C} = \int \bar{c}(a, z)\bar{f}(a, z)dadz$; since both the idiosyncratic consumption levels $\bar{c}(a, z)$ and the distribution $\bar{f}(a, z)$ are determined by $\bar{i}$, $\bar{Q}$ and $\bar{N}$, we can then retrieve the following parsimonious functional formulation for $\bar{C}$:

$$\bar{C} = C(\bar{i}, \bar{Q}, \bar{N}) \tag{2.8.1}$$

Given $\bar{D}^r$, $\bar{i} = i^*$ and the stationary price of energy $\bar{P}_E^*$, equations (5.3.3),(5.7.1),(2.7.2),(5.7.2), (5.8.1),(5.5.3) define a system of six equations in six variables: $\bar{\pi}, \bar{Y}_H, \bar{N}, \bar{C}, (\bar{Y}_{ft} + \bar{Y}_{nt}), \bar{Q}$.

If mortgages are 0 ($\bar{D}^r = 0$), the model shocks are small enough in size to guaranteee that the dynamics revert to the *initial* steady state: heterogeneous agents small open economy models can indeed feature stable steady states thanks to the convergence property of the asset distribution (beyond Auclert et al. (2023a), see also Nuño and Thomas (2022) de Ferra et al. (2020)[15]).

However, allowing for $\bar{D}^r > 0$ leads to convergence to a different final steady state from the initial one, due to the different final mortgage stock D^r, if inflation π_t is different from 0 at any point in time.

Notice that the discussion so far relies on the assumption of no structural parametric changes over the dynamics of the model, which would mechanically lead to a different final steady state. This however will be the case for the quantitative analysis of section 4 and 5, which will postulate a different final stationary interest rate both in the domestic and foreign economy, $\tilde{i} = i^* + \tilde{\xi} > \bar{i} = \bar{i}^* + \bar{\xi}$ (with $\bar{\xi}$ and $\tilde{\xi}$ being respectively the initial and final stationary value for ξ_t) providing an additional reason behind the attainment of a different final steady state, in addition to the inflation-driven adjustment of the mortgage stock.

3 Trading off appreciations with mortgage costs

In this section I analyse the impact of an energy price shock on crossectional household income, and later will introduce the trade-off faced by monetary policy in its reaction. Starting from a steady state configuration for the domestic economy, I will take into account an unexpected and temporary rise in the price of energy p^*_{Et} ($P^*_{Et} > \bar{P}^*_E$ and $P^*_{Es} = \bar{P}^*_E$ for $s \in (t, \infty)$). Given the results obtained in model outline, we can express the real income of a mortgage-holding household with states a, z (i.e. $\delta a_t + z_t w_t n_t - D^r_t i^d_t + \Pi_t$), net of the coupon payment δa_t, as follows:

$$z p_H(Q_t, p_{Et}) Y_{Ht} - (1-\omega) D^r_t i^d_t (i_{ss} \in [t, t+S))) \tag{3.0.1}$$

Where labor income is a function of domestic output Y_{Ht} and and the real price of domestic good $p_H(Q_t, p^*_{Et})$, while the mortgage rate i^d_t is expressed as a function of all the future short-term interest rates until $t + S$ (i.e. $i^d_t(i_{ss} \in [t, t+S))))$. The equilibrium expression $(1-\omega) D^r_t i^d_t$ stands for mortgage payment net of revenues Π_t. Notice that I decide not to include coupon payments δa_t in the income specification (3.0.1), as they do not depend directly on the energy price variation, nor on the interest rate policy (they depend instead

[15] Then it is not needed to resort to debt-elastic interest rates as commonly done in representative agent models without international risk sharing.

indirectly on these shocks through the endogenous response of the household in adjusting its asset stock a_t).

A jump in p^*_{Et} makes domestic goods relatively more attractive than energy, increasing overall world demand for domestic goods relatively to demand for the foreign ones. This effect is captured in equation (5.7.1) , and has a positive impact on domestic output Y_{Ht} (expenditure switching channel, ES). On the other side, an increase in p^*_{Et} lowers the firms' revenue per unit of output, and then wages, i.e. the term $p_H(Q_t)$ in equation (3.0.1) (terms of trade channel, TT). This last effect is produced by the higher price of energy relative to domestic goods, which passes through on domestic real wages.

If ES is stronger, households will enjoy a higher current wage income, while if TT dominates they will suffer from a current wage income loss. By looking at equation (5.7.1), with elasticities ϵ and η low enough the effect of energy price on demand for domestic good is muted: therefore the expenditure switching channel is dominated by the terms of trade channel. This is the case I will focus from now onwards, as it allows the energy price shock to induce a real income loss (as in Auclert et al. (2023b)).

Let us now assume rigid prices and zero wedges in the UIP[16]: $\pi_s = \xi_s = 0 \ \forall s > t$. Therefore, following the discussion of Section 2.8, the dynamics following the energy shocks are such to make the economy revert to the initial steady state (since the real value of mortgages does not change over time), with real exchange rate level $\bar{Q}$. If the central bank reacts to the shock by producing an increase in the interest rate i_t by a contractionary monetary policy, that implies $dQ_t < 0$ by the UIP condition (5.5.3). In order to have this movement being consistent with a reversion to the initial steady state, Q_t needs to jump at the onset of the shock: the economy experiences a real appreciation. Intuitively, the domestic currency temporarily soars before depreciating over time back to its steady state level: this reduces the incentive to invest in domestic assets and restores indifference between the two countries' investment opportunities. This will be hence labelled as the *UIP channel* of an interest rate hike. The real exchange rate appreciation in turn passes through the real domestic wages by the firms' pricing condition (5.2.3), restoring some purchasing power for the household:

[16]This is obtained by assuming fully rigid nominal wages, so non-energy good prices P_{Dt}, together with the fact that $P^*_{Es} = \bar{P}^*_E$ for $s > t$.

analytically, in equation (3.0.1),the real wage term $p_H(Q_t, p^*_{Et})$ is negatively affected by the shock to p^*_{Et} but positively affected by the increase in Q_t. The interest rate hike fights the fall in real income by a domestic real appreciation.

However, the rise in the interest rate i_t affects real income (3.0.1) also through a higher outflows in terms of mortgage payment, as the aggregate mortgage rate i^d_t rises due to the increase in the short-term interest rate (by the mechanisms unraveled in equations (2.1.8) and (2.1.9) (*debt-cost* channel of an interest rate increase). The effects of an interest rate hike on consumption of mortgagors poses a trade-off to central bank's policy: on one side, the whole households' crossection suffers a weaker real income loss, on the other, mortgagors incur into a higher cost of debt.

The key aspect, however, is that whether the interest rate hike is frontloaded or smoothed can make a lot of difference to mitigate this trade off. Indeed you can achieve an appreciation of the current exchange rate even if the interest rate hike is smoothed over time. Let us consider the policy maker willing to attain the level $Q_t = Q^* > \bar{Q}$. The forward iteration of the UIP condition (5.5.3) up to infinity yields:

$$\ln Q^* - \ln \bar{Q} = \int_t^\infty (i_\tau - i^*) d\tau \tag{3.0.2}$$

So the current real exchange rate depends on the whole sum of future interest rates.

The question to be posed here is whether the trade-off between current appreciation and mortgage cost increase can be relaxed by distributing the latter over a protracted time span, leveraging the forward looking nature of Q_t. This can be engineered by an increase in the *future* interest rates short term rates $\int_t^\infty (i_\tau - i^*) d\tau$ (from now onwards, I will refer to this policy as *monetary smoothing*); notice that this would nevertheless come at the expense of Q_t and i_t being persistently above steady state beyond t, when it would be not anymore needed.

What does this interest rate smoothing strategy implies for the current variation in the mortgage cost, $i^d_t - i^*$? I will answer to this question by considering first two simple extreme cases ($S \to 0$ and $S = \infty$), and then I will analyze the general case for any fixed term horizon.

1. Case $S \to 0$ (short maturity mortgages). The fixed rate behaves as a variable rate ($i_t^f = i_t$) (we can see that by plugging the limit $S \to 0$ inside (2.1.8)). So, by equation (2.1.9), the variation in mortgage cost ($i_t^d - i^*$) boils down to $i_t - i^*$. A smoothed pattern for the policy rate i_t over time maps exactly into the same pattern for i_t^d, so interest rate smoothing is extremely effective in shifting the mortgage cost burden of an appreciation forward in time.

2. Case $S \to \infty$ (long maturity mortgages). The size of the sub-mortgages getting their interest rate updated in the interval dt, i.e. (dt/S), goes to 0. The aggregate fixed mortgage interest rate at t is the average of the previously renewed mortgage rates down to time $t - S$ (set at i^* since the economy was in steady state before t) and the current renewed rate at the forward looking value $\frac{1}{S} \int\limits_{[t,t+S)} i_\tau d\tau$ (see equation (2.1.8)). Therefore, the variation in i_t^f (i.e. $(\dot{i})_t^f$ is given by:

$$(\dot{i})_t^f = \frac{1}{S}\left(\frac{1}{S}\int\limits_{[t,t+S)} i_\tau d\tau - i^*\right) \tag{3.0.3}$$

   where for $S \to \infty$, the expression above equals zero. Hence, by (2.1.9), the overall variation in mortgage cost is $(\dot{i})_t^d = \frac{D^v}{D}(\dot{i})_t$. The total mortgage rate deviation is pinned down only by variable rate mortgages variations. Therefore, the rationale to implement interest rate smoothing is more limited and given exclusively by the aim to smooth out variable rate mortgage cost increases over time.

The two simple cases above represent two extreme cases with respect to the extent to which interest rate smoothing shifts ahead the mortgage cost burden: significantly in the case $S \to 0$ and minimally in the case $S \to \infty$. Hence it is reasonable to expect that this policy would be more desirable the lower is the mortgage horizon S, as showed below. Consider the variation at t of mortgage cost (according to equation (2.1.9)):

$$(\dot{i})_t^d = \left(1 - \frac{D_v}{D}\right)(\dot{i})_t^f + \frac{D^v}{D}(\dot{i})_t \tag{3.0.4}$$

We can then substitute for (3.0.3):

$$\dot{(i)}_t^d = \left(1 - \frac{D_v}{D}\right) \frac{1}{S^2} \int\limits_{[t,t+S)} (i_\tau - i^*) d\tau + \frac{D^v}{D} \dot{(i)}_t \tag{3.0.5}$$

Substituting for (3.0.2) we obtain:

$$\dot{(i)}_t^d = \left(1 - \frac{D_v}{D}\right) \frac{1}{S^2} \left(\ln Q^* - \ln \bar{Q} - \int\limits_{[t+S,\infty)} (i_\tau - i^*) d\tau \right) + \frac{D^v}{D} \left(\ln Q^* - \ln \bar{Q} - \int\limits_{(t,\infty)} (i_\tau - i^*) d\tau \right) \tag{3.0.6}$$

And finally rearranging, we obtain:

$$\dot{(i)}_t^d = \left[\left(1 - \frac{D_v}{D}\right) \frac{1}{S^2} + \frac{D^v}{D} \right] (\ln Q^* - \ln \bar{Q}) - \underbrace{\left[\left(1 - \frac{D_v}{D}\right) \frac{1}{S^2} \int\limits_{[t+S,\infty)} (i_\tau - i^*) d\tau + \frac{D^v}{D} \int\limits_{(t,\infty)} (i_\tau - i^*) d\tau \right]}_{\text{smoothing effect}} \tag{3.0.7}$$

Equation (5.5) provide the key analytical result to understand why monetary policy smoothing can relax the trade-off between appreciation of Q_t and increase in i_t^d. Adopting a smoothed policy allows to achieve the target Q^* at the expense of a lower mortgage rate i_t^d - effect captured in the term $\int\limits_{t+S}^{\infty} (i_\tau - i^*) d\tau$ (the raise in interest rates beyond the fixed mortgage term $t + S$ entails indeed no effect on the currently updating fixed rate i_t^f) and in the term $\int\limits_{(t,\infty)} (i_\tau - i^*) d\tau$ (the raise in interest rates beyond t has not effect on the current variable rate i_t).

For a higher mortgage term S (higher maturity), interest rate smoothing is less effective in mitigating the increase in mortgage costs (the impact becoming minimal for $S \to \infty$, as discussed previously). This is due to:

1. the impact of future monetary contraction on today's rate i_t^d is active for a longer time span $[t, t + S]$ (analytically, the"innocuous" forward guidance term $\int\limits_{t+S}^{\infty} (i_\tau - i^*) d\tau$ shrinks).

2. a smaller fraction of mortgages are updated at t, so shifting the debt cost burden ahead in time is quantitatively less important in the determination of i_t^d (analytically, this is

given by the smaller term $\frac{1}{S}$).

The results indicate that smoothing the interest rate path during an energy shock is beneficial from a welfare perspective, as it allows for real exchange rate appreciation while reducing the immediate pressure on mortgage costs. By avoiding sharp rate hikes, policymakers can mitigate the financial burden on households during the shock period.
However, this approach has a long-term cost. Prolonged monetary accommodation leads to higher future mortgage rates due to delayed monetary tightening, extending beyond the energy shock. This creates a forward guidance challenge, where the policymaker must assess whether short-term relief outweighs the future burden. A detailed quantitative analysis, as outlined in the next sections, is necessary to determine the overall welfare impact of this trade-off.

4 A quantitative application to the UK economy

4.1 The UK case in data

The surge in energy prices starting from 2021 had significant consequences for the UK economy. As depicted in Figure 4.1, the industrial energy price index for electricity, gas, and other fuels surged by approximately 150% from 2021 to 2023. This surge in energy prices translated into a surge in CPI inflation, which peaked at 11% in 2023. Real wages, as illustrated in Figure 4.1, experienced a fall from the second half of 2021 onwards, resulting in a decline in the purchasing power of workers and households. In response to the inflation surge brought on by the increase in energy prices, the Bank of England responded decisively. Between 2021 and 2024, the bank significantly raised nominal interest rates, climbing from 0.25% to approximately 5%. This shift in nominal interest rates held implications for mortgages' cost dynamics. Notably, approximately a quarter of the total outstanding mortgage stock were poised to conclude their fixed-rate terms between the final quarter of 2022 and the culmination of 2023, getting their interest rate revised upwards and impacting negatively on households' finances; moreover, a 12% of the total outstanding mortgage stock is made

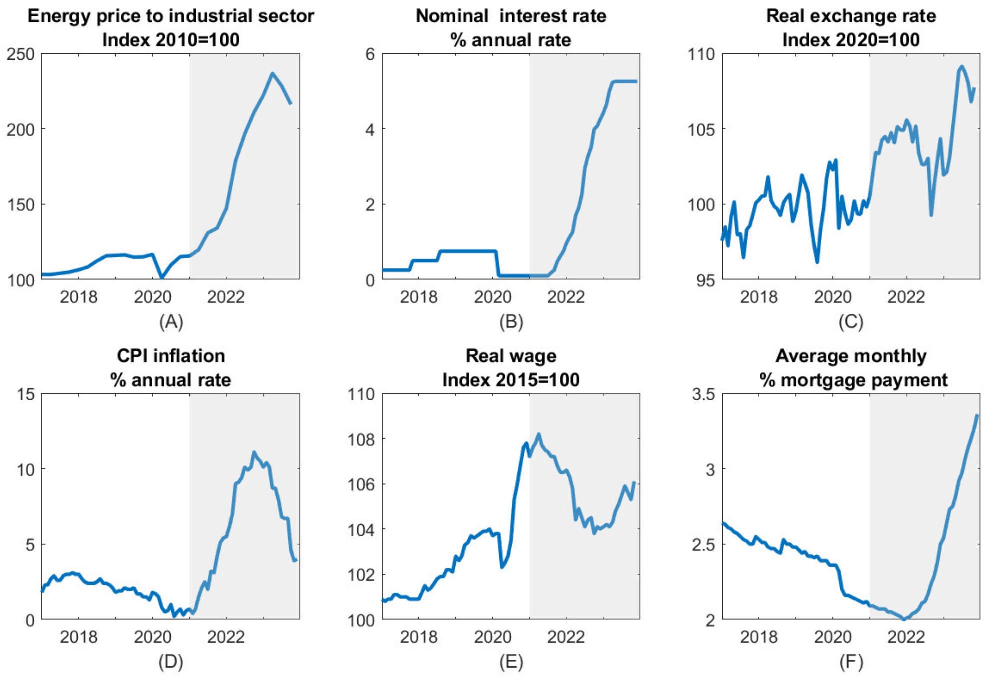


Figure 4.1: Energy prices to industry (quarterly data), policy rate, real exchange rate, CPI inflation rate, real wage and aggregate mortgage rate. Source: Office for National Statistics, BoE and FRED database)

up by variable rate mortgages[17]. These features of the mortgage market determined a discernible increase in the aggregate economy-wide mortgage rate, which climbed from 2% to almost 3.5% between 2021 and 2024. The facts presented above demonstrate the challenging trade-off faced by the Bank of England. Striking a balance between restoring real wage values and keeping borrowing rates moderate for mortgages was a complex task: while the former objective required a tight monetary policy to contain inflation, the latter was calling for a loose interest rate setting.

In what follows, I will further dig into the relevance of the increase in mortgage rates in affecting crossectional consumption. Leveraging data from the “Understanding Society” survey, a longitudinal panel that tracks information across various households in the UK over time, I explore the dynamics within two interview waves: 2020-2021 and 2021-2022.

In particular, I restrict the the analysis to households interviewed both in 2021 and in 2022, in order to track their consumption behavior over time. I include in the sample only households

[17]Source: Office of National Statistics

categorized as either housing mortgagors or outright homeowners. Households with tenure status changing between the two interview waves are also excluded, leading to a final sample of 2,477 households. The survey associates to each household its food consumption consumed at home, in addition to income, demographic and geographical characteristics.

Table 2: Descriptive Statistics (monthly), households in 2021 interview wave

Per capita food consumption (£)			Income (£)		% mortgagors
Decile	Mean	Std	Mean	Std	
Bottom 20%	98.6	23.2	4'099	2'242	63%
Bottom 40%	124.7	31.9	4'084	2'348	61%
Bottom 60%	148.1	43.1	4'143	2'649	59%
Bottom 80%	174.2	59.4	4'137	2.742	56%
100%	222.7	133.9	4.128	2.779	54%

Due to the importance of distributional outcomes of a mortgage cost surge in the current framework, it is convenient to express descriptive statistics of the sample with respect to different subsamples of the distribution of food consumption in the pre-energy shock interview wave (i.e. 2021). The total sample of households is indeed split into 5 subsamples according to the position held by each household in the 2021 consumption distribution, namely the bottom 20%,40%,60%,80%,100% of the distribution. I restrict my analysis to the variation in annual food consumption, due to the limited range of expenditure items captured in the survey. For each household, I compute the percentage variation in per-member household food consumption (given by the ratio between household food consumption and household size $C_f(i,t) = food\_consumption(i,t)/size(i,t)$) between 2022 and the initial wave response:

$$\Delta_{c,f}(i, 2022) = \left[\frac{C_f(i, 2022)}{CPI\_food(2022)} - \frac{C_f(i, 2021)}{CPI\_food(2022)}\right] \Bigg/ \frac{C_f(i, 2021)}{CPI\_food(2022)} \tag{4.1.1}$$

where $C_f(i, 2022)$ is the food consumption value for household i reported in 2022, and $C_f(i, 2021)$ is the value stated by the same respondent in the previous 2021 interview. The variations is adjusted for changes in the food price index, in order to track only movements in real expenditure for food.

In order to capture distributional effects of mortgage cost increases along the households' crossection, I regress the consumption variation $\Delta^j_{c,f}(i, 2022)$ on a dummy $I_M(i)$, which assumes value 1 if the household owns its house through a mortgage and 0 if it is an owner outright; in the regression I control for the total net household real income variation between the two interview waves, $\Delta income(i, 2022) = \frac{income(i,2022)}{CPI(2022)} / \frac{income(i,2021)}{CPI(2021)}$. An additional vector X of regressors include government office regions as a geographical controls, and both number of children and number of people in working age as demographical controls. The empirical specification, for each quintile $\mathcal{Q}^j$ of the consumption distribution for $C(i, 2021)$, writes:

$$\Delta^j_{c,f}(i, 2022) = \beta^j_0 + \beta^j_1 * I_M(i) + \beta^j_2 * \Delta income(i, 2022) + \beta^j_3 X_t(i) + \varepsilon(i) \qquad (4.1.2)$$
$$\forall\ i \text{ s.t. } C_f(i, 2021) \leq \mathcal{Q}^j(C_f(i, 2021))$$

The results up are summed up in Table 3.

