Three Essays on Prices and Selection

Inauguraldissertation zur Erlangung der Würde eines

Doctor rerum oeconomicarum

der Wirtschafts- und Sozialwissenschaftlichen Fakultät

der Universität Bern

vorgelegt von

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von Luzern, LU

2014

Die Fakultät hat diese Arbeit am 20. März 2014 auf Antrag der beiden Gutachter Prof. Dr. Winand Emons und Prof. Dr. Natalia Fabra als Dissertation angenommen, ohne damit zu den darin ausgesprochenen Auffassungen Stellung nehmen zu wollen.

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Preface

This thesis is a collection of three separate papers. Chapter 3 is based on joint work with Marc Blatter.

Although the whole thesis consists of theoretical essays in Industrial Organization, regarding their content the three chapters are quite heterogeneous: Chapter 1 is on quality and price discrimination; Chapter 2 discusses job markets with posted salaries; and Chapter 3 analyzes exclusivity agreements in upstream markets.

Nevertheless, all chapters seek answers to the same broad set of questions: How do firms set prices when their pricing decisions affect the composition of their demand? Moreover: What is the resulting allocation of buyers once all firms use prices as selection devices? This second question is of particular relevance when it comes to the evaluation of market outcomes and policy measures. From a Utilitarian point of view, prices themselves are irrelevant. Each dollar spent by a buyer ends up in the pocket of a seller. If, however, prices are such that a transaction does not take place at all, although the willingness-to-pay exceeds the marginal cost of production, the welfare implications are detrimental. Furthermore, even if all customers end up buying, their allocation may be inefficient. That is, there could exist another allocation where the sum of consumer surplus and producer surplus is higher than in the market equilibrium.

To reach such conclusions, it is essential to allow for settings where firms are able to set prices. As we discuss in the following, this does not require that firms have monopoly power. The presence of transportation cost (Chapter 1), asymmetrical information (Chapter 2), or additional means of competition (Chapter 3) provides firms with sufficient degrees of freedom. Of course, the existence of competitors (a feature of all our settings) limits their price-setting power: By defining the buyers' outside opportunities, competitors set the benchmark on how much a firm can charge. But beyond this, the customers' and trading partners' willingness-to-pay can also be affected by instruments under control of the firm. By selling products in qualitatively different varieties, for instance, a firm influences the goodness of fit between its customers' preferences and the products it sells. Or in a labor market, by refusing to hire applicants in the wake of an unpromising job talk, an employer can raise the salary it promises ex ante. Eventually, in the market for an intermediate good, by asking for exclusivity in a trade relationship, a retailer may obtain a better position in the downstream market, which in turn