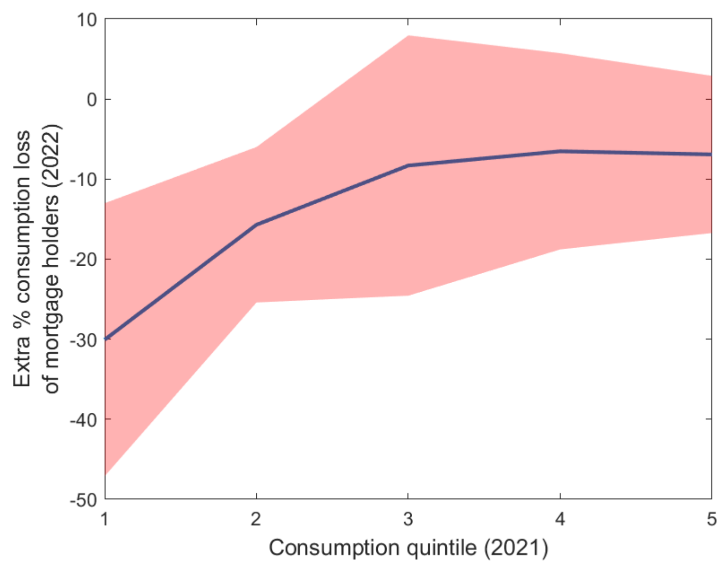


Figure 4.2: Coefficient β^j of mortgagor dummy in the regression for consumption variation $\Delta^j_c(i, 2022)$, with all controls, for households lying below each 2021 wealth quintile D_j. Shaded area: 90% confidence bandwidth

Consistently with the prediction of heterogeneous agents literature, households which are able to afford lower consumption levels have also a low capacity to financially absorb income shocks (like a mortgage cost increase). We can indeed notice how the coefficients of the

Table 3: Regression results for consumption variation $\Delta_c^j(i, 2022)$

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.3006***	-0.1570***	-0.0830	-0.0653	-0.0692
	(0.1306)	(0.0590)	(0.0988)	(0.0745)	(0.0597)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.2916***	-0.1525***	-0.0852	-0.0634	-0.0717
	(0.1034)	(0.0590)	(0.0983)	(0.0742)	(0.0595)
Demographic controls	Yes	Yes	Yes	Yes	Yes
Regional controls	No	No	No	No	No
Mortgagor	-0.4210***	-0.2750***	-0.1088	-0.0575	-0.0228
	(0.0894)	(0.0510)	(0.0846)	(0.0636)	(0.0513)
Demographic controls	No	No	No	No	No
Regional controls	Yes	Yes	Yes	Yes	Yes
Mortgagor	-0.4156***	-0.2732***	-0.1095	-0.0570	-0.0249
	(0.0892)	(0.0510)	(0.0841)	(0.0633)	(0.0511)
Demographic controls	No	No	No	No	No
Regional controls	No	No	No	No	No
Δ% income control	Yes	Yes	Yes	Yes	Yes
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Observations	495	991	1486	1982	2477

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

"Mortgagor" dummy increase in size and significance as we consider subsamples closer to the bottom of the consumption distribution. In particular, controlling for locations and demographic characteristics, the the bottom 20% and 40% of the distribution displayed respectively a 30% and 16% consumption loss of mortgagors with respect to outright owners - with a statistical significance of 1%; the other samples (bottom 60%, 80% and 100%) feature instead lower and not significant consumption effect from mortgage holding. Overall, controlling for geographical locations does not change significantly the estimated impact of mortgage holding, while controlling for demographics drops this impact from 42% to 30%, suggesting that household's composition is a determinant of the mortgagor/owner outright status of the household, as well as of its consumption variation over the 2021-2022 time span. In what follows, I will tailor the calibration of the model to calibrate the empirical estimates in the case with all controls (first line of Table 3), reported graphically in Figure 4.2.

4.2 Calibration

The main channels of effect of real exchange rate policy in the model are the "open economy" dimension, that generates the adverse effects of the price of energy on domestic real wages through a terms-of-trade effect, and the "mortgage" dimension, which mediates the transmission of contractionary interest rate policy on crossectional consumption through the mortgage cost variation faced by households. Therefore my calibration strategy aims at matching salient features of the UK economy along both these dimensions.

Parameters. Following the calibration of Chan, Diz, and Kanngiesser (2023), tailored to the UK economy, I set the energy share in production α_e to 0.05 and the elasticity between labor and energy ϵ to 0.15, the price elasticity of world demand for domestic exports η to 0.35, and the export share α to 0.25. The time step Δ is 1/3 (monthly unit periods). The slope of the Phillips curve is set to 0.0049 as in Auclert et al. (2023a). The real wage stabilization motive $\zeta = 25$ guarantees that the pressure on nominal wages in the labor market is such to push inflation to a 8% peak above the steady state. By a proper choice of value for foreign consumption C^*, I obtain an initial steady state real exchange rate $\bar{Q}$ equal to 1, which serves to mediate the effect of the energy shock on real wages down to a -3% at the beginning of

Table 4: Calibrated parameters

Parameter	Definition	Value	Source/Target
		Households	
ρ	Household discount factor	0.05	Auclert et al. (2023b) - HANK with energy shocks
σ	Household risk aversion	1	Chan et al. (2023) - Quantitative model for UK
ϕ	Inverse Frisch elasticity	2	Auclert et al. (2023b) - HANK with energy shocks
$\mu(z)$	Mean of the diffusion process	$0.3(1-z)$	Literature
ς^2	Variance of the diffusion process	4	Shape of crossectional Δ consumption (section 4.4)
δ	Amortization rate, LT bonds	0.021	Nuño and Thomas (2022)
$\bar{a}$	Borrowing limit	-0.02	Literature
		Mortgages	
D	Mortgage stock	-50	Magnitude of crossectional Δ consumption (section 4.4)
S	Mortgage duration	66	Aggregate mortgage rate path
D_v/D	% variable rate mortgages	12%	ONS UK
ω	% mortgagors	54%	Understanding society survey (2021)
		Labour Unions	
ε	Labor demand elasticity	10	Literature
κ	Slope of Phillips curve	0.0049	Auclert et al. (2023a) - HANK open economy
ζ	Real wage stabilization	25	Inflation peak $\approx$ 8% above pre-crisis mean
		Firms and international trade	
α_e	Energy share in production	0.05	5% energy share in production
ϵ	CES degree energy-labour in production	0.15	UK estimates
η	Elasticity of world demand for domestic goods	0.35	Chan et al. (2023) - Quantitative model for UK
α	Foreign preference for domestic exports	0.25	Export share ≈ 0.25
C^*	Foreign consumption	1.29	$\bar{Q} = 1.4$ such that real wage fall $\approx$ 3%
		Monetary Policy	
$\bar{i} + \bar{\xi}$	Initial steady state interest rate (with $\bar{\xi} = 0$)	0.5% yearly	Pre-energy crisis path
$\tilde{i} + \tilde{\xi}$	Final steady state interest rate	3% yearly	Post-energy crisis path (BoE projections)

2023, in line with data (see Figure 4.4)[18].

With regards to the household crossectional dimension, I follow Chan et al. (2023) quantitative model for energy shock effects on the UK in setting $\sigma = 1$, while I set $\phi = 2$ and $\rho = 0.05$ as in the open economy HANK calibration of Auclert et al. (2023a); the borrowing limit is close to 0 ($\bar{a} = -0.2$) according to literature's standard practice. The long-term bonds amortization rate δ is set to 0.021, consistent with a bond duration of 4.5 years (see Nuño and Thomas (2022)). The fraction of mortgagors replicates the data for the full "Understanding Society" survey sample (54%, see Table 2). The average mortgage duration S is set to 5.5 years to match the aggregate mortgage rate path as in Figure 4.4, and the fraction of variable rate mortgages is set to 0.12 according to the most recent Office of National Statistics (ONS) data. The mortgage stock is calibrated at $D = -50$, to match the magnitude of the consumption effect of mortgages as in Figure 4.2 (as carried out in section 4.4). In order

[18]Different values for $\bar{Q}$ have indeed the consequence of scaling up and down the whole path for Q_t (reported in Figure 4.3 in percentage deviations from steqady state), with mitigating or amplificating effect on the domestic price of energy P^*_{Et}/Q_t.

to replicate not only the magnitude of the curve of effects in Figure 4.2, but also its shape, while assuming a standard calibration value $\mu(z) = 0.3(z^{mean} - z)$ as in Achdou et al. (2021), I set the variance of the diffusion $\varsigma^2 = 4$. The high idiosyncratic risk indeed creates a strong precautionary motive in a household the closer it is to the borrowing limit, letting its consumption absorb the mortgage cost shock in order not to affect the precautionary asset buffer.

Shocks. The model is fed with an energy price shock and an interest rate policy following a lognormal time profile starting from $t = 01/2022$ and and tracking the data pattern, as showed in Figure 4.3. The right tail of the lognormal model input for i_t (i,.e. at the right of the *argmax* of the curve) is truncated when the implied value for i_t would fall below the final steady state $\tilde{i}$, i.e. at year September 2027: from then onwards, i_t is set at the new level $\tilde{i}$. The lognormal time profile is described in the following equations:

$$P^*_{Et} = \bar{P}^*_E + K_e Lognormal_{\mu_e,\sigma_e^2}(t) \ \forall t > 04/2022 \tag{4.2.1}$$

$$i_t = \begin{cases} \bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t) \ \forall t \in [01/2022, \text{argmax}(i_t)] \\ \max\{\bar{i} + K_i Lognormal_{\mu_i,\sigma_i^2}(t) \ , \ \tilde{i}\} \ \forall t > \text{argmax}(i_t) \end{cases} \tag{4.2.2}$$

where $K_e = 29.2, K_i = 1.07, \mu_e = 3.25, \mu_i = 4.1, \sigma_e = 0.7, \sigma_i = 1$ are parameters set to match closely the data counterpart. Policy rate data are retrieved from realized and expected future interest rates (from BoE monetary policy committee's projections): the latter point out to a gradual interest rate cut - already initiated in April 2024 - to be implemented at a progressively slower pace. Consistently with this assumption, I assume a final "landing" stationary value for the BoE rate i_t of 3% (annualized). As mentioned previously, in order to obtain $\tilde{\pi} = 0$ in the final steady state, I assume the final stationary value for the foreign interest rate $i^* + \tilde{\xi}$ to be equal to an annualized 3% as well (see equation (5.5.3)). The initial steady state interest rate $\bar{i}$ is instead set at 0.5% annualized, consistently with the pre-energy crisis existing policy rate data (see Figure 4.3, centre plot).

As far as energy price is concerned, the observed counterpart is given by the quarterly overall fuel cost to industry index (detrended with respect to the same time span of the exchange rate series ,i.e. 2017-2021) - recalling that the energy enters the model as a domestic firms' input.

Q_t is set to steady state until December 2021 - the onset of the energy crisis shock. Afterwards it is computed as the filtered version of the real exchange data series[19], whose computation details are left in the appendix. Using UIP condition (5.5.3), I can backward-engineer the pattern of UIP shocks ξ_t such that the imposed time profile of Q_t is consistent with the data input. In this way, even though Q_t is endogenous in the model, I can successfully reproduce its path in simulation. The pattern of Q_t reported in Figure 4.3 is characterized by a different final steady state: as discussed in section 2.8, this is due to the fact that the final steady state displays a different interest rate in both economies :$\tilde{i} = i^* + \tilde{\xi} > \bar{i} = i^* + \bar{\xi}$, where $\tilde{\xi}$ is the final stationary value for ξ_t.

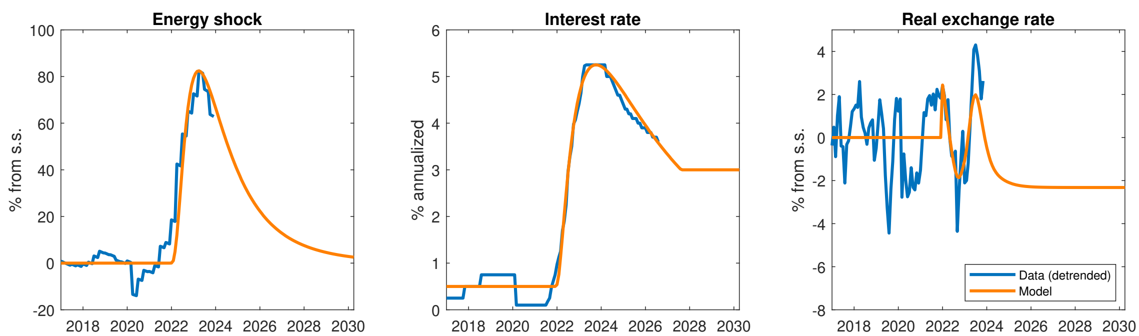


Figure 4.3: Input of the model: Energy shock, interest rate, real exchange rate, vs. data (Re-elaboration from series by Office for National Statistics, BoE and FRED). Energy shock and real exchange rate removed trends are computed on the 2017-2021 time sample, while the interest rate is presented in raw data.

4.3 Model validation at the aggregate level

The model is solved under perfect foresight, by looping over the final steady state real mortgage stock $\bar{D}^r$, and aggregate consumption, building on the solution method by Achdou et al. (2021) - details are reported in the appendix. The impulse responses for inflation, real wage and aggregate mortgage rate i_t^d are reported in Figure 4.4 and compared with the data counterparts, which are build from the dataset of BoE and ONS; UK inflation from 01/2022 (CPI) is presented in % points and absolute difference from the 2% BoE target; the aggregate mortgage rate is presented in % points and absolute difference from the plateau

[19] Real exchange data for UK are recovered by the FRED database.

reached in 2021 after a steady decrease ongoing since year 2016 (see Figure 4.1) . Real wages are presented in percentage deviation from a trend computed on a shorter time span (2017-2019) due to the impact of the pandemic period on the variable's path. The magnitude and hump-shape (resp. u-shape) of CPI inflation (resp. real wage) is successfully replicated by the model output, with inflation peaking at 9% above the steady state level, and real wages falling beyond 3%. The aggregate mortgage rate follows a similar upward trend as the data, while deviating by up to 0.5 percentage point.

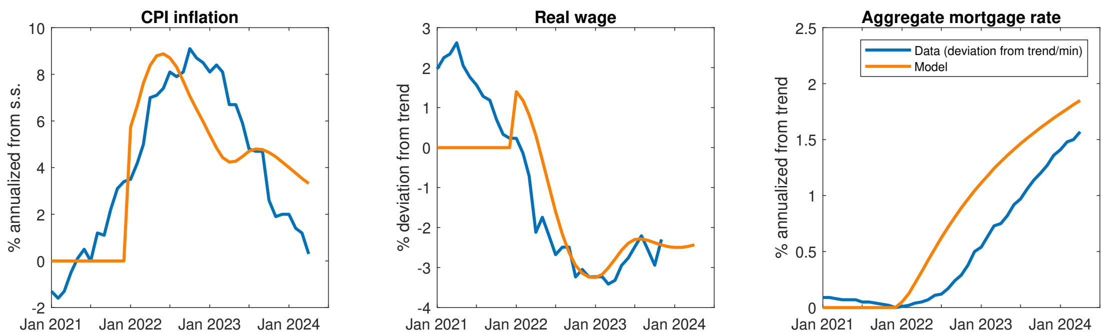


Figure 4.4: Output of the model: Inflation, real wage and aggregate mortgage rate i_t^d, vs. data (Re-elaboration from series by Office for National Statistics and BoE). CPI inflation is take in difference from the pre-crisis 2%. The real wage is presented with the linear trend removed, calculated based on the 2017-2019 pre-COVID time sample. Aggregate mortgage rates is showed in absolute differences from the 2% 2021 plateau.

4.4 Model validation in the crossectional consumption response

So far calibration choices were not discussed in detail with respect to the per-household mortgage stock D and the variance of the diffusion process ς^2. The goal of this section is to present a calibration choice of these parameters, suitable to let the models replicate the difference in the 2021-2022 percentage consumption variation between the mortgagors and non-mortgagors (Figure 4.2). The features of the diffusion process of idiosyncratic shocks are indeed a paramount element of the model to determine the differences in precautionary saving across households according to their position held in the initial consumption distribution - and hence, the difference in consumption responses to the increase in mortgage costs.

The main challenge that needs to be addressed by the validation method consists in producing a discrete sample of mortgage and non-mortgage households with food consumption and income variations between 2021 and 2022, in order to implement a regression of the type (4.1.2) on the simulation output.

4.4.1 A model-generated crossectional effect of mortgages

In order to compare the effect of the mortgage cost increase on crossectional household consumption with the data output in Table 3, I need to perform a regression of the same type on the data delivered by the model: that requires, for each household starting at node a, z in period 12/2021, to identify the model implied variation of food consumption between 2021 and 2022 $\Delta^j_{c,f}(2022)(a, z)$ and variation of income $\Delta income(2022)(a, z)$, which I formulate as the ratio of the average expected consumption and income in 2022, on their initial steady state value (as of January 2021), given the initial state node a, z:

$$\Delta^j_{c,f}(2022)(a, z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[c_{ft}|c_{f,ss} = c_f(a, z)\right]}{c_{f,ss}} - 1 \tag{4.4.1}$$

$$\Delta^j_{y}(2022)(a, z) = \frac{\frac{1}{12}\sum_{t\in 2022} E\left[y_t|y_{ss} = y(a, z)\right]}{y_{ss}} - 1 \tag{4.4.2}$$

where income y_t is defined as the resources flow accrued to the household:

$$y_t = \delta a_t + z_t w_t n_t + \Pi_t - D^r_t i^d_t \tag{4.4.3}$$

The asset a and shock z are discretized along grids with dimension I and J respectively, which deliver discretized vectors $\{g_t\}_t, \{C_{ft}\}_t, \{y_t\}_t$ with size $I * J \times 1$, which comprise respectively the density, food consumption and income for each state node a, z. Following Achdou et al. (2021), we can also derive for each period a transition matrix $\mathcal{G}^{t+1}_t$ such that $g_{t+1} = \mathcal{G}^{t+1}_t g_t$; therefore, by multiplying the transition matrices from $t = 12/2021$ to any $t \in 2022$, we obtain the transition matrix that map g_{ss} to g_t:

$$g_t = \mathcal{G}^t_{ss} g_{ss} \tag{4.4.4}$$

Each column of the matrix $\mathcal{G}^t_{ss}$ (hence, each row of the transpose $(\mathcal{G}^t_{ss})^T$) represents the distribution of outcomes in t conditional on state a, z in staedy state. Then I can recover the expected consumption (resp., income) in 2022, conditional on the household being characterized by states a, z in steady state (i.e. in 12/2021), and hence the variations introduced in equations (4.4.1)-(4.4.2):

$$\Delta_{c,f}^{j}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{ft}}{c_{f,ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{ft}\right] - \ln c_{f,ss} \tag{4.4.5}$$

$$\Delta_{y}^{j}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * y_{t}}{y_{ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * y_{t}\right] - \ln y_{ss} \tag{4.4.6}$$

At this stage, discretizing g_{ss} into a frequency allows to obtain a countable number of households - indexed by i - each one with consumption $c_{ss}(i)$. I can then rank the resulting sample of model household according to $c_{ss}(i)$, to obtain the initial discretized distribution of consumption. Notice that, alongside the derivation carried out in this section, I can also formulate an expression for the variation in total consumption basket c_t, analogous to (4.4.5)

$$\Delta_{c}^{j}(2022)(a,z) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{t}}{c_{ss}} - 1 \approx \ln\left[\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{t}\right] - \ln c_{ss} \tag{4.4.7}$$

4.4.2 From total nondurable to food consumption

The total nondurable consumption values for each model household ($c_t(i)$) are necessary inputs to generate the model-implied idiosyncratic food consumption levels and hence to draw a comparison with the empirical section results', as previously discussed. Given the consumption aggregator (2.1.2) and the result $p_{ft} = p_t$, food consumption is given by:

$$c_{ft}(i) = \varphi_t c_t(i) \tag{4.4.8}$$

Similarly to Aguiar and Bils (2015)[20], for any t I can carry out a first order approximation of $c_t(i)$ around $c_\tau(i)$, where τ is any benchmark date:

$$\ln c_{ft}(i) - \ln c_{f,\tau}(i) = (\ln\varphi_t - \ln\varphi_\tau) + (\ln c_t(i) - \ln c_\tau(i)) \tag{4.4.9}$$

[20]The authors perform instead a linear approximation around the crossectional average of $c_t(i)$.

Therefore the equation boils down to:

$$\ln c_{ft}(i) = \Phi_t + \ln c_t(i) \tag{4.4.10}$$

where Φ_t is a time varying coefficient. Let us assume that φ_t (and then Φ_t) varies only on a yearly basis; taking time difference between the expected value of 2022 consumption and 12/2021 (steady state), we get:

$$\Delta_{c,f}(i, 2022) = \Delta\Phi_{2022} + \Delta_c(i, 2022) \tag{4.4.11}$$

where the quantities $\Delta_{c,f}(i, 2022)$ and $\Delta_c(i, 2022)$ are defined respectively by (4.4.5) and (4.4.7), and $\Delta\Phi_{2022}$ is given by $\Delta\Phi_{2022} = \Phi_{2022} - \Phi_{2021}$.
Let us now run a regression on the model output, mirroring the empirical counterpart (4.1.2), with the exception of being performed on total nondurable consumption instead of exclusively food:

$$\Delta_c^j(i, 2022) = \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i, 2022) + \varepsilon(i) \tag{4.4.12}$$
$$\forall\ i \text{ s.t. } c_{12/2021}(i) \leq \mathcal{Q}^j(c_{12/2021})$$

where $\Delta y(i, 2022)$ is the percentage income variation of household i between 12/2021 and year 2022 (given by expression (4.4.6), $I_M(i)$ is the previously defined indicator function for mortage holders, and $\mathcal{Q}^j(c_{12/2021})$ is the $j-th$ quintile of the steady state model consumption distribution. Results are summarized in Table 5.

Table 5: Regression results for consumption variation $\Delta_c^j(i, 2022)$ (model output)

Variable	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	-0.2406	-0.1172	-0.0730	-0.0747	-0.0649
Bottom % of $C(i, 2021)$	20%	40%	60%	80%	100%
Δ% income control	Yes	Yes	Yes	Yes	Yes

Note: All values are significant as the regression is performed on the whole model population

Once having estimated the coefficients γ_0^j, γ_1^j, γ_2^j, we substitute for the linear prediction

(4.4.12) inside (4.4.11):

$$\Delta_{c,f}(i,2022) = \Delta\Phi_{2022} + \gamma_0^j + \gamma_1^j * I_M(i) + \gamma_2^j * \Delta y(i,2022) + \varepsilon(i) \tag{4.4.13}$$

The coefficient γ_1^j provides the impact of mortgage holding on 2021 -2022 on food consumption variation, a model counterpart of the empirical estimate of β_1^j retrieved in section 4.1 and plotted in Figure 4.2 for each sample of the model consumption distribution in 12/2021. Note that, while the model accounts percentage variations in food consumption deviating from the ones in total consumption by the factor $\Delta\Phi_{2022}$, the average *difference* between percentage consumption variations of mortgagors and non-mortgagors is the same both with respect to food and total consumption, and measured by the factor γ_1^j.

Table 6: Consumption variation $\Delta_{c,f}^j(i,2022)$. Model vs. Data.

Variable		**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
Mortgagor	Data	-0.3006***	-0.1570***	-0.0830	-0.0653	-0.0692
		(0.1306)	(0.0590)	(0.0988)	(0.0745)	(0.0597)
	Model (γ_1^j)	-0.2406	-0.1172	-0.0730	-0.0747	-0.06494
Bottom % of $C(i,2021)$		20%	40%	60%	80%	100%

Note: Standard errors in parentheses. *Significant at the 10% level. **Significant at the 5% level. ***Significant at the 1% level

Figure 4.5 compares the crossectional effects of the mortgage cost increase as from the simulation's outcome, to the empirical counterpart illustrated in Figure 4.2, and to the outcome which would arise in a setting with near-zero idiosyncratic shock ($\varsigma^2 = 0.0001$). The model replicates closely the negative relationship between the position held in the consumption distribution at the end of 2021 (i.e. in steady state) and the extra-consumption loss with respect to owners outright over the crossection of mortgagors, with households at the bottom of the distribution suffering most in food consumption terms. No confidence bandwidths arise in the model-based regression, as the latter is performed on the whole model population. In the near-zero idiosyncratic shock case, the heterogeneity dimension of the model is shut down, as all agents have nearly the same propensity to consume: consequently the impact on consumption of the mortgage cost increase is equal across all quintiles of the steady state

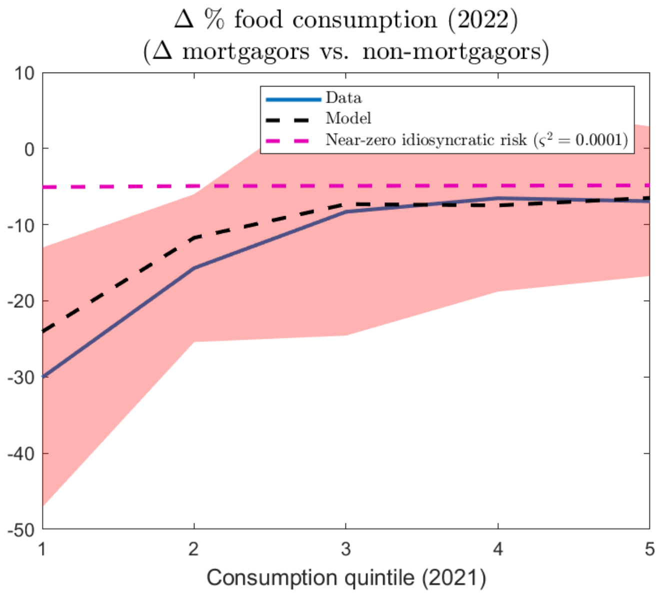


Figure 4.5: Coefficient β^j of mortgagor dummy in the regression for food consumption variation $\Delta^j_{c,f}(i, 2022)$, for households lying below steady state consumption deciles $\mathcal{Q}^j$. Shaded area: 90% confidence bandwidth of the empirical results. Model vs Data.

distribution (around 7% loss with respect to non-mortgagors). Therefore the heterogeneity dimension of the model is a key element to match the stronger impact of the shock at the bottom of the steady state consumption distribution; however, for higher quintiles the effects become increasingly similar in magnitude, due to the consumption smoothing behavior of households in HANK being more aligned to the ones in the complete markets environment, thanks to the higher wealth stock working as a buffer against idiosyncratic shocks.

5 Smoothing interest rate policy

5.1 The equilibrium effect of the benchmark BoE policy

The impulse response of the variables under the interest rate set by the BoE (from onwards labelled as i_t^{bmk}, where "bmk" being short for "benchmark"), which were showcased in the previous section, underlie a real appreciation effect that fights the real income loss due to the energy price shock, along the lines discussed in section 3. Through the UIP condition, a persistent increase in the interest rate produces an upward shift of the whole real exchange rate path. In order to show that, Figure 5.1 compares the equilibrium pattern for CPI

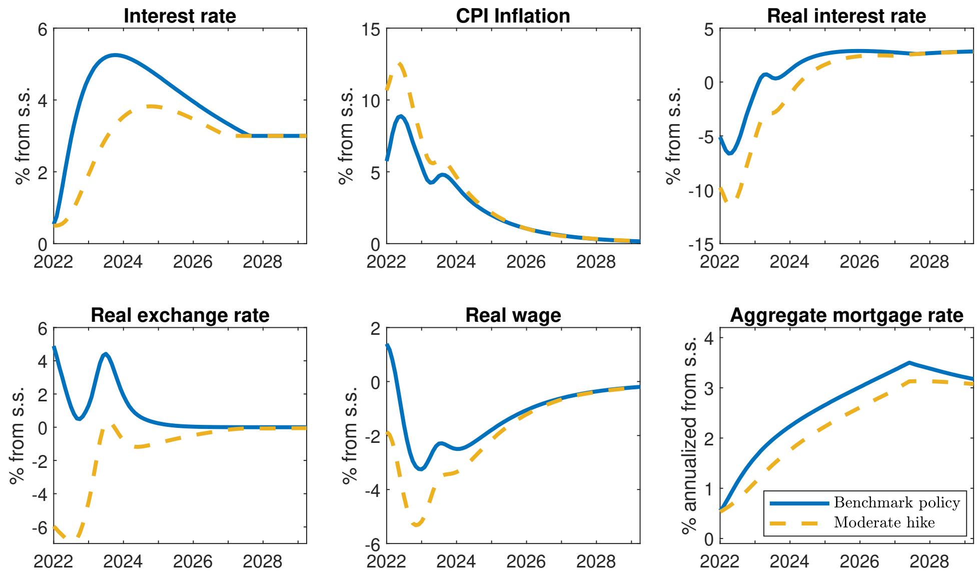


Figure 5.1: Impulse response functions to the energy shock. Benchmark policy vs. Moderate hike.

inflation, nominal and real ($i_t - \pi_t$) interest rate, real exchange rate, real wage and aggregate mortgage rate to the one that would materialize with a milder interest rate policy ("moderate hike", in short mh) implemented. Such alternative policy is constructed as imposing the parameters $\sigma_i^{mh} = 0.75$ and $K_i^{mh} = 0.7$ (lower than the $\sigma_i = 1$ and $K_i = 1.07$ of the benchmark). While lowering σ_i reduces the mass in the tails of the interest rate path, the decrease in K_i shrinks the whole path downwards. The parametrization allows to implement the landing on the new steady state interest rate in the same year of the benchmark (2027), while mitigating the hike especially in the first stages of the crisis.

The lower nominal interest rate hike translates into a stronger drop in the real interest rate, as the former makes up less for inflation. Through UIP, this not only implies a lower appreciation in the real exchange rate, but even a depreciation: Q_t falls by more than 6% from its steady state level. The combined effect of the energy shock and the real depreciation determines a stronger fall in the real wage (down to an extra 2% over 2022) in the moderate hike case with respect to the benchmark scenario. Nevertheless, the milder rise in the nominal interest rate allows to produce a lower path for the aggregate mortgage rate (by around 0.4% for five years from the onset of the shock).

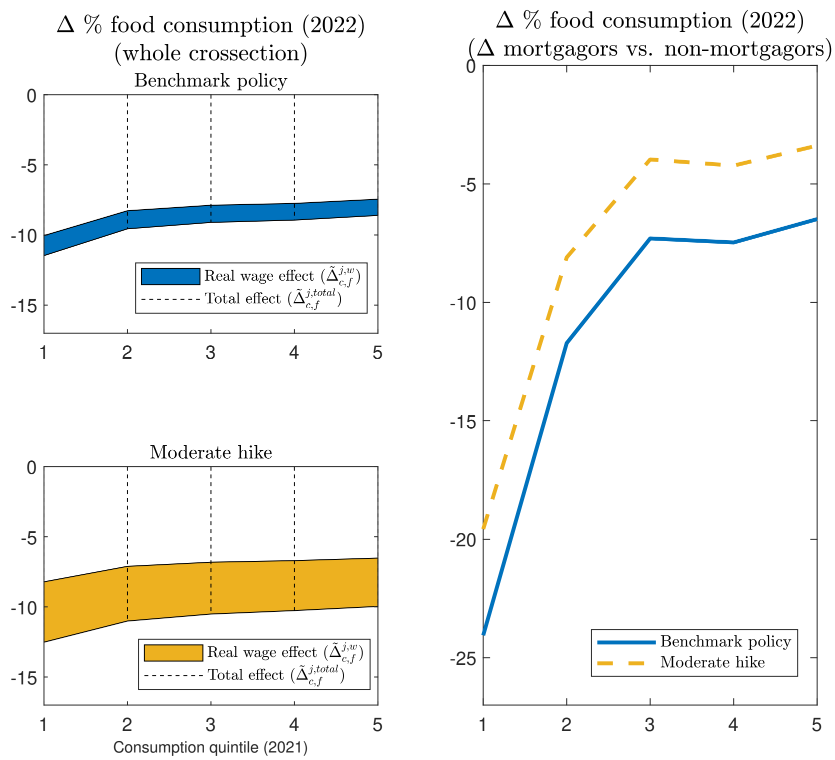


Figure 5.2: Left: food consumption % fall over 2022 for each consumption quintile of the 12/2021 consumption distribution (total $\Delta^{total}_{c,f}$ and decomposed by real wage effect $\Delta^{w}_{c,f}$). Right: 2022 Δ% consumption difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.4.11)). Benchmark policy vs. Moderate hike.

Figure 5.2 showcases the average % variation in food consumption between steady state (12/2021) and year 2022, isolating the effect of real wage fall alone, for the households lying below each j quintile of the steady state consumption distribution. The overall variation in consumption ($\Delta^{total}_{c,f}$) is defined as the average of the total expected extra variation in consumption over the crossection with respect to a scenario without any aggregate shock. The effect of wages is isolated by subtracting from this variation the one that would be obtained by exogenously fixing the real wages to steady state in the partial equilibrium outcome of the households' block. The total $\Delta^{total}_{c,f}$ and real wage-driven $\Delta^{w}_{c,f}$ variations can then be defined as follows:

$$\tilde{\Delta}_{c,f}^{j,total}(2022) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t})^{T} * C_{ft}}{c_{f,ss}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,no\ shock})^{T} * C_{ft}^{no\ shock}}{c_{f,ss}} \tag{5.1.1}$$

$$\tilde{\Delta}_{c,f}^{j,w=\bar{w}}(2022) = \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,\ w=\bar{w}})^{T} * C_{ft}^{w=\bar{w}}}{c_{f,ss}} - \frac{\frac{1}{12}\sum_{t\in 2022}(\mathcal{G}_{ss}^{t,no\ shock})^{T} * C_{ft}^{no\ shock}}{c_{f,ss}} \tag{5.1.2}$$

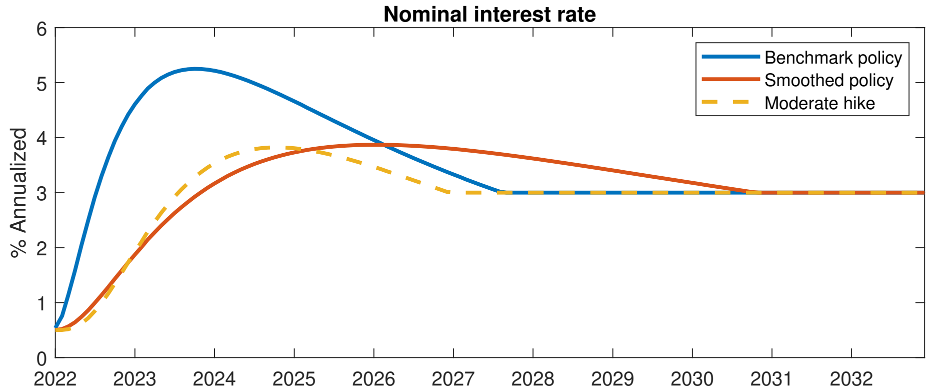
$$\tilde{\Delta}_{c,f}^{j,w}(2022) = \tilde{\Delta}_{c,f}^{j,total}(2022) - \tilde{\Delta}_{c,f}^{j,w=\bar{w}}(2022) \tag{5.1.3}$$

where $\mathcal{G}_{ss}^{t}$ and C_{ft} are respectively the transition matrix from 12/2021 to time t, and the food consumption level.

Figure 5.2 well captures the trade-off implied by the "moderate hike" policy between real exchange rate appreciation and mortgage costs. The alternative policy produces a worse impact of energy shock on consumption through a real wage fall - as real exchange rate appreciation is milder - by approximately 5% with respect to the 2% of the benchmark, across all consumption quintiles (see left plots). However, the lower interest rate involves a better performance in terms of consumption of mortgagors, who can enjoy a weaker increase in the aggregate mortgage rate and see their consumption inequality gap with non-mortgagors being reduced by approximately 4 percentage points across all consumption quintiles.

5.2 A smoothed interest rate policy alternative

In what follows, I will introduce a new candidate policy ("smoothed policy", in short *sm*), which assumes a lognormal profile specified in the same way as in the benchmark policy (equation (4.2.2)),except for the "location" parameter μ_i^{sm} and the scaling coefficient K_i^{sm} which are such that $\mu_i^{sm} \neq \mu_i$ and $K_i^{sm} \neq K_i$, while keeping σ_i^{sm} equal to the benchmark σ_i^{bmk}. In order to make this alternative policy *smoother* than the benchmark one, I impose the assumption $\mu_i^{sm} > \mu_i$: an increase in the location parameter indeed reduces the height of the peak and shifts the whole distribution to the right, as it is illustrated in Figure 5.3, where the smoothed policy path is compared to the benchmark and the moderate hike previously considered in Figure 5.1. Furthermore, the scaling size parameter is set higher than the benchmark ($K_i^{sm} > K_i$) such to generate a prospective cumulate sum of interest



Benchmark policy	σ_i^{bmk}	1	μ_i^{bmk}	4.1	K^{bmk}	1.07
Smoothed policy	σ_i^{sm}	1	μ_i^{sm}	4.9	K^{sm}	1.69
Moderate hike	σ_i^{mh}	0.75	μ_i^{mh}	4.1	K^{mh}	0.7

Figure 5.3: Benchmark policy vs. smoothed policy and moderate hike. Graaphical path and functional form

rates $\int_{01/2022}^{\infty} i_t dt$ sizable enough to determine the same effect on the real exchange appreciation through UIP as the one produced by the benchmark policy. This can be seen in the bottom-left plot of Figure 5.4, where until 2024 the real exchange rate path under the smoothed policy is quantitatively similar to the benchmark, implying a similar patterns of real wages as well, which experience a 3% fall with respect to steady state over 2023. This in turn implies that the effect of the energy shock on consumption through the real wage is equal between the benchmark and smoothed policy (in both cases comprising a 2% food consumption loss), as shown in the left graph of Figure 5.5. On the other side, the mortgage rate i_t^d in the smoothed policy takes lower values until 2026, with a peak reduction of up to 0.7 percentage points. As highlighted in the bottom-right plot, this translates into a significantly lower consumption drop for mortgagors with respect to non-mortgagors (by 4% in the 2021-2022 time window): smoothed policies are successful in partially closing the inequality gap between the two types of agents, without affecting the performance in terms of real exchange rate appreciation during the energy crisis.

By comparing the smoothed policy with the simple moderate hike, we can observe that the equally mild initial rise in the interest rate implemented by both policies delivers an equal relief on mortgagors' consumption (right graph of Figure 5.5); however, only the smoothed policy is able to achieve that without affecting negatively the real exchange rate, since it sustains it at the same level implied by the benchmark policy through the 3-years further protracted interest rate hike. Therefore, the smoothed policy overperforms the moderate hike and matches the benchmark policy in reducing the consumption loss due to real wage fall.

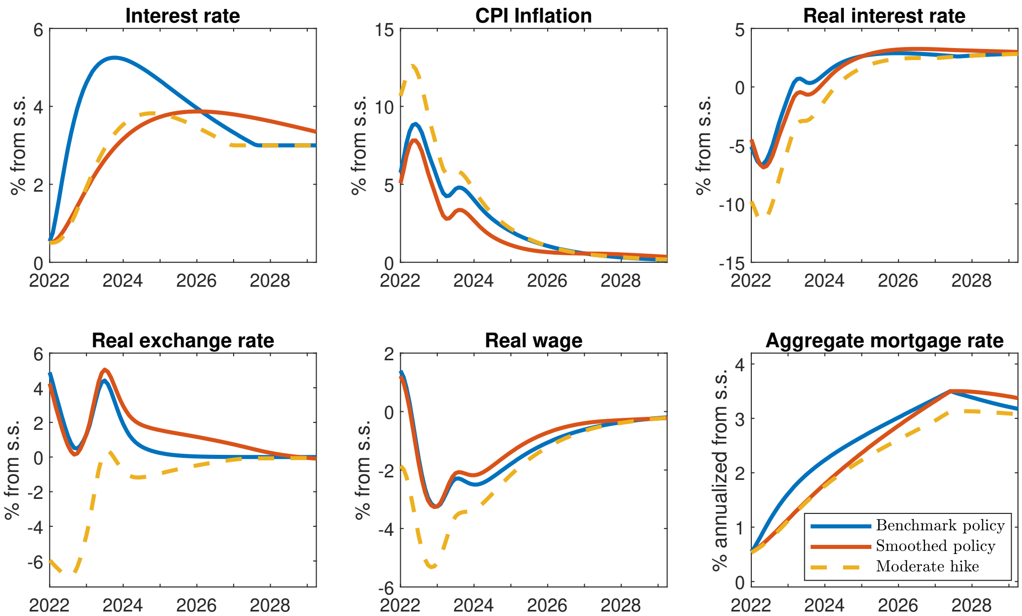


Figure 5.4: Impulse response functions to the energy shock. Benchmark policy vs. Smoothed policy and Moderate hike.

From the discussion above, we can see how the quantitative results confirm the theoretical prescriptions coming from the stylized model of section 3: a smoothing motive of the interest rate policy relaxes the trade-off between real appreciation and mortgage cost increase, which instead was still relevant in the simple moderate hike case: the consumption loss due to real wage fall is indeed the approximately the same between the benchmark and the smoothed policy, while the latter can achieve more moderate mortgage rates and therefore a lower impact on consumption of mortgagors.

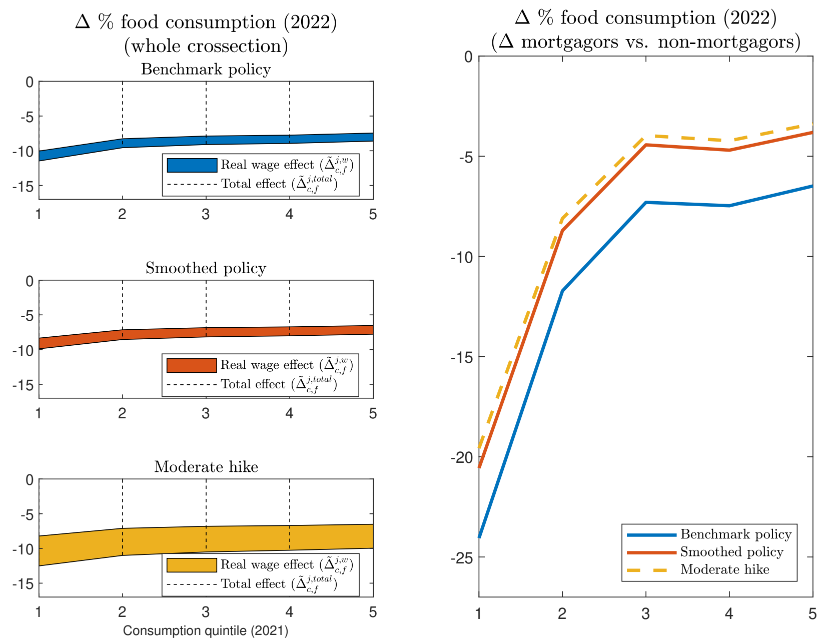


Figure 5.5: Left: food consumption % fall over 2022 for each consumption quintile of the 12/2021 consumption distribution (total $\Delta^{total}_{c,f}$ and decomposed by real wage effect $\Delta^{w}_{c,f}$). Right: 2022 % consumption fall difference between mortgage and non-mortgagors ($\Delta_{c,f}$ from equation (4.4.11)). Benchmark policy vs. Smoothed policy and Moderate hike.

5.3 Welfare implications

As a further step with respect to the policy experiment carried out so far, I proceed to investigate the welfare implications of adopting the smoothed interest rate policy. Given the perfect foresight nature of the model, the discounted welfare of any household is embedded in its value function $V^m_{t_0}(a,z)$ or $V^{nm}_{t_0}(a,z)$, where t_0 is the time index for the first period of the simulation, and m and nm are respectively indexes for mortgagor and an non-mortgagor household. The analysis of the previous section pointed out that interest rate smoothing, during initial stages of the energy crisis, relieves the mortgage cost burden without giving up real wage defence; however, the interest rate remains higher for a longer time, making real exchange and wages' appreciation more persistent until 2028 - and less needed, as the energy price would have already decreased substantially (see Figure 4.3); moreover the interest rate smoothing involves an undesirable longer protraction of high mortgage rates, as can be observed in the bottom-right plot of Figure 5.4, where i^d_t under the smoothed policy overtakes the one produced by the benchmark policy from year 2028 onwards. The adverse effect of this kind of "forward guidance" intervention needs to be taken into account in order to quantitatively evaluate the welfare implications of the smoothing policy: such implications are nonetheless encoded in the initial level of the value functions $V^{m}_{t_0}(a,z)$ or $V^{nm}_{t_0}(a,z)$, which can be averaged across the initial idiosyncratic shocks to obtain average value functions per asset level $V^{m}_{t_0}(a)$ and $V^{nm}_{t_0}(a)$. Figure 5.6 reports on the left an "inequality" measure given by the difference between $V^{m}_{t_0}(a)$ and $V^{nm}_{t_0}(a)$: mortgagors are worse off than non-mortgagors in both policy scenarios, due to mortgage costs burdening both over the dynamics and in the final steady state; however, implementing the smoothed policy allows to reduce inequality between the two household class, thanks to its mitigation effect on mortgage rates. In the current scenario a policymaker caring about inequality would then consider the smoothed policy as a "less costly" measure, from a welfare perspective, to tackle the impact of the energy shocks on the economy. Total utilitarian welfare, defined as the average discretized value function at time 0, i.e. $\sum\limits_{t=12/2021}^{T} \beta^t \sum\limits_{a,z} g_t(a,z)v(a,z)$ (with $\beta = \frac{1}{1+\rho\Delta}$ and T being the last simulation period) increases from -5.7639 to -5.7501, pointing out that the reduction in inequality is not achieved at the expense of a lower economywide utility. In order to substantiate the welfare increase in terms of consumption unit, I compute in the benchmark scenario the consumption subsidy that would need to be accrued to every household over 2022, taking the equilibrium consumption and labor choices as given, in order to yield the same total welfare of the smoothed policy outcome. In other terms, I seek to compute the subsidy k^* such that:

$$\sum_{t=12/2021}^{\infty} \beta^t \sum_{a,z} g_t(a,z) u(c_t^{bmk}(1+k_t), n_t^{bmk}) = \sum_{t=12/2021}^{\infty} \beta^t \sum_{a,z} g_t(a,z) u(c_t^{sm}, n_t^{sm}) \tag{5.3.1}$$

$$\text{with } k_t = \begin{cases} k^* \text{ if } t \leq 12/2022 \\ 0 \text{ if } t > 12/2022 \end{cases}$$

The resulting 2022 subsidy k^* is equal to 1.1%, implying that the consumption path of all the households (and consequently aggregate consumption) would need to be shifted upward over 2022 by this percentage amount in order to guarantee the achievement of the same total welfare as in the smoothed policy case (see right plot of Figure 5.6).

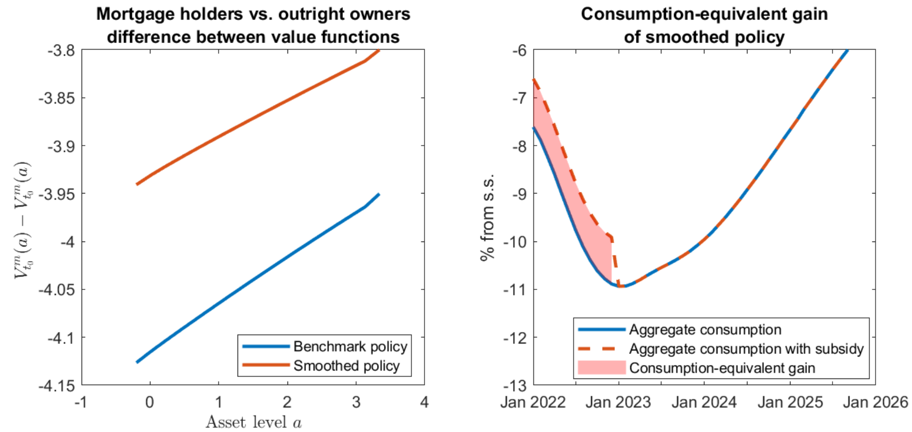


Figure 5.6: Value functions at the first simulation period, for each household class and asset level, Benchmark policy vs. Smoothed policy. (left plot). Consumption-equivalent gain of adopting the Smoothed policy (right plot)

5.4 Testing the model implications: increasing fixed mortgage horizon

A corollary policy prescriptions coming from the discussion of section 3 is that a shorter time horizon S for fixed rates' renewal leads to a stronger effect of interest rate smoothing in relaxing the trade-off between exchange rate appreciation and mortgage costs, due to the higher sensibility of the current mortgage rate to future short term interest rate variations. I can test this implication in the current quantitative setting, by assessing the impulse responses under the same shock and three candidate policies of last section, with the exception of S being now set to three years instead of the 5.5 years calibrated so far. Note that the pressure of contractionary policy on real exchange rate determination (through UIP (5.5.3)) is the same as in the previous section, as the policy paths for i_t are the same as the ones considered before. On the other side, given the lower stickiness of fixed rate mortgages, the overall mortgage rate i_t^d displays for all the three policies a stronger reaction in magnitude with respect to the baseline, with the benchmark policy's mortgage rate peak amounting to 4% (as opposed to the 3.5% peak of the baseline), as showed in Figure 5.7.

What is now the impact of the different policies on mortgage rates? As a result of the increased influence of the fixed-rate mortgage channel on the overall mortgage channel i_t^d, the impact of rising mortgage costs on mortgagors' consumption is amplified in both the policy

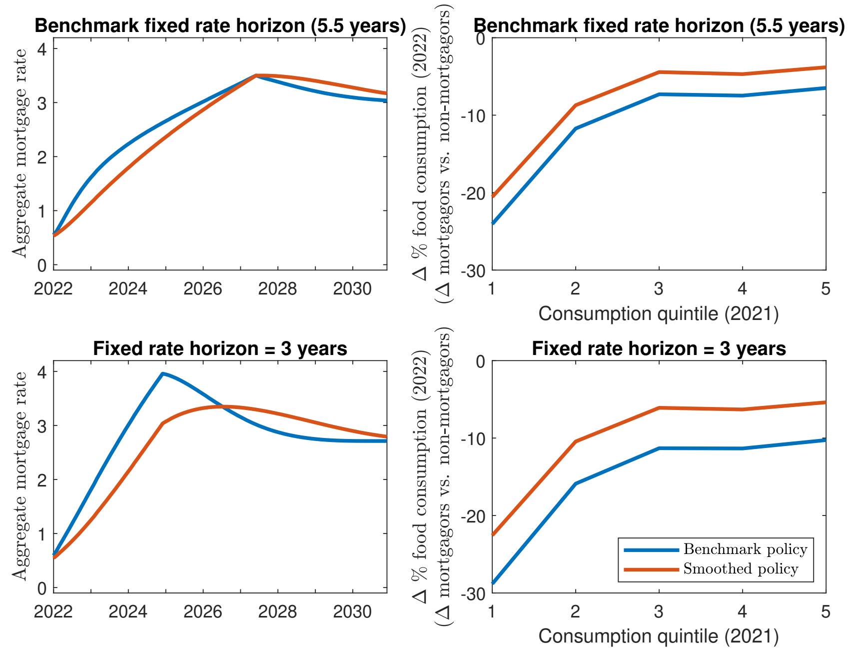


Figure 5.7: Policy's impact of mortgages for different fixed rate time horizons. Benchmark vs. Smoothed policy.

options (see the bottom-right plot), with the consumption effects of mortgage cost increases now being different by 7 percentage points between the two policies (with respect to the 4% difference of the benchmark case). This confirms the analytical prediction outlined in section 3.

6 Conclusion

The trade-off between shielding the real wage of households and maintaining moderate costs for mortgagors in response to an energy price shock through an interest rate hike presents a complex challenge. While an increase in the interest rate can protect the purchasing power of households via real exchange rate appreciation, it also leads to higher mortgage rates. The benchmark contractionary policy implemented by the Bank of England (BoE) during 2022-2023 resulted in significant consumption losses for mortgagors, particularly those at the lower end of the consumption distribution. To address these challenges, this paper has explored an alternative strategy that employs milder and prolonged interest rate hikes. This

approach achieves the same real exchange rate appreciation but allows for the spread of mortgage cost increases over an extended period, thereby mitigating the immediate burden on mortgagors. The effect is decreasing in the length of fixed rate mortgage contracts.
This strategy presents a balanced approach to monetary policy, that would lead to more equitable welfare outcomes in the face of energy price shocks. A natural extension for this paper would therefore consist in a fully microfounded normative analysis, in the spirit of the literature about optimal policy in HANK.

A Appendix

B Derivation of the New Keynesian Phillips curve

Following Wolf (2023), I assume that unions seek to maximize the *utility the average household* [21], i.e, a fictitious agent consuming the average amount over the household's crossection, and subject to the same supply schedule across labor varieties - set by the unions. The utility is evaluated net of an inflation and real wage stabilization cost Ψ_t. The maximization problem writes:

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau \left(\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\} - \Psi_t\right)\right] = \tag{B.0.1}$$

$$\max \int_{\tau \geq 0} \exp\left[-\rho\tau \left(\{u\left(C_{t+\tau}\right) - v\left(N_{t+\tau}\right)\} - \frac{\psi}{2}\pi_t^{W2} N_{t+\tau} - \frac{\zeta}{2}\frac{(\varepsilon-1)\tilde{N}u'(\tilde{C})}{\tilde{w}}\left(w_{k,t+\tau} - \tilde{w}\right)^2\right)\right] \tag{B.0.2}$$

subject to 1) the average real labor earning at time $t+\tau$ being given by:

$$Z_{t+\tau} = \frac{1}{P_{t+\tau}} \int_0^1 W_{kt+\tau} \left(\frac{W_{kt+\tau}}{W_{t+\tau}}\right)^{-\varepsilon} N_{t+\tau} dk \tag{B.0.3}$$

[21] This is a convenient assumption to model the way union aggregates preferences, because it allows to abstract inflation dynamics from distributional outcomes; an alternative is to assume maximization of the average utility of households for some arbitrary weights

2) the envelope condition:

$$\frac{\partial C_{t+\tau}}{\partial W_{kt+\tau}} = \frac{\partial Z_{t+\tau}}{\partial W_{kt+\tau}} = \frac{1}{P_{t+\tau}} \int_0^1 W_{kt+\tau} \left(\frac{W_{kt+\tau}}{W_{t+\tau}} \right)^{-\varepsilon} N_{t+\tau} dk \tag{B.0.4}$$

and 3) the effect of the kth-variety nominal wage W_{kt} on labor supply, that , due to the N_{kt} determination $N_{kt} \equiv \int_0^1 \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t dk$, and the symmetry $N_{kt} = N_t$, writes:

$$\frac{\partial N_t}{\partial W_{kt}} = \frac{\partial N_{kt}}{\partial W_{kt}} = -\varepsilon \frac{N_{kt}}{W_{kt}} = -\varepsilon \frac{N_t}{W_{kt}} \tag{B.0.5}$$

The problem can be formulated as a Hamilton-Bellman-Jacobi equation:

$$\rho J(W,t) = \max_{\pi^w} \left[\{ u(C_t) - v(N_t) \} - \frac{\psi}{2} \pi_t^2 N_t - \frac{\zeta}{2} \frac{(\varepsilon - 1) \bar{N} u'(\bar{C})}{\bar{w}} (w_{k,t+\tau} - \bar{w})^2 \right] + J_W(W,t) W \pi^W + J_t(W,t) \tag{B.0.6}$$

where $J(W,t)$ is the real value of a union with wage W. Taking the envelope and first order conditions and imposing symmetry across all k, we get:

$$J_W(W,t)W = \psi \pi^W N \tag{B.0.7}$$

$$\left(\rho - \pi^W \right) J_W(W,t) = \frac{\varepsilon}{W} \left[N v'(N) - \frac{\varepsilon - 1}{\varepsilon} N w u'(C) - \zeta \frac{\bar{N}}{\frac{\varepsilon}{\varepsilon - 1} \bar{w}} u'(C) (w - w) w \right] + \tag{B.0.8}$$

$$+ J_{WW}(W,t) W \pi^W + J_{Wt}(W,t) \tag{B.0.9}$$

Differentiating (B.0.7) with respect to time gives

$$J_{WW}(W,t)\dot{W} + J_{Wt}(W,t) = \frac{\psi \bar{N} \pi^{\dot{W}}}{W} + \frac{\psi \dot{N} \pi^W}{W} - \frac{\psi \pi^W \bar{N}}{W} \frac{\dot{W}}{W} \tag{B.0.10}$$

Substituting the above expression and (B.0.7) inside (B.0.8) we obtain the Phillips curve as presented in section 5.3 (equation (5.3.3)), with $\kappa^W = \frac{\varepsilon}{\psi}$.

C Equilibrium conditions

The model equilibrium is described by the following set of conditions:

$$\rho V_t(a,z) = \max_{a_t,c_t}\left[\frac{c_t^{1-\sigma}}{1-\sigma} - \chi\frac{n_t^{1+\phi}}{1+\phi} + s_t(a,z)\frac{\partial V_t}{\partial a}\right] + \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t}$$
$$c_t(a,z)^{-\sigma} = \frac{\partial V_t(a,z)}{\partial a}$$
$$s_t(a,z) = \begin{cases} \frac{\delta a_t + z_t w_t n_t + d_t - c_t - D_t^r i_t^d + \Pi_t}{X_t} - (\delta+\pi_t)a_t & \text{if mortgagor} \\ \frac{\delta a_t + z_t w_t n_t + d_t - c_t + \Pi_t}{X_t} - (\delta+\pi_t)a_t & \text{if non-mortgagor} \end{cases}$$
$$\frac{\partial f_t(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t(a,z)f_t(a,z)] - \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2}$$
$$C_t = \int_0^1 c_t(a,z)f_t(a,z)dadz$$
$$w_t\frac{1}{A} = p_H(Q_t, P_{Et}^*)$$
$$\dot{D}_t^r = -D_t^r\pi$$
$$i_t^d = \frac{D^f}{D}i_t^f + \frac{D^v}{D}i_t$$
$$i_t^f = \frac{1}{S}\int_0^S i_t^f(s)ds$$
$$i_t^f(s) = i^f_{\tau\in[t,t+S)}(s) = \frac{1}{S}\int_{[t,t+S)} i_\tau d\tau$$
$$\pi_t^W = \frac{1}{\rho - \dot{N}_t/N_t}\left[\kappa\left(\chi N_t^\phi - \frac{\varepsilon-1}{\varepsilon}w_t C_t^{-\sigma} - \zeta\frac{\varepsilon-1}{\varepsilon}\frac{\tilde{N}}{N_t}\tilde{C}^{-\sigma}(w_t-\tilde{w})\frac{w_t}{\tilde{w}}\right) + \dot{\pi}_t^W\right]$$
$$X_t = \int_t^\infty \delta e^{-\left[\int_t^s i_s + \delta(s-t)\right]}ds$$
$$i_t - \pi_t = i^* - \pi^* - \frac{dQ_t}{Q_t} + \xi_t$$
$$Y_{Ht} = (1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t,p_{Et}^*)^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha(p_F(Q_t)/p_D(Q_t,p_{Et}^*))^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}(Y_{ft}+Y_{nt}) +$$
$$\alpha\left(p_H(Q_t,p_{Et}^*)Q_t\right)^{-\eta}C^*$$

$$Y_{ft} + Y_{nt} = C_t$$
$$Y_{Ht} = AN_t$$
$$N_t = n_t$$
$$\Pi_t = \omega D_t^r i_t^d$$
$$d_t = Y_{Ht} - AN_t$$
$$w_t = W_t/p_t$$
$$\pi_t = \dot{p}_t/p_t$$
$$\pi_t^W = \dot{W}_t/W_t$$

Which in order, are: the Hamiltonian-Bellman-Jacobi equation, the optimality condition of the household, the drift function, the Kolmogorov-Forward equation, the definition of aggregate consumption, the domestic good producers' pricing, the evolution of the real mortgage stock, the definition of the mortgage rate, of the fixed mortgage rate, and of the fixed rate of submortgage s. Then we have the Phillips curve, the pricing of long term bonds, the UIP condition, the market clearing conditions and the mortgage revenues rebating rule. Finally we have the definition of dividends, real wage, price inflation and wage inflation.

D Real exchange rate path in the benchmark scenario

Q_t is set to the initial steady state $\bar{Q}$ until April 2022 - the onset of the energy crisis shock. Afterwards it is denoted by Q'_t and it is computed as the filtered version of the real exchange data series (Figure 4.3). Q'_t is made up by the following two subsets: 1) $\bar{Q}$+the detrended real exchange rate index for UK for $01/2022 < t < 07/2023$ (denoted by $\hat{Q}^{data}_t$). The linear trend is computed according to the pre-energy crisis period 01/2017-04/2022. I choose 2017 as starting year for the trend computation sample, when the time series for the real exchange rate presents a structural break due to Brexit. 2) a diffusion process Q_t for $t \geq 08/2023$ with no innovation, persistence $\rho = 0.85$, and with starting point $Q'_{08/2023} = \hat{Q}^{data}_{08/2023}$. This represents the normalization "tail" of monetary contraction following the decline of energy price pressures.

$$Q'_t = \tilde{Q} + \hat{Q}^{data}_t \quad 01/2022 < t < 07/2023 \tag{D.0.1}$$

$$dQ'_t = (\rho - 1)(Q_t - \tilde{Q}) \quad \forall t \geq 08/2023 \tag{D.0.2}$$

Where Q' is the final steady state real exchange rate. The input Q_t is given then by:

$$Q_t = \bar{Q} \quad \forall t < 01/2022 \tag{D.0.3}$$

$$Q_t = filter(Q'_t) \quad \forall t \geq 01/2022 \tag{D.0.4}$$

E Solution algorithm

E.1 Steady state

Under the benchmark policy, the model is solved numerically with the method presented in Achdou et al. (2021), by iteration over the aggregate consumption value. Prior to considering the solution over the dynamics it is necessary to solve for the final steady state of the model (for given D^r) through the following steps:

1. Use the calibrated value for $\tilde{Q}$ to obtain the wage

$$w = p_H(\tilde{Q}, p_E^*)/A \tag{E.1.1}$$

2. As discussed in section, 2.8, steady state requires $\pi = 0$. Therefore, since the real wage is constant, also nominal wage inflation is 0, $\pi^W = 0$.

3. Imposing stationarity in the Phillips curve (5.3.3), we get

$$N = \left(\frac{\varepsilon - 1}{\varepsilon} w \tilde{C}^{-\sigma} \frac{1}{\chi}\right)^{\frac{1}{\phi}} \tag{E.1.2}$$

4. Solve the household problem by iteration on the HJB equation (see Nuño and Thomas (2022) for the case with long-term bonds). Notice $i^d = i$ in the initial steady state and $i^d = i'$ in the final steady state.

5. Compute aggregate consumption $C = \int_a \int_z c(a, z) da dz$

6. From the equilibrium condition (5.7.1)-(2.7.2), we obtain:

$$\begin{aligned} A\tilde{N} =& (1 - \alpha_E) \left(\frac{1 - \alpha_E p_E(\tilde{Q}, p_E^*)^{1-\epsilon}}{1 - \alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}} (1 - \alpha) \left(\frac{1 - \alpha(p_F(\tilde{Q})/p_D(\tilde{Q}, p_E^*))^{1-\eta}}{1 - \alpha}\right)^{-\frac{\eta}{1-\eta}} \tilde{C} \\ &+ \alpha \left(p_H(Q, p_E^*)\bar{Q}\right)^{-\eta} C^* \end{aligned} \tag{E.1.3}$$

   From which we can retrieve the value for foreign consumption C^* consistent with the stationary equilibrium

Once computed the final steady state, I already exploited the degree of freedom provided by C^*, so , in order to compute the initial steady state, as well as a different final steady state characterized by a different D, I need to solve the system of equations (E.1.1),(E.1.2),(E.1.3), together with aggregate demand (equation (5.8.1))

$$\bar{C} = C(\bar{i}, \bar{Q}) \tag{E.1.4}$$

that is a system of four variables ($\bar{w}$,$\bar{Q}$,$\bar{N}$,$\bar{C}$) in four equations. Since (E.1.4) has to be solved numerically as in points 4-5, I proceed as follows:

1. Guess $\bar{C}$

2. Use (E.1.1),(E.1.2),(E.1.3) to get $\bar{Q}$, $\bar{w}$,$\bar{N}$

3. Use $\bar{w}$ and to solve for the households' optimization and aggregate into an updated guess $\bar{C}'$ ((as in point 4 and 5 of final steady state computation))

4. Update the guess:
$$\bar{C} = \bar{C} + \vartheta^S(\bar{C}' - \bar{C}) \tag{E.1.5}$$
until convergence of the quantity $|\bar{C} - \bar{C}'|$ to a threshold small enough. The sign and magnitude of the coefficient ϑ depends on the parameters of the model. For my parametrization and initial guess for $\bar{C}$, imposing a positive ϑ^S leads to an explosive feedback-loop between $\bar{C}$ and $\bar{w}$, while a negative ϑ^S (=0.1) allows to reach convergence.

E.2 Dynamics

Let us now turn the attention to the solution over the dynamics following an unexpected shock to p^*_{Et}, under perfect foresight. The algorithm unfolds as follows:

- Assume a long time horizon T for the discretized variables' path
- Start with the inputs for i_t, Q_t, p^*_{Et}
- Compute $\{w_t\} = \{p_H(Q_t, p^*_{Et})/A\}$
- Use the sequence $\{i_t\}$ to compute the path for long-term bond prices $\{X_t\}$

Then go through the following loop

1. Guess a value for the final steady state real mortgage stock $D^{r\prime}$ and compute the final steady state through the same steps showcased in the second part of section E.1

2. Guess a value for $\{C_t\}$

3. Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (5.7.1)-(2.7.2))

4. Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π^W_t backward, starting from $\pi^W_T = 0$

5. Compute $\pi_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi^W_t}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$)

6. Use the UIP condition (5.5.3) to back out the path of wedges $\{\xi_t\}$ such that the assumed values for Q_t are consistent with the resulting inflation path $\{\pi_t\}$

7. Starting from $D^r_{-1} = D$, use $\{\pi_t\}$ to compute the path for the real mortgage stock $\{D^r_t\}$ up to time T (leading to a final value D^r_T not necessarily equal to the guess $D^{r\prime}$

8. At each t, compute $i^d_t = \frac{D^f}{D} i^f_t + \frac{D^v}{D} i_t$. Following the assumptions of section 2.1, we can express $i^f_t = \frac{1}{S}((S-1)i^f_t + \frac{1}{S}\sum_{\tau=0}^{\infty} i_{t+\tau})$ (S needs to be $\in \mathbb{N}$).

9. Solve the household problem with long term bonds holding (see Nuño and Thomas (2022) backward, starting from the value functions of the final steady state computed in point 1.

10. Compute the new path for aggregate consumption $\{C'_t\} = \{\sum a, zc_t(a,z)dadz\}$

11. Update C_t as $C_t = (1-\vartheta)C_t + \vartheta C'_t$ for an arbitrary coefficient $\vartheta \in (0,1)$

12. Iterate until convergence of $\max |\{C_t\} - \{C'_t\}|$ to some low threshold value.

13. Update D'_r as $D'_r = (1-\vartheta^D)D'_r + \vartheta^D D^r_T$ for an arbitrary coefficient $\vartheta^D \in (0,1)$

14. Iterate until convergence of $\max |D^{r\prime} - D^r_T{}'|$ to some low threshold value.

E.3 Alternative policies

In order to solve the model for the alternative policies, we do not take anymore $\{Q_t\}$ as an input and back out the UIP wedges $\{\xi_t\}$ consistent with equilibrium, but we instead take $\{\xi_t\}$ as exogenous and solve for $\{Q_t\}$. In order to accomplish this task, I augment the model with an inner loop over the *real interest rate*, in order to determine the inflation path given the policy on i_t. The modified algorithm writes:

- Assume a long time horizon T for the discretized variables' path
- Start with the inputs for i_t and p^*_{Et}
- Use the sequence $\{i_t\}$ to compute the path for long-term bond prices $\{X_t\}$

1. Guess a value for the final steady state real mortgage stock $D^{r\prime}$ and compute the final steady state through the same steps showcased at the beginning of the current section.
2. Guess a value for $\{C_t\}$
3. Go through the following loop
   (a) Guess a path for the real interest rate $r_t \equiv i_t - \pi_t$
   (b) Obtain the implied path for inflation $\pi_t = i_t - r_t$
   (c) Substitute for $\{r_t\} \equiv \{i_t - \pi_t\}$ and the wedges $\{\xi_t\}$ inside the UIP condition (5.5.3) for every t. Iterate the condition backward, starting from $Q_T = \bar{Q}$, to recover the path for Q_t.
   (d) Compute $\{w_t\} = \{p_H(Q_t, p^*_{Et})/A\}$
   (e) Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (5.7.1)-(2.7.2))
   (f) Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π^W_t backward, starting from $\pi^W_T = 0$
   (g) Compute the implied inflation from energy prices and labor market forces: $\pi'_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi^W_t}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$)
   (h) Update the real rate at each t according to the variation between inflation determined by the guess and the resulting inflation from the last point:
   $r_t = r_t - \vartheta(\pi'_t - \pi_t)$ for an arbitrary coefficient $\vartheta \in (0, 1)$
   (i) iterate until convergence of $\max |\{r_t\} - \{r'_t\}|$ to some low threshold value.
4. Go through the point 7-14 as for the benchmark policy algorithm, and iterate until convergence of $\max |D^{r\prime} - D^{r}_T{}'|$ to some low threshold value.

F Extension: price stickiness

In this section I study a version of the model where price stickiness characterize firms instead of unions. The extension is relevant to explore the robustness of the results of the model in presence of slow pass-through of energy prices onto final product prices. I will hereafter assume the wage adjustment cost ψ equal to 0, while introducing an alternative layer of price rigidity at the final producers' level.

For simplicity I assume away food and non food duality in the final good supply, which instead is now given by a range of varieties from 0 to 1 - each variety being produced by a different firm in a monopolistically competitive market. In presence of Rotemberg's price adjustment costs, the recursive problem of a final producer writes:

$$(\rho^F - \pi)J(p,t) = \max_{\pi} \left(\frac{p}{P_t} - m_t \frac{1}{\tau^F}\right)\left(\frac{p}{p_t^j}\right)^{-\upsilon} C_t - \frac{\tilde{\psi}}{2}\pi^2 C_t + J_p(p,t)p\pi + J_t(p,t) \tag{F.0.1}$$

where ρ_t^F is the discount factor of any firm, $J(p,t)$ is the real value of a firm with price p, P_t is the price level, C_t is aggregate consumption, m_t is the real marginal cost, τ^F is a government's subsidy, and $\tilde{\psi}$ is a coefficient measuring the extent of price adjustment costs. The first order and envelope conditions for the firm are

$$J_p(p,t) = \frac{\tilde{\psi}\pi C}{p}$$

$$\left(\rho^F - \pi\right) J_p(p,t) = -\left(\frac{p}{P} - m\frac{1}{\tau^F}\right)\upsilon\left(\frac{p}{P^j}\right)^{-\upsilon-1}\frac{C}{P} + \left(\frac{p}{P}\right)^{-\upsilon}\frac{C}{P} + J_{pp}(p,t)p\pi + J_{tp}(p,t)$$

By perfect competition within food and non-food industry, and symmetry among firms, we will have $p = P$, and hence

$$J_p(p,t) = \frac{\tilde{\psi}\pi C}{p} \tag{F.0.2}$$

$$\left(\rho^F - \pi\right) J_p(p,t) = -(1 - m\frac{1}{\tau^F})\upsilon\frac{C}{p} + \frac{C}{p} + J_{pp}(p,t)p\pi + J_{tp}(p,t) \tag{F.0.3}$$

Differentiating (F.0.2) with respect to time gives

$$J_{pp}(p,t)\dot{p} + J_{pt}(p,t) = \frac{\tilde{\psi}C\dot{\pi}}{p} + \frac{\tilde{\psi}\dot{C}\pi}{p} - \frac{\theta C}{p}\frac{\dot{p}}{p}\pi$$

Substituting into condition (F.0.3) and dividing by $\theta C/p$ gives

$$\left(\rho^F - \frac{\dot{C}}{C}\right)\pi = \frac{1}{\theta}(-(1-m\frac{1}{\tau^F})\upsilon + 1) + \dot{\pi}$$

Since profits of firms are accrued to households, we assume discounting of firms is weighted by the marginal utility of the latter, i.e. $\rho^F = \rho + (\sigma-1)\frac{\dot{C}}{C}$, that implies:

$$\left(\rho + (\sigma-1)\frac{\dot{C}}{C}\right)\pi = \frac{1}{\theta}(-(1-m\frac{1}{\tau^F})\upsilon + 1) + \dot{\pi} \tag{F.0.4}$$

Notice that the marginal cost is given by the following Dixit-Stigliz price aggregator

$$m_t = \tau^F \left((1-\alpha_e)\left(\left((1-\alpha)\left(\frac{p_{h,t}}{p_t}\right)^{1-\eta} + \alpha\left(\frac{p_{ft}}{p_t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\varepsilon} + \alpha_e\left(\frac{p_{e,t}}{p_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{F.0.5}$$

Importantly, now that perfect competition is ruled out, a profit term Π_t^F coming from the dividends of the final producers is rebated to households; following Wolf (2021), I assume that this dividend term is weighted by household productivity (i.e. it enters the budget constraint as $\Pi_t^F z$, summing to Π_t^F over the crossection, thanks to assumption (5.1.15)); this allows not to have cyclical inequality implied by the dividends' rebating scheme. For simplicity, I assume that profit are zero in steady state thanks to a proper level of τ^F - financed by firm's profit itself in la lump-sum fashion, as in Corbellini (2024b).

The algorithm solution needs to be updated from its version of section E.2: point 3 is replaced by a joint computation of the equilibrium w_t, m_t, N_t , by a system of market clearing conditions (5.7.1)-(5.7.2), marginal cost expression (F.0.5) and the union's first order condition (5.3.3) with $\psi = 0$. The results in terms of m_t is in turn propedeutical to compute inflation π_t at each point in time using (F.0.4), which replaces point 4.

Figure F.1 reports the impulse response functions of the economy in presence of sticky prices and flexible wage setting.

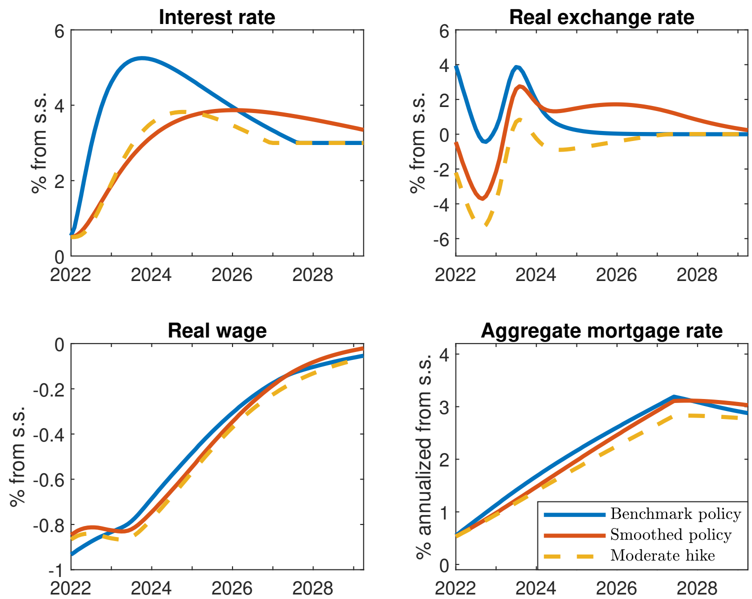


Figure F.1: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Sticky prices with flexible wage setting.

By looking at the top-centre and bottom-left subplot, we can note how also in this case the implementation of the smoothed policy yields quantitative results which confirm the theoretical implications of the model, even though significantly smaller: the real exchange rate is better stabilized with respect to the moderate hike case, with the aggregate mortgage rate closely tracking the moderate hike counterpart until 2024.

G Sensitivity Analysis

The following robustness checks aims at generating the impulse responses to the same energy shock analyzed in the body of the text, under different parametrizations of the key quantities determining the extent of the appreciation-mortgage cost trade-off, namely the mortgage stock amount D and the steady state real exchange rate $\bar{Q}$, that is determined through leaving a degree of freedom on the foreign consumption parameter C^*, as discussed in section 4.2.

Real exchange movements are a key force behind the monetary policy trade-off explored by the model. Varying values of $\bar{Q}$ effectively result in scaling the entire path of Q_t (shown in Figure 4.3 as percentage deviations), which either dampens or enhances the domestic energy price P^*_{Et}/Q_t and then the impact on real wages. Figure G.1 reports the impulse response functions of the economy under a lower value for $\bar{Q}$, i.e. $\bar{Q} = 1.1$ - recovered by the steady state computation by choosing a proper value of foreign consumption C^*.

As the real exchange rate is characterized by different path in absolute levels but not in percentage deviations with respect to the steady state, the impulse response of Q_t is unchanged with respect to the baseline scenario of Figure 5.4. Real wages instead react more strongly than in the baseline analysis, with the benchmark policy implying a peak fall in w_t of 4% - stronger than the 3% trough illustrated in Figure 5.4. Again, the intuition in terms of trade-off relaxation by the smoothing of the interest rates follows exactly as in the analysis in the body of the paper.

Next, I consider an alternative calibration of the mortgage stock D: I compare the effects of mortgage cost increase on consumption under the benchmark scenario ($D = -50$), with the case of a calibration $D = -30$ (Figure G.2). The mechanisms and trade-off of the baseline scenario are yet unaltered.

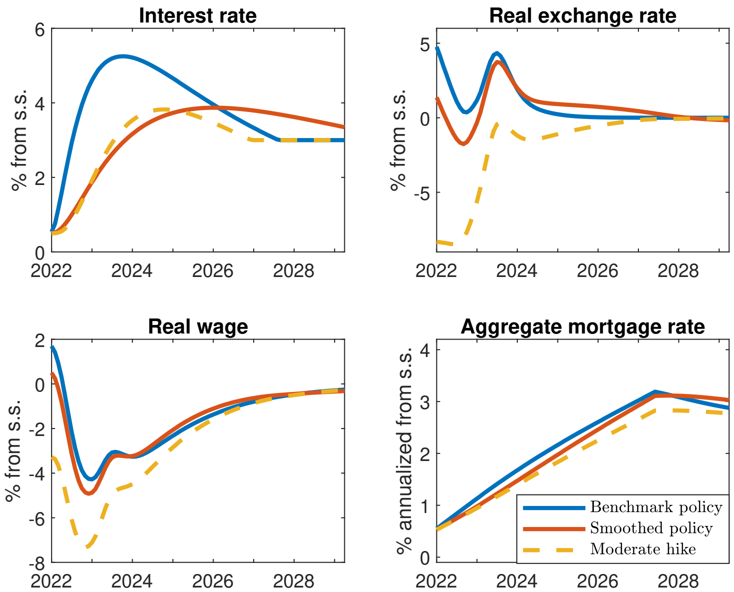


Figure G.1: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Case $\bar{Q} = 1.1$

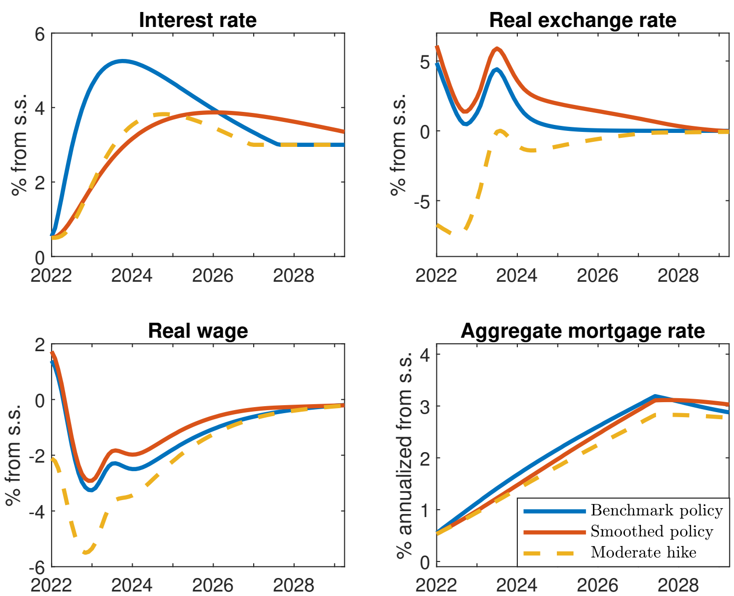


Figure G.2: Impulse response functions to the benchmark energy shock. Benchmark vs. Smoothed policy and Moderate hike. Case $D = -30$

Chapter 3

Solving for Optimal Monetary Policy in HANK by a Discretize-then-Optimize Algorithm: a 2-stage Iterative Approach

1 Introduction

In recent years, heterogeneous agents New Keynesian (HANK) models have emerged as a pivotal framework for understanding macroeconomic dynamics in economies characterized by incomplete insurance markets, hence by households with different consumption reactions to idiosyncratic and aggregate shocks.

The introduction of agents' heterogeneity in a new keynesian environment gives rise to different marginal propensity to consume across households, creating deviations in the response of aggregate variables from the benchmark representative agent framework, as well as cyclical consumption inequality. These elements point out to the importance of a normative analysis in HANK. The study of optimal policy in these environments is a challenging objective, as the Ramsey planner needs to optimize over a variety of individual histories which branch out to an infinite amount in the time limit - making impossible to apply the standard dynamic optimization techniques used in representative agents models.

Continuous time methods, used in Nuño and Moll (2018) and Nuño and Thomas (2022), solve the problem by optimizing over the time-varying *distribution* of states itself, instead of over the single agents' histories. This requires functional differentiation under the form of *Gateaux derivatives*, that implies a degree of mathematical complexity that becomes particularly challenging when treating models with both a Ramsey problem and aggregate externalities. Smirnov (2023) uses calculus of variations to address this task. A solution to reducing mathematical burden in normative continuous time HANK has been proposed by Nuño et al. (2023), which use a *discretize-then-optimize* algorithm. This method consists in setting up the the Ramsey problem in continuous time, and then take the first order conditions once having discretized the problem: this allows to take first order conditions with respect to the distributions exactly as done for all the other variables, ruling out the need to resort to Gateaux differentiation.

The solution algorithm of Nuño et al. (2023) prescribes to solve the entire system of equilibrium equations at once by a guess-and-verify approach on the policy instrument (the interest rate), requiring a heavy numerical task to be accomplished by the machine. In my paper, I show that the numerical burden can be eased by splitting the system of equations into two subsets: the planner's constraints, i.e. agents' optimal conditions and market clearing (which can be solved by the standard routine by Achdou et al. (2021)), and the planner's first order conditions. All the sources of nonlinearities are contained in the first block, so the second block can be solved by a first-order approximation and Blanchard-Kahn approach. Once described the algorithm, I apply my method to a HANK setting built on Auclert et al. (2023b), to describe optimal monetary policy response to energy price shocks in a small open economy - a particularly relevant topic for the current policy debate.

My contribution to the literature is to further reduce the computational weight of normative HANK models, with respect to what continuous time papers as Nuño and Moll (2018), Nuño and Thomas (2022) and Nuño et al. (2023) have attained so far. The value added of this branch of models to the general optimal policy literature in HANK lies indeed in the capability to reduce the numerical complexity through additional analytical intermediate results that can be obtained in a continuous time setting. Alternative models have pursued instead more numerical-intensive strategies to compute optimal policies. Crossectional consumption inequality measure and the impact of idiosyncratic choices on aggregate variables are the elements that in principle would require keeping track of all cumulated households' states histories in a normative analysis: Dávila and Schaab (2023) and Wolf (2023) propose a *sequence space jacobian* computation that allows to recover the evolution of both those elements over time without need to track all the idiosyncratic histories. The method, grounded on the workhorse algorithm by Auclert, Bardóczy, Rognlie, and Straub (2021), requires to perform small perturbation to a steady state configuration of the model, to compute numerically and under matrix forms both the micro-to-aggregate effects and crossectional inequality, which evolve by interacting with aggregate variables. Bhandari et al. (2021) instead take functional (Fréchet) derivatives of the discrete distribution by introducing small noise perturbations, and then uses such derivatives in the Ramsey problem's optimization.

The paper unfolds as follows: in section 1 I present the algorithm for a general class of models and derive the discretized version and first order conditions, in line with Nuño et al. (2023); in section 2 I describe the solution algorithm, and in section 3 I show the application to energy shock management.

2 Discretization of a generic Ramsey problem for a heterogeneous agents model

The following framework is taken from Nuño et al. (2023), and sums up the problem of a Ramsey planner in a heterogeneous agents environment.

$$\max_{Z_t,u_t(x),\mu_t(x),v_t(x),r_t} \quad \int_0^\infty \exp(-\varrho t) f_0\left(Z_t\right) dt \tag{2.0.1}$$

s.t. $\forall t$

$$\dot{X}_t = f_1\left(Z_t, r_t\right) \tag{2.0.2}$$

$$\dot{U}_t = f_2\left(Z_t, r_t\right) \tag{2.0.3}$$

$$0 = f_3\left(Z_t, r_t\right) \tag{2.0.4}$$

$$\tilde{U}_t = \int f_4\left(x, u_t(x), Z_t, r_t\right) \mu_t(x) dx \tag{2.0.5}$$

$$\rho v_t(x) = \dot{v}_t(x) + f_5\left(x, u_t(x), Z_t, r_t\right) + \sum_{i=1}^{I} b_i\left(x, u_t(x), Z_t, r_t\right) \frac{\partial v_t(x)}{\partial x_i} + \sum_{i=1}^{I}\sum_{k=1}^{I} \frac{\left(\sigma(x)\sigma(x)^\top\right)_{i,k}}{2} \frac{\partial^2 v_t(x)}{\partial x_i \partial x_k}, \forall x \tag{2.0.6}$$

$$0 \quad = \quad \frac{\partial f_5}{\partial u_{j,t}} + \sum_{i=1}^{I} \frac{\partial b_i}{\partial u_{j,t}} \frac{\partial v_t(x)}{\partial x_i}, \quad j = 1, \ldots, J, \forall x \tag{2.0.7}$$

$$\dot{\mu}_t(x) = -\sum_{i=1}^{I} \frac{\partial}{\partial x_i}\left[b_i\left(x, u_t(x), Z_t, r_t\right) \mu_t(x)\right] + \frac{1}{2}\sum_{i=1}^{I}\sum_{k=1}^{I} \frac{\partial^2}{\partial x_i \partial x_k}\left[\left(\sigma(x)\sigma(x)^\top\right)_{i,k} \mu_t(x)\right], \forall x \tag{2.0.8}$$

$$X_0 = \bar{X}_0 \tag{2.0.9}$$

$$\mu_0(x) = \bar{\mu}_0(x) \tag{2.0.10}$$

$$\lim_{t\to\infty} U = \bar{U}_\infty \tag{2.0.11}$$

$$\lim_{t\to\infty} v(x) = \bar{v}(x)_\infty \tag{2.0.12}$$

where r_t is the policy variables (e.g. the interest rate), x is the individual state vector , u individual control vector with J elements , $u(x)$ is the vector of controls as function of individual state, $\mu(x)$ is the density of agents at x, and $v(x)$ is the value function. X (with size N_X) is the vector of aggregate states, U (with size N_U) is the vector of aggregate intertemporal controls, $\tilde{U}$ is the vector of aggregator controls[22] and Z_t (with size N_Z) is the vector of all aggregate variables - including purely contemporaneous aggregate controls. b is the drift function of x, f_0 is the welfare function and f_1, f_2, f_3 are the aggregate equilibrium conditions; f_4 is an aggregator function and f_5 is individual utility.

Line (2.0.1) outlines the planner's objective function. Equations (2.0.2)-(2.0.4) set forth the aggregate equilibrium conditions covering aggregate states, jump variables, and contemporaneous variables (for example, an aggregate labor supply condition). Equation (2.0.5) establishes a link between aggregate and individual variables (for example, the sum between capital holding of individual households summing up to aggregate capital). Equations (2.0.6) and (2.0.7) describe the individual agent's value function and first-order conditions, which need to be met across the entire individual state vector x. The Kolmogorov Forward equation (2.0.8) dictates the agent distribution's evolution.

The discretized version of the problem is obtained as follows. I consider a time step of size Δ. For any variable k_t, its time variation dk_t is approximated by the discrete version $\Delta[k_t] = \frac{k_{t+1}-k_t}{\Delta}$ if the variable is forward looking (for instance, the value function $v_t(a,z)$), and by $\Delta[k_t] = \frac{k_t - k_{t-1}}{\Delta}$ if it is backward looking (for instance, the distribution $f_t(a,z)$). The idiosyncratic state is discretized by a evenly-spaced grid of size $[N_1, \ldots, N_I]$ where $1,..,I$ are the dimensions of the state x. The state step size is Δx_i. We define $x^n \equiv (x_{1,n_1}, \ldots, x_{i,n_i}, \ldots, x_{I,n_I})$, where $n_1 \in \{1, N_1\}, \ldots, n_I \in \{1, N_I\}$. Due to state constraints and/or reflecting boundaries, the dynamics of idiosyncratic states are constrained to the compact set $[x_{1,1}, x_{1,N_1}] \times [x_{2,1}, x_{2,N_2}] \times \ldots \times [x_{I,1}, x_{I,N_I}]$. We also define $x^{n_i+1} \equiv (x_{1,n_1}, \ldots, x_{i,n_i+1}, \ldots, x_{I,n_I})\,, x^{n_i-1} \equiv (x_{1,n_1}, \ldots, x_{i,n_i-1}, \ldots, x_{I,n_I})$ $f_t^n \equiv f\left(x^n, u_t^n, Z_t\right), f_t^{n_i-1} \equiv f\left(x^{n_i-1}, u_t^n, Z_t\right)$ and $f_t^{n_i+1} \equiv f\left(x^{n_i+1}, u_t^n, Z_t\right)$. The upwind derivatives ∇ or $\hat{\nabla}$ for the discretized functions v_t^n, μ_t^n are defined as:

[22]an exampe of an aggregator variable is the Dixit-Stigliz basket of consumption varieties in the New Keynesian model $(\int c(i)^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$, with η being the elasticity of substitution among varieties

$$\nabla_i\left[v_t^n\right] \equiv \left[\mathbb{I}_{b_{i,t}^n>0}\frac{v_t^{n_i+1}-v_t^n}{\Delta x_i}+\mathbb{I}_{b_{i,t}^n<0}\frac{v_t^n-v_t^{n_i-1}}{\Delta x_i}\right] \tag{2.0.13}$$

$$\hat{\nabla}_i\left[\mu_t^n\right] \equiv \left[\frac{\mathbb{I}_{b_{i,t}^{n_i+1}<0}\mu_t^{n_i+1}-\mathbb{I}_{b_{i,t}^n<0}\mu_t^n}{\Delta x_i}+\frac{\mathbb{I}_{b_{i,t}^n>0}\mu_t^n-\mathbb{I}_{b_{i,t}^{n_i-1}>0}\mu_t^{n_i-1}}{\Delta x_i}\right] \tag{2.0.14}$$

The discretized problem writes:

$$\max_{Z_t,u_t^n,\mu_t^n,v_t^n,r_t} \quad \sum_t \beta^t f_0\left(Z_t\right) \tag{2.0.15}$$

$$\text{s.t. } \forall t$$

$$\frac{X_{t+1}-X_t}{\Delta t} = f_1\left(Z_t,r_t\right) \tag{2.0.16}$$

$$\frac{U_{t+1}-U_t}{\Delta t} = f_2\left(Z_t,r_t\right) \tag{2.0.17}$$

$$0 = f_3\left(Z_t,r_t\right) \tag{2.0.18}$$

$$\tilde{U}_t = \sum_{n=1}^{N} f_4\left(x^n,u_t^n,Z_t,r_t\right)\mu_t^n \tag{2.0.19}$$

$$\rho v_t^n = \frac{v_{t+1}^n-v_t^n}{\Delta t}+f_5\left(x^n,u_t^n,Z_t,r_t\right)+\sum_{i=1}^{I} b_i\left(x^n,u_t^n,Z_t,r_t\right)\nabla_i\left[v_t^n\right]+\frac{1}{2}\sum_{i=1}^{I}\left(\sigma_i^2\right)^n \triangle_i^2\left[v_t^n\right], \forall n \tag{2.0.20}$$

$$0 \quad = \quad \frac{\partial f_{5,t}^n}{\partial u_{j,t}^n}+\sum_{i=1}^{I}\frac{\partial b_{i,t}^n}{\partial u_{j,t}^n}\nabla_i\left[v_t^n\right], \quad \forall j,n \tag{2.0.21}$$

$$\frac{\mu_{t+1}^n-\mu_t^n}{\Delta t} = -\sum_{i=1}^{I}\hat{\nabla}_i\left[b_{i,t}^n\mu_t^n\right]+\frac{1}{2}\sum_{i=1}^{I}\triangle_i\left[\sigma_i^2\mu_t^n\right] \tag{2.0.22}$$

Where the proper initial condition specified in the continuous time problem still hold. The problem can be formulated in the form of a Lagrangian, where the objective (2.0.15) is maximized under constraints (2.0.16)-(2.0.22).

The total number of constraints is $N_Z+N\times 3$, where N_Z are respectively the numbers of aggregate state equation embedded in (2.0.16),(2.0.17),(2.0.18),(2.0.19); while $N\times 3$ is the number of constraints represented by equations ((2.0.20)-(2.0.22)), the number of possible idiosyncratic state combinations being equal to N.

As we are dealing with a Lagrangian, we can take first order conditions with respect to each of the discretized endogenous variables of the model, i.e. each of the $N_Z + 1$ aggregate variables (states and the unique policy variable) plus each of the $N \times 3$ idiosyncratic variables (i.e. value function v_t^n, density μ_t^n and individual control u_t^n, for every combination of states); the appendix of Nuño et al. (2023) provides algebraic computation of each of these conditions.

3 Timeless perspective

In the problem outlined in the previous section, some endogenous variables are forward looking (for example, the value function v_t^n). Evaluating the first order conditions at time 0 delivers a $t = -1$ multiplier showing up in the expressions, for each forward looking variable: in order to solve the problem, we must assign a value to these multipliers. I choose to adopt a timeless perspective, consistently with Dávila and Schaab (2023), that involves assigning these multipliers the steady state value, i.e. the value they would attain in the time limit if optimal policy were implemented long ago and in absence of aggregate shocks. In this case, time 0 multipliers represent the value of the policymaker to fulfill past promises when the economy is hit by an unexpected aggregate shock at time 0.

4 The algorithm

4.1 Leveraging linearity of the Ramsey first order conditions

Once taken the first order conditions, the resulting system is composed by the $N_Z + N \times 3$ constraints of the problem, the first order condition with respect to the policy variable r_t, plus all the other $N_Z + N \times 3$ other first order conditions (i.e. with respect to all the aggregate and idiosyncratic variables). Due to the high number of non-linearities embedded in the drift and upwind functions, log-linearization around the steady state is not viable. Nuño et al. (2023) rely on the "perfect foresight" built-in Dynare routine, that considers the system simulated for T periods, with reversion to steady state by time T, and computes the solution through a Newton-type solver. This method turns out to be computationally demanding for models with no blocks solvable by closed form.

My approach builds on the observation that all the first order conditions of problem (2.0.15)-(2.0.22) are *linear* in the multipliers, due to the fact that the latter enter the Lagrangian multiplied by the constraints of the model, according to the following structure:

$$\mathcal{L} = \sum_t \beta^t \left[f_0(Z_t) + \xi_t' m_t \right] \tag{4.1.1}$$

where ξ_t is a vector of multipliers and m_t is a vector comprising constraints (2.0.16)-(2.0.22). The overall system of equations can therefore by rewritten in three blocks: 1) the *nonlinear* block, that comprises the constraints of the Ramsey planner, i.e. equations (2.0.16)-(2.0.22); 2) the first order condition with respect to the policy variable r_t; 3) the *linear* block, which gathers all the other first order conditions of the Ramsey problem, under the form of a *linear* system in the multipliers:

$$A_t \xi_{t+1} + B_t \xi_t + C_t \xi_{t-1} + d_t = 0 \quad \forall t \tag{4.1.2}$$

where the matrices A_t, B_t, C_t and the vector d_t are time-varying and with entries given by parameters and non-multiplier variables, while ξ_t is the vector of the $N_Z + N \times 3$ multipliers. Let us define ξ_t^F as the forward looking multipliers, ξ_t^B as the backward-looking multipliers, ξ_t^C as the purely contemporaneous multipliers. Leveraging linear algebra, equation (4.1.2) can be simplified into the following dynamic system:

$$\begin{bmatrix} \xi_{t+1}^F \\ \xi_t^B \end{bmatrix} = P_t \begin{bmatrix} \xi_t^F \\ \xi_{t-1}^B \end{bmatrix} + p_t \tag{4.1.3}$$

$$\xi_t^C = Q_t \begin{bmatrix} \xi_t^F \\ \xi_{t-1}^B \end{bmatrix} \tag{4.1.4}$$

$$\xi_t = \begin{bmatrix} \xi_t^F \\ \xi_t^C \\ \xi_t^B \end{bmatrix} \tag{4.1.5}$$

Where again matrices P_t, Q_t and vector p_t are filled with parameters and non-multiplier variables.

Let us first assume that the time varying matrices and vectors of the system above are all known: the system can be solved starting straight from (4.1.3), which delivers the sequences of ξ_t^F and ξ_t^B, which in turn allow to solve for ξ_t^C through (4.1.4) and finally for the overall ξ_t through (4.1.5). It is here important to remark that the system (4.1.3) presents almost the same structure of generalised state space form, solvable through Blanchard-Kahn method, if only P_t and p_t were constant over time. In order to transform the system in this sense, I take a first order approximation of equation (4.1.3) around the steady state, obtaining the state space model:

$$\begin{bmatrix} \hat{\xi}_{t+1}^F \\ \hat{\xi}_t^B \end{bmatrix} = P \begin{bmatrix} \hat{\xi}_{t+1}^F \\ \hat{\xi}_t^B \end{bmatrix} + \tilde{p}_t \tag{4.1.6}$$

where "hat" variables are deviations in level, P is the P_t matrix evaluated at steady state and $\tilde{p}_t = \xi$ is a vector of residual time-varying terms defined as follows:

$$\tilde{p}_t = \begin{bmatrix} \hat{P}_{11t} & \hat{P}_{12t} & \dots & \dots & \dots \\ \hat{P}_{21t} & \dots & & & \\ \dots & & \dots & & \\ \dots & & & \dots & \\ \dots & & & & \dots \end{bmatrix} \xi + \hat{p}_t \tag{4.1.7}$$

System (4.1.6) can be solved in an computationally effective way through Schur decomposition (see Klein (2000)). The condition for a unique solution as in the Blanchard-Kahn approach, requires Assumption 4.1.1 holding:

Assumption 4.1.1. *The number of eigenvalues of matrix P higher than one in modulus is equal to the number of forward-looking multipliers*

Summing up the solution algorithm for the linear block of the Ramsey problem, provided that we know matrices A_t, B_t, C_t, Q_t and the vector d_t, and given the "timeless" intial conditions $\xi_{-1}^B = \xi_B$, we can compute the state-space formulation for forward and backward looking variables in deviations ((4.1.6)) by applying Schur decomposition (see Klein (2000)). We can then retrieve the sequences of ξ_t^F and ξ_t^B in levels by applying $\xi_t^F = \xi^F + \hat{\xi}_t^F$ and $\xi_t^B = \xi^B + \hat{\xi}_t^B$, where ξ^F and ξ^B are the vectors of steady state forward and backward looking multipliers ; finally, through equations (4.1.4)-(4.1.5), we recover the whole sequence of multipliers ξ_t in levels. Of course, due to the linear approximation carried out to apply Schur decomposition, the resulting solution for $\{\xi_t\}_t$ is a linear approximation of the "true" solution as well.

4.2 A two-stages iterative approach

In the previous section I showed how a sizable part of the equilibrium equations of the models are efficiently and simply solved once having considered their linearity in the co-states (multipliers) variables. I presented the solution strategy by assuming in the first place that the matrices A_t, B_t, C_t, Q_t and the vector d_t in equations (4.1.3)-(4.1.4) are known. However this is not the case, since their entries are made up by non-multiplier variables which are endogenous as well. These variables would be in principle recovered by solving separately the first block of the paper, i.e. the Ramsey planner's constraints (equations (2.0.16)-(2.0.22)). This block can indeed be solved separately from the first order conditions' block, *given* a path for the policy variable r_t. Equations (2.0.16)-(2.0.22) are indeed equilibrium conditions of an economy without optimal policy being put in place, and can be solved by a time-effective guess-and-update routine on an aggregator variable $\tilde{U}_t$, as in Achdou et al. (2021) (the typical example for this kind of looping solution is the iterated update of aggregate capital in the Aiyagari model, starting from an initial guess).

Therefore so far I established that, for a given path for r_t, the constraints block of the problem can be solved by nonlinear guess-and-verify techniques, delivering the entries of the matrices A_t, B_t, C_t, Q_t and d_t, which allow in turn to solve the linear first order conditions block (up to a first order approximation) and obtain the value of the multiplier at each point in time. The entire set of equilibrium equations, except for the first order condition with respect to r_t, is therefore solved in *two stages* for a given path for the policy variable r_t. If we initiate the path for r_t by an initial guess, we can use the still unaddressed condition, i.e. the derivative with respect to r_t, in order to update the guess, and then launch again the two-stages algorithm. In this sense, my solution approach is *iterative*. Defining as $g(r_t)$ the derivative of the Ramsey lagrangian with respect to to r_t the update of the solution is

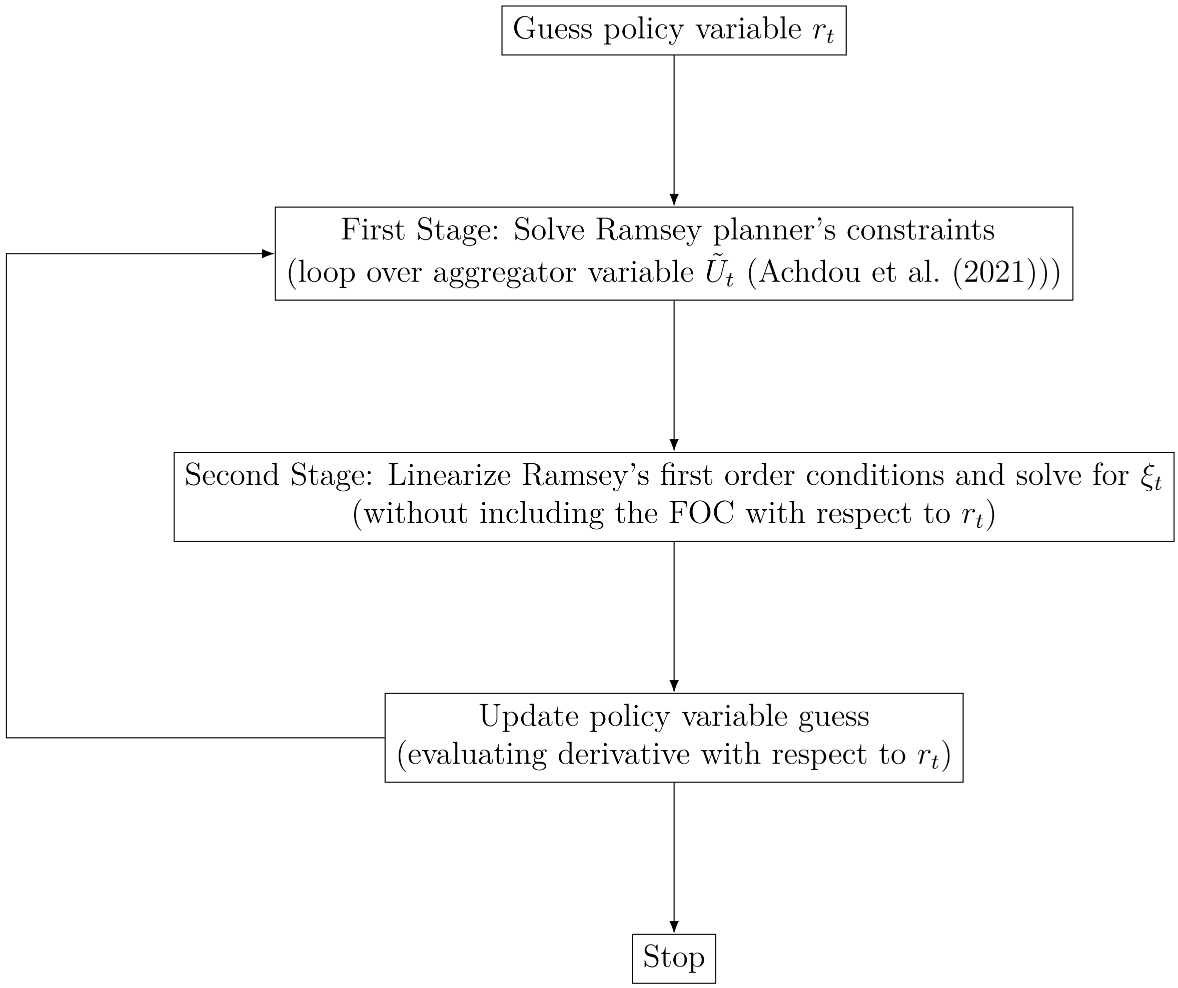


Figure 4.1: The two-stages iterative algorithm

carried out by the following rule:

$$r_t' - r_t = K\phi(r_t)\ \forall t \tag{4.2.1}$$

where K is a positive parameter, and $\phi(r_t)$ is the first order condition of problem (2.0.15)-(2.0.22) with respect to r_t. When the derivative is positive, it means that there is scope for a further interest rate rise a time t to increase welfare, while when $g(r_t) < 0$, a higher welfare is achieved by decreasing r_t. The algorithm is iterated until some measure of accuracy of satisfaction of first order conditions is high enough (for instance I check whether $\sum_{t=0}^{T} |(g(r_t)|$ is close enough to 0).

5 An application to energy price shocks in a HANK open economy

In this section I provide an application of the solution algorithm to a general framework building on Auclert et al. (2023a), that explores the role of monetary policy in addressing foreign energy price shock in a small open HANK economy.

5.1 Domestic households

A small open economy (the "domestic" economy) is populated by a unit mass of households, heterogeneous with respect to their wealth and their labor productivity. Households display Greenwood–Hercowitz–Huffman preferences (GHH), characterized by utility from consumption and disutility from labor[23] . They also bear quadratic costs from wage inflation. The discounted utility of a generic household i in economy j reads:

$$E_0 \int_0^\infty e^{\rho t} \left[\frac{1}{1-\sigma} \left(c_t - \chi \frac{n_t^{1+\phi}}{1+\phi} \right)^{1-\sigma} - \frac{\Psi}{2} \pi_t^{W2} \right] dt \tag{5.1.1}$$

where ρ is the subjective discount rate, and σ is the coefficient of risk aversion. Wage inflation π_t^W is given by $\pi_t^W = dW_t/W_t$, where W_t is the nominal wage in the domestic economy. Consumption level of the household is given by a CES aggregator of consumption of a non-energy good, c_{HFt}, and consumption of the energy good supplied by the foreign economy, c_{Et}:

$$c_t = [(1-\alpha_E)^{\frac{1}{\epsilon}} c_{HFt}{}^{\frac{\epsilon-1}{\epsilon}} + \alpha_E^{\frac{1}{\epsilon}} c_{Et}{}^{\frac{\epsilon-1}{\epsilon}}]^{\frac{\epsilon}{1-\epsilon}} \tag{5.1.2}$$

Where ϵ is the elasticity of substitution between energy and non-energy goods. The CES utility function gives rise to the standard formulation for the domestic price index:

$$p_t = [(1-\alpha_E) p_{HFt}^{1-\epsilon} + \alpha_E p_{Et}^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{5.1.3}$$

[23]A discussion of the advantages of adopting GHH preferences in this setting is carried out in section 5.3.

where p_{Ht} and p_{Et} are respectively the prices of the non-energy and energy goods.

The non-energy good c_{HFt} is in turn itself a CES aggregator of a home-produced good c_{Ht} and foreign-produced good c_{Ft}:

$$c_{HFt} = [(1-\alpha)^{\frac{1}{\eta}} c_{Ht}{}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{Ft}{}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{5.1.4}$$

Where η is the elasticity of substitution between the domestic and foreign good. The price of the non-energy good can be derived as:

$$p_{HFt} = [(1-\alpha)p_{Ht}^{1-\eta} + \alpha p_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}} \tag{5.1.5}$$

Labor supply n_t is a bundle of a unit mass of labor varieties k supplied by the household:

$$n_t = \int n_{kt} dk \tag{5.1.6}$$

where each variety's supply n_{kt} is determined by a union, whose optimization problem will be discussed later.

Households can invest in a risk-free asset a_t; asset holding evolves according to:

$$\dot{a}_t = z_t w_t n_t + r_t a_t - \frac{p_{HFt}}{p_t} c_{HFt} - \frac{p_{Et}}{p_t} c_{Et} + d_t \tag{5.1.7}$$

where $w_t \equiv W_t/p_t$ is the real wage, d_t is the dividend amount rebated to the household, and z_t is an idiosyncratic productivity shock that follows a diffusion process with parameters $\mu(z), \varsigma^2$. The law of motion of a_t is constrained by the following borrowing limit:

$$a_t \geq \bar{a} \tag{5.1.8}$$

Household aims at maximizing lifetime utility (5.1.1) by choosing consumption and asset holding under constraints (5.1.7) and (5.1.8). The household also solves an intratemporal optimal consumption variety selection problem, which delivers the standard demand formulation for energy, and domestic and foreign goods:

$$c_{Et} = \alpha_E \left(\frac{p_{Et}}{p_t}\right)^{-\epsilon} c_t \tag{5.1.9}$$

$$c_{Ht} = (1-\alpha_E)\left(\frac{p_{HFt}}{p_t}\right)^{-\epsilon} (1-\alpha)\left(\frac{p_{Ht}}{p_{HFt}}\right)^{-\eta} c_t \tag{5.1.10}$$

$$c_{Ft} = (1-\alpha_E)\left(\frac{p_{HFt}}{p_t}\right)^{-\epsilon} \alpha \left(\frac{p_{Ft}}{p_{HFt}}\right)^{-\eta} c_t \tag{5.1.11}$$

The intertemporal problem of the household can be formulated recursively under the form of a Hamiltonian-Bellman-Jacobi equation for household with productivity realization z and asset holding a:

$$\rho V_t(a,z) = \max_{a_t,c_t}\left[\frac{1}{1-\sigma}\left(c_t - \chi\frac{n_t^{1+\phi}}{1+\phi}\right)^{1-\sigma} - \frac{\Psi}{2}\pi_t^{W2} + s_t(a,z)\frac{\partial V_t}{\partial a}\right] + \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2} + \frac{\partial V_t(a,z)}{\partial t} \tag{5.1.12}$$

The drift function $s(a,z)$ is given by $s_t(a,z) = z_t w_t n_t + r_t a_t - \frac{p_{HFt}}{p_t}c_{Ht} - \frac{p_{Et}}{p_t}c_{Et} + d_t$. Optimality leads to the standard first order condition for consumption:

$$c_t(a,z)^{-\sigma} = \frac{\partial V_t(a,z)}{\partial a} \tag{5.1.13}$$

We can also define the joint density of wealth and productivity $f_t(a,z)$ (and the corresponding cumulative distribution function $F_t(a,z)$). Its dynamics over time are governed by a Kolmogorov-forward equation:

$$\frac{\partial f_t(a,z)}{\partial t} = -\frac{\partial}{\partial a}[s_t(a,z)f_t(a,z)] - \mu(z)\frac{\partial V_t}{\partial z} + \frac{\varsigma^2}{2}\frac{\partial^2 V_t}{\partial z^2} \tag{5.1.14}$$

I will assume that the process for z is normalized such that the idiosyncratic productivity realizations aggregate to one:

$$\int_0^1 z f_t(a,z)dz = 1 \tag{5.1.15}$$

Lastly, let us define aggregate consumption in the domestic economy:

$$C_t = \int_0^1 c_t(a,z) f_t(a,z) dadz \tag{5.1.16}$$

5.2 Firms

The domestic good is a produced by a competitive mass of firms[24] which operate under a technology linear in aggregate labor N_t and aggregate productivity A:

$$Y_{Ht} = AN_t \tag{5.2.1}$$

Aggregate labor N_t is a Dixit-Stigliz aggregator of labor varieties:

$$N_t = \left(\int N_{kt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{5.2.2}$$

where N_{kt} is the aggregate labor demand for variety k. The zero profit condition equates the real wage per unit of output to the price of the domestic good:

$$w_t \frac{1}{A} = \frac{p_{Ht}}{p_t} \tag{5.2.3}$$

Notice that this implies zero dividends ($d_t = 0$). Firms also face an optimal choice of the labor variety mix, leading to the standard optimal labor variety demand:

$$N_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon} N_t \tag{5.2.4}$$

where W_{kt} is the nominal wage in labor market k.

[24] Auclert et al. (2023a) instead assumes a monopolisticaly competitive sector with flexible prices. This allows to obtain endogenous markups under domestic currency export pricing - an extension here not explored

5.3 Unions

Each union k determines the labor supply n_{kt} - equal across all households - standing ready to satisfy labor demand:

$$n_{kt} = N_{kt} \tag{5.3.1}$$

The union chooses the nominal wage W_{kt} at which it supplies labor in order to maximize the average utility of agents, where inflation disutility is measured by a coefficient ψ that needs not to be equal to the households' "true" inflation cost Ψ[25]:

$$\int_0^1 \left(\frac{1}{1-\sigma} \left(c_t - \chi \frac{n_t^{1+\phi}}{1+\phi} \right)^{1-\sigma} - \frac{\Psi}{2} \pi_t^{W2} \right) dF(a,z) \tag{5.3.2}$$

As shown in the appendix, solving the maximization problem and imposing symmetry across unions gives rise to a standard New Kynesian Phillips curve in the labor market:

$$\rho \pi_t^W = \frac{\varepsilon}{\psi} N_t \left(\chi N_t^{\phi} - \frac{\varepsilon - 1}{\varepsilon} w_t \right) + d\pi_t^W \tag{5.3.3}$$

From the equation above it can be seen how an increase in labor demand raises households' marginal disutility of labor and drives unions to revise nominal wages upwards, causing wage inflation to rise. Notice that the adoption of GHH preferences at the household level allows to factor out any role for heterogeneity in the New Keynesian Phillips curve (otherwise the union should take into account the average marginal utility from consumption of households, as in Auclert, Rognlie, and Straub (2018)). Wolf (2023) follows a similar approach, by assuming, instead of GHH, that the union maximizes the welfare of a hypothetical household consuming the average level if consumption in the economy.

5.4 Foreign economy

The rest of the world is populated by a representative household with constant exogenous consumption C^*. It has exactly symmetrical preferences to the domestic households, except for the fact that it does not consume energy. Their consumption basket is therefore symmetric

[25] Allowing for different inflation costs of households and unions is a handy feature for computational purposes, as discussed in section 6.3

to (5.1.4)

$$c_t = [\alpha^{\frac{1}{\eta}} c_{Ht}{^*}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_{Ft}{^*}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{1-\eta}} \tag{5.4.1}$$

Exported domestic goods are priced in foreign currency, and the law of one price holds. Given these assumptions, foreign households feature a demand for the domestic good which mirrors (5.1.10):

$$c^*_{Ht} = \alpha \left(\frac{p^*_{Ht}}{p^*_t} \right)^{-\eta} C^* \tag{5.4.2}$$

Where p^*_{Ht} and p^*_t are the home good price and the foreign price level in foreign currency, respectively. The foreign price index p^*_t is given by the standard CES formulation, symmetric to (5.1.3):

$$p^*_t = [(1-\alpha) p^{*1-\eta}_{Ft} + \alpha p^{*1-\eta}_{Ht}]^{\frac{1}{1-\eta}} \tag{5.4.3}$$

with p^*_{Ft} being the price of the foreign good in foreign currency; I assume p^*_{Ft} to be itself a Dixit-Stigliz aggregator of a mass of varieties N^*, i.e. $p^*_{Ft} = \left(\int\limits_0^N \tilde{p}^{*1-\eta}_{Ft}(n) dn \right)^{\frac{1}{1-\eta}}$. For $N \to \infty$, imposing symmetry across the foreign varieties' prices $\tilde{p}^*_{Ft}(n)$ implies $p^*_{Ft} \to p^*_t$ - namely, the foreign economy is "big" with respect to the domestic one, so its price index is not affected by domestic economy's price fluctuations.

Monetary policy in the foreign economy ensures full price stability:

$$p^*_t = p^*_{Ft} = 1 \tag{5.4.4}$$

where I normalize p^* to 1. Applying the law of one price, we obtain:

$$p^*_{Ht} = p_{Ht} / \delta_t \tag{5.4.5}$$

$$p_{Ft} = p^*_{Ft} \delta_t = \delta_t \tag{5.4.6}$$

where δ_t is the nominal exchange rate. Defining the real exchange rate as $Q_t = \frac{p_t}{\delta_t p^*_t} = \frac{p_t}{\delta_t}$, we can rewrite foreign demand (5.4.2) as:

$$c^*_{Ht} = \alpha \left(\frac{p_{Ht}}{p_t} Q_t \right)^{-\eta} C^* \tag{5.4.7}$$

From the equation above, it can be noticed how a real appreciation (i.e. an increase in Q_t), leads foreign consumers to express a lower demand for domestic goods, which become relatively less convenient.

In the light of the foreign price stability and law of one price assumptions, and using the definition $Q_t = \frac{p_t}{\delta_t}$ we can also rearrange the domestic price index (5.1.3) formulation to obtain the real price of energy and the domestic and foreign goods as a functions of real exchange rate Q_t and energy price p^*_{Et}:

$$\frac{p_{Et}}{p_t} = p^*_{Et}/Q_t \equiv p_E(Q_t, p^*_{Et}) \tag{5.4.8}$$

$$\frac{p_{Ht}}{p_t} = \left[\frac{1}{1-\alpha}\left(\frac{1-\alpha_E p_E(Q_t, p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{\frac{1-\eta}{1-\epsilon}} - \frac{\alpha}{1-\alpha} p_F(Q_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv p_H(Q_t, p^*_{Et}) \tag{5.4.9}$$

$$\frac{p_{Ft}}{p_t} = 1/Q_t \equiv p_F(Q_t) \tag{5.4.10}$$

The real price of energy p_{Et}/p_t depends positively on the foreign nominal price of energy p^*_{Et}, and negatively on the real exchange rate Q_t: domestic goods' appreciation indeed makes imported energy relatively cheaper. p_{Ft}/p_t depends negatively on the real exchange rate as well: real appreciations indeed reduce the price of the foreign good relatively to the domestic one. The real price of the domestic good, p_{Ht}/p_t, depends negatively on both the real price of energy and the real price of foreign goods: therefore, a real appreciation (i.e. and increase in Q_t) boosts the real price of domestic goods by making energy and foreign goods relatively cheaper. An increase in energy price p^*_{Et} instead lowers p_{Ht}/p_t by reducing the relative price of domestic goods with respect to energy.

5.5 Financial intermediaries

The real risk-free asset claims in the domestic economy are supplied by competitive risk-neutral financial intermediaries, which can invest in two types of nominal assets: a domestic one, yielding i_t return in domestic currency, and a foreign one, yielding i^*_t in foreign currency. Following Galí (2020), assume a behavioral wedge in expectations, such that the expected log real exchange rate of $\log Q_{t+\Delta}$ is systematically biased to the level $\kappa \log Q_{t+\Delta}$: for a proper choice of κ, the assumption guarantees the correct number of explosive eigenvalues in the system of first order conditions of the discretized problem (Assumption 4.1.1). In discrete time, the expected log-gain for the financial intermediaries in investing in the foreign asset would be given by the log-foreign interest rate corrected by the log-change in the real exchange rate (recalling that foreign inflation is equal to 0):

$$\begin{aligned} & i^* - (\kappa \log Q_{t+1} - \log Q_t) = \\ & i^* - (\kappa(\log Q_{t+1} - \log Q_t) - (\kappa - 1) \log Q_t) \end{aligned} \tag{5.5.1}$$

The intermediaries seek to maximise profit, given by the total revenue from the investment in the domestic and foreign asset, net of the payment due to the household (i.e. the real rate r_t times the aggregate investment of domestic agents); formally, the period t problem of the intermediary writes:

$$\begin{aligned} \max_{h_t} \Bigg[& h_t(i_t - \pi_t) \int a dF_t(a,z) + (1 - h_t) \left[i_t^* - (\kappa(\ln \dot{Q}_t) - (\kappa - 1) \ln Q_t) \right] \int a dF_t(a,z) \\ & - r_t \int a dF_t(a,z) \Bigg] \end{aligned} \tag{5.5.2}$$

where h_t and $1 - h_t$ denote resepectively the fraction of aggregate investment allocated in home and foreign assets, and $\pi_t = dP_t/P_t$ and $\pi_t^* = dP_t^*/P_t^* = 0$ (the latter inflation term being 0 by the foreign price stability assumption). Notice that the effective return of the foreign asset takes into account the movements in the inverse of the real exchange rate over time, $(\ln \dot{Q}_t)$. To rule out arbitrage opportunities (which would imply an optimal choice of $h_t \to \infty$ or $h_t \to -\infty$), the return from the two assets need to be equal (uncovered interest parity, "UIP"):

$$i_t - \pi_t = i_t^* - (\kappa(\ln \dot{Q}_t) - (\kappa - 1) \ln Q_t) \tag{5.5.3}$$

By perfect competition intermediaries make zero profit. Therefore, equating the argument of objective (5.5.2) to zero, and substituting for condition (5.5.3), we can pin down r_t as equal to the nominal domestic rate net of inflation:

$$r_t = i_t - \pi_t \tag{5.5.4}$$

5.6 Central bank

Public sector is in charge to set the nominal interest rate on the domestic risk free asset i_t. Following Auclert et al. (2023a), I assume that the central bank sets

$$i_t = \pi_t + r'_t \tag{5.6.1}$$

where r'_t is the targeted real interest rate.

5.7 Equilibrium

Given a path for the interest rates i_t and energy prices p^*_{Et}, an initial distribution of wealth and productivity $f_0(a, z)$, and foreign consumption C^*, a competitive equilibrium is defined as a path for households' choices (a_t,c_{Ht},c_{Ft},c^*_{Ht}), firms' choices (N_t), unions' choices (n_t,π_t), prices ($r_t, p_H(Q_t, p^*_{Et}), p_E(Q_t, p^*_{Et}), p_F(Q_t), w_t, Q_t$), aggregate quantities (Y_{Ht},C_t) and distributions ($f_t(a, z)$, consistent with the Kolmogorov forward dynamics (5.1.14)) such that households and firms optimize, and the following market clearing conditions in the goods and labor market are satisfied:

$$\begin{aligned}
Y_{Ht} =&(1-\alpha_E)\left(\frac{p_{HFt}}{p_t}\right)^{-\epsilon}(1-\alpha)\left(\frac{p_{Ht}}{p_{HFt}}\right)^{-\eta}c_t + \alpha\left(\frac{p^*_{Ht}}{p^*_t}\right)^{-\eta}C^* = \\
=&(1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t,p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha p_F(Q_t)^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}C_t + \alpha\left(p_H(Q_t,p^*_{Et})Q_t\right)^{-\eta}C^*
\end{aligned} \tag{5.7.1}$$

$$Y_{Ht} = AN_t \tag{5.7.2}$$

$$N_t = n_t \tag{5.7.3}$$

where (5.7.1) is market clearing in the domestic consumption good's market[26], (5.7.2) is market clearing the labor market, and (5.7.3) stands for the assumptions of households complying with the unions' choices in setting their labor supply (by symmetry among unions, $n_{kt} \equiv n_t \ \forall k$). The goods market clearing condition (5.7.1) in particular is given by the sum

[26] Condition (5.7.1) is retrieved by substituting for p_{HFt}/p_t and p_{Ht}/p_{HFt} by using the price indexes (5.1.3) and (5.1.5) and results (5.4.8)-(5.4.10).

of domestic demand (the first term on the right hand side) and foreign demand (the second term on the right hand side). Notice that, for the elasticities $\epsilon \to 0$ and $\eta \to 0$, the impact of energy price and the real exchange rate on demand goes to zero as well, as substitution effects among goods are muted.

5.8 Steady state

In steady state $p_H(Q)$ and w are determined uniquely by the steady state Q. Therefore, by (5.1.7), each household's consumption in home and foreign good $c_H(a,z), c_F(a,z)$ - and also the overall consumption basket $c(a,z)$ - is determined uniquely by Q, the steady state real rate r and the states a, z. This implies that the drift function $s(a,z)$ depends only on r and Q. Then, by setting to 0 the left hand side of (5.1.14), we can obtain the whole steady state distribution $f(a,z)$ as a function of r and Q.

Aggregate consumption C is defined as $C = \int c(a,z)f(a,z)dadz$; since both the idiosyncratic consumption levels $c(a,z)$ and the distribution $f(a,z)$ are determined by r and Q, we can retrieve the following parsimonious functional formulation for C:

$$C = C(r, Q) \tag{5.8.1}$$

Given r, equations (5.3.3),(5.7.1),(5.7.2),(5.8.1) and (5.5.3) define a system of five equations in five variables: π, Y_H, N, C, Q. The model is calibrated such that, whenever is hit by a temporary shock to policy (i_t) or exogenous variables (i_t^*), the dynamics revert to the *initial* steady state: heterogeneous agents small open economy models can indeed feature stable steady states thanks to the convergence property of the asset distribution (beyond Auclert et al. (2023a), see also Nuño and Thomas (2022) de Ferra et al. (2020)[27]).

[27]Then it is not needed to resort to debt-elastic interest rates as commonly done in representative agent models with incomplete markets

6 Optimal monetary policy

6.1 The optimal policy problem

The monetary authority sets a path for the nominal interest rate i_t in order to maximise the average utility of all the equally weighted households. The Ramsey problem is given, under the Hamiltonian form, by

$$\mathcal{L} = \int_0^\infty e^{\rho t} \left[\int_0^1 \left(\frac{1}{1-\sigma} \left(c_t(a,z) - \chi \frac{n_t^{1+\phi}}{1+\phi} \right)^{1-\sigma} - \frac{\Psi}{2} \pi_t^{W2} \right) dF(a,z) da dz + \xi_t' m_t \right] dt \quad (6.1.1)$$

where m_t is a vector of constraints and ξ_t is a vector of multipliers. The entries of vector m_t are the equilibrium conditions derived in the previous section: the Kolmogorov-forward equation (5.1.14), the Bellman equation (5.1.12), the intertemporal optimality condition (5.1.13), market clearing conditions (5.7.1), (5.7.2), (5.7.3), domestic wage and price determination (5.2.3) and (5.4.9), the new Keynesian Phillips curve (5.3.3), UIP (5.5.3), zero profit condition of the intermediaries (5.5.4), and monetary policy (5.6.1).

The discretized version of problem (6.1.1) is a Lagrangian of which I can derive first order conditions with respect to all the variables of the problem. According to the notation introduced in section 2, I label by N the number of gridpoints spanning every possible combination of idiosyncratic states a, z; moreover, $N_Z = 4$ stands for the number of aggregate variables N_t, π_t, w_t, Q_t (after simple rearrangements that allow to remove C_t and Y_{Ht} from the final set of erquilibrium conditions). The discretized problem is reported below:

$$\mathcal{L} = \sum_0^\infty \beta^t \left[\sum_{i,j} f_{i,j}^t \left(\frac{1}{1-\sigma} \left(c_{i,j}^t - \chi \frac{N_t^{1+\phi}}{1+\phi} \right)^{1-\sigma} - \frac{\Psi}{2} \pi_t^{W2} \right) + \xi_t' m_t \right] \quad (6.1.2)$$

where $\beta = 1/(1+\rho\Delta)$, and $f_{i,j}^t$ and $c_{i,j}^t$ are respectively the discretized distribution and consumption level for states $a = i, z = j$. The vector of constraints m_t is given by the discretized equilibrium conditions, as follows:

$$
m_t = \begin{bmatrix}
\forall i,j \begin{cases}
\frac{f^t_{i,j}-f^{t-1}_{i,j}}{\Delta} - & \Big[-\frac{f^t_{i,j}[N_t w_t z_j + r a_i - c^t_{i,j}]^+ - f^t_{i-1,j}[N_t w_t z_j + r a_{i-1} - c^t_{i-1,j}]^+}{\Delta a} \\
& -\frac{f^t_{i+1,j}[N_t w_t z_j + r a_{i+1} - c^t_{i+1,j}]^- - f^t_{i,j}[N_t w_t z_j + r a_i - c^t_{i,j}]^-}{\Delta a} \\
& -\frac{f^t_{i,j}\mu(z_j) - f^t_{i,j-1}\mu(z_{j-1})}{\Delta z} + \frac{f^t_{i,j+1}\varsigma^2 + f^t_{i,j-1}\varsigma^2 - 2 f^t_{i,j}\varsigma^2}{2(\Delta z)^2} \Big]
\end{cases} \\
\forall i,j \begin{cases}
\frac{v^{n+1}_{i,j}-v^{n}_{i,j}}{\Delta} + \rho v^{n+1}_{i,j} - & \Big[u\left(c^n_{i,j}\right) + \frac{v^{t+1}_{i+1,j}-v^{t+1}_{i,j}}{\Delta a}\left(N_t w_t z_j + r a_i - c^t_{i,j}\right)^+ + \\
& + \frac{v^{t+1}_{i,j}-v^{t+1}_{i-1,j}}{\Delta a}\left(N_t w_t z_j + r a_{i-1} - c^t_{i-1,j}\right)^- \\
& + \frac{v^{t+1}_{i,j+1}-v^{t+1}_{i,j}}{\Delta z}\mu^+_j + \frac{v^{t+1}_{i,j}-v^{t+1}_{i,j-1}}{\Delta z}\mu^-_j + \frac{\varsigma^2}{2}\frac{v^{t+1}_{i,j+1}-2v^{n+1}_{i,j}+v^{t+1}_{i,j-1}}{(\Delta z)^2} \Big]
\end{cases} \\
\forall i,j \Big\{ \left(c^t_{i,j} - \chi\frac{N_t^{1+\phi}}{1+\phi}\right)^{-\sigma} - \left[\frac{v^t_{i+1,j}-v^t_{i,j}}{\Delta a}[N_t w_t z_j + r a_i - c^t_{i,j}]^+ + \frac{v^t_{i,j}-v^t_{i-1,j}}{\Delta a}[N_t w_t z_j + r a_i - c^t_{i,j}]^-\right] \\
A N_t - \Big[(1-\alpha_E)\left(\frac{1-\alpha_E p_E(Q_t,p^*_{Et})^{1-\epsilon}}{1-\alpha_E}\right)^{-\frac{\epsilon}{1-\epsilon}}(1-\alpha)\left(\frac{1-\alpha p_F(Q_t)^{1-\eta}}{1-\alpha}\right)^{-\frac{\eta}{1-\eta}}\sum_{i,j} f^t_{i,j} c^t_{i,j} + \\
+\alpha\left(p_H(Q_t,p^*_{Et})Q_t\right)^{-\eta} C^* \Big] \\
w_t - A p_H(Q_t, p^*_{Et}) \\
\rho\pi^W_t - \left[\frac{\varepsilon}{\psi}N_t\left(\chi N_t^{\phi} - \frac{\varepsilon-1}{\varepsilon}w_t\right) + \frac{\pi^W_{t+1}-\pi^W_t}{\Delta}\right] \\
r_t - \left[i^*_t - (\kappa(\ln \dot{Q}_t) - (\kappa-1)\ln Q_t)\right]
\end{bmatrix}
$$

where $v^t_{i,j}$ is the value function for states i, j at time t. The entries of m_t are given by (in order): the Kolmogorov-Forward condition, the HBJ equation, the optimality condition of households. These are the $N \times 3$ conditions (recalling that N is equal to the number of all possible ij combinations). The remaining $N_Z = 4$ constraints are the market clearing in the goods-labor markets, the firms' pricing condition, the Phillips curve and the UIP condition. The resulting set of first order conditions of problem (6.1.2) includes N conditions for consumption $c^t_{i,j}$, and the same number of equations for $v^t_{i,j}$ and $f^t_{i,j}$ (summing to $N \times 3$ conditions). Moreover, N_Z conditions hold for each of the aggregate variables, plus one first order condition for the policy variable r_t. The system of first order conditions at time t can be summed up in the following compact form (see equation (4.1.2)):

$$
A_t\xi_{t+1} + B_t\xi_t + C_t\xi_{t-1} + d_t = 0 \quad \forall t \tag{6.1.3}
$$

where $A_t, B_t.C_t, d_t$ are matrices filled with non-multiplier variables or parameters.

6.2 Timeless perspective

In expression (6.1.1) all the forward-looking variables (i.e, $V_t(a,z), \pi_t, Q_t$) show up both in m_t and m_{t-1}: therefore, for $t = 0$, some elements of ξ_{-1} show up in the system. Following the "timeless penalties" approach described in section 3, I set $m_{-1} = m$, so that the initial multipliers of the forward-looking variables are at their steady state values. This is the timeless perspective which considers optimal policy as having been implemented since far in the past.

From the discussion above, in order to solve for the dynamics of the model we need first to solve it in steady state to obtain the stationary vector of costates m, containing the initial condition for the multipliers of the forward looking variables. In line with section 2, the steady state system display $2 \times (N_Z + N \times 3) + 1$ equations in $2 \times (N_Z + N \times 3) + 1$ unknowns (the $+1$ standing for the steady state policy variable $\bar{r}$).

Now that, once solved for the steady state of the model under the Ramsey plan, we are able to retrieve the initial multiplier vector $m_{-1} \equiv m$; therefore now it is possible to solve the system of first order conditions and constraints also over the dynamics, as we computed the necessary initial conditions on multipliers.

6.3 The algorithm

The goal of the algorithm is to solve the system of $2 \times (N_Z + N \times 3) + 1$ (constraint and first order conditions) to recover the pattern of the variables and the multipliers over time.

The starting point is to guess the pattern for the real rate r_t - which, using the $N_Z + N \times 3$ constraints contained in vector m_t, in turn delivers the pattern for all the $N_Z + N \times 3$ non-multiplier variables ($c_t(a,z)$,$V_t(a,z), q_t$, etc.). As discussed in section 2, to accomplish this task I adapt the standard routine by Achdou et al. (2021) to the current framework:

- Compute $\{N_t\}$ as a function of $\{Q_t\}, \{p^*_{Et}\}, \{C_t\}$ (see equilibrium conditions (5.7.1)-(5.7.3)). Note that $\{Q_t\}$ can be computed separately at once trough (5.5.3), given the sequence for $\{r_t\}$

- Use $\{C_t\}, \{N_t\}, \{w_t\}$ to compute π_t^W backward, starting from $\pi_T^W = 0$

- Compute $\pi_t = \frac{w_{t-1}}{w_t}\frac{1}{\pi_t^W}$ $\forall t \leq T$ (notice $w_{-1} = \bar{w}$)

- Solve the household problem backward, starting from the value functions in steady state.

- Compute the new path for aggregate consumption $\{C_t'\} = \{\sum a, zc_t(a,z)dadz\}$

- Update C_t as $C_t = (1-\vartheta)C_t + \vartheta C_t'$ for an arbitrary coefficient $\vartheta \in (0,1)$

- Iterate until convergence of $\max |\{C_t\} - \{C_t'\}|$ to some low threshold value.

Once computed the value of all the non-multipliers variable through the routine above ($N_t, \pi_t, w_t, Q_t,$ plus the $c_{i,j}^t, v_{i,j}^t, f_{i,j}^t$ for each of the N gridpoints), I can fill the entries of the matrices A_t, B_t, C_t, d_t of the system given by the $N_z + N \times 3 + 1$ first order conditions of the planner:

$$A_t\xi_{t+1} + B_t\xi_t + C_t\xi_{t-1} + d_t = 0 \ \ \forall t \tag{6.3.1}$$

where A_t, B_t, C_t is a $(N_z+N\times3+1)\times(N_z+N\times3+1)$ matrix and d_t is a $(N_z+N\times3+1)\times1$ vector; both have entries filled with of "parameters" (i.e. their entries contains either properly said parameters, or the guessed values of the non-multipliers variables). ξ_t is a $(N_z+N\times3+1)$ vector of multipliers. System (6.3.1) can be solved along the lines of section (4.1). In particular we can segment vector ξ_t into multipliers of forward-looking (V_t, q_t, π_t), backward looking (f_t), and contemporaneous variables to obtain subsystems of the type (4.1.3)-(4.1.4). The whole model is solved by the guess and update routine on r_t, starting from the $r_t = r \ \forall t$ guess, then going through the iteration on C_t to compute non-multiplier variables, and then solving (6.3.1) to retrieve the first order condition with respect to r_t and update its value accordingly, as outlined in Figure 4.1.

7 Simulation

Optimal monetary policy in the current setting is characterized by the tension between two forces: on ones side the need to sustain consumption by implementing a mild interest rate policy, and on the other side a monetary contraction motive, that allows to fight the real depreciation induced by the energy shock, by acting on the UIP margin (equation (5.5.3)). This trade-off is analyzed in depth in Corbellini (2024a), as a key determinant of the incentives of the policymaker in face of an energy shock. The remainder of this sections outlines the results in terms of optimal monetary policy when the model described so far is hit by a shock to energy price p^*_{Et}.

7.1 Calibration

I consider a time step $\Delta = 1/3$, implying monthly data. The asset continuum is discretized into $I = 30$ gridpoints, where $\bar{a} = -1$ and $amax = 5$, while the shock values z are discretized into 10 gridpoints. The coefficient of relative risk aversion σ and the inverse Frisch elasticity of labor supply ϕ are both set at 2. The discount factor ρ is 0.04, while $i^* = 0.03$. The diffusion process is characterized by $\varsigma = 1.5^2$ and $-\mu(z) = -\log(1/\exp(0.05))(z^{mean} - z)$. The labor disutility coefficient χ is set at 0.01, and the trade elasticities ϵ and η are set respectively at 0.1 and 0.51, following Auclert et al. (2023b). I assume a coefficient $\varepsilon/\psi = 0.02$ in the New Keynesian Phillips curve - a standard calibration choice. The steady state real price of energy $\bar{p}_E/Q$ is set to the high value 15: that, for presentation purposes, minimizes the extent of the "appreciation" channel of a monetary contraction compared to the debt-cost channel, providing an interesting result of in terms of desirability of a monetary loosening, displayed in the next section. The shock process is log-autoregressive with a low persistence parameter $\rho_E = 0.5$:

$$\log(p^*_{Et}) = \rho \log(p^*_{Et-1}) + \varepsilon_t \tag{7.1.1}$$

where ε_{t_0} is such to raise the initial price above steady state by 10% i.e. $\varepsilon_{t_0} = \log(0.5\bar{p}_E)$.

7.2 Results

Figures 7.1 and 7.2 report respectively the impulse response functions of aggregate variables and crossectional consumption for each level of wealth to the shock specified above. The outcome is compared to an inertial policy of the type considered in Auclert et al. (2023b), where the real interest rate is kept constant by the monetary authority.

The higher price of energy reduces the real value of wages; due to a low elasticity between energy and non-energy consumption ϵ, this effect on income is not balanced by a higher demand for domestic goods, therefore aggregate and crossectional consumption falls (as in Auclert et al. (2023b) and Corbellini (2024a)). In the optimal policy scenario, the central bank finds more convenient to sustain aggregate consumption by a real interest rate cut, which benefits more the household holding a lower asset level, rather then appreciating the real exchange rate by an interest rate hike. The consumption gain is present at all levels of wealth as, showed in the crossectional outcome figures. In particular, the first wealth decile, made up by borrowers, is benefited by lower debt costs. Inflation and real wages remain substantially unchanged from the inertial policy scenario, due to the modest effect on the real exchange rate of the implemented interest rate policy. Wage inflation slightly decreases in both policy cases, due to the fall in consumption and hence labor demand.

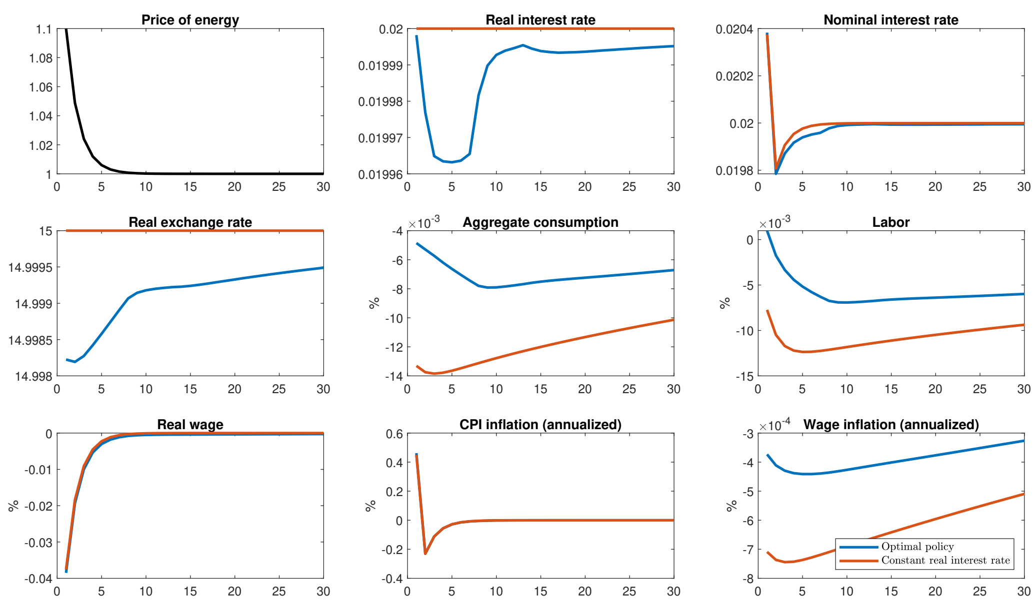


Figure 7.1: Optimal policy vs. Constant real interest rate. Aggregate variables impulse response

Figure 7.3 illustrates the welfare gain - in terms of of implementing optimal policy with respect to constant real rate policy: the gain is expressed simply as the difference between the average value function for each asset level at time 0, without considering the commitment

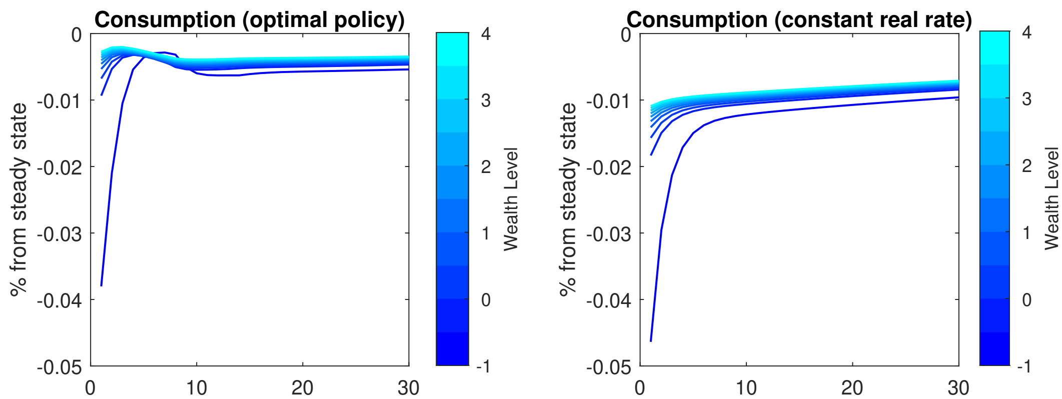


Figure 7.2: Optimal policy vs. Constant real interest rate. Crossectional consumption impulse response

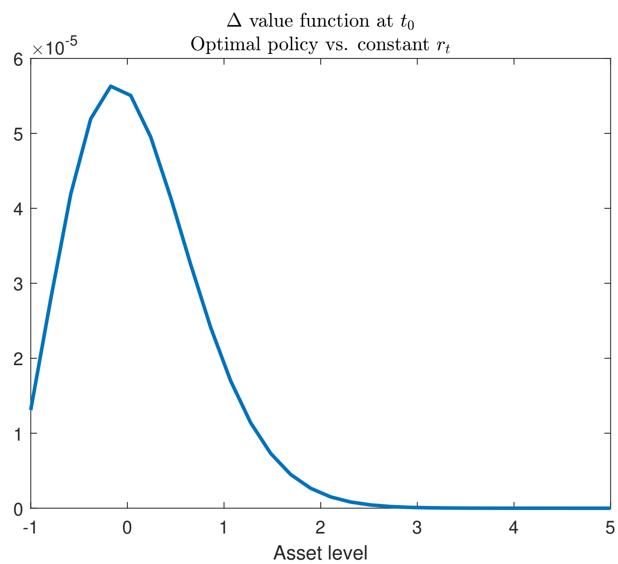


Figure 7.3: Optimal policy vs. Constant real interest rate. Welfare gains in terms of average value function across shocks z for each discretized asset level a

value of timeless penalties. Optimal policy benefits all the households at each asset level, consistently with the distributional outcome showed in Figure 7.2.

8 Conclusion

This paper presented an algorithm to compute optimal policy in HANK through a discretize-then-optimize routine in the spirit of Nuño et al. (2023). The contribution of the paper relies on exploiting the linearity of the first order conditions of the Ramsey planner in the co-states to deliver less-computationally intensive solution method. The algorithm proves effective in handling non-stylized models, as a small open economy HANK subject to energy shocks.

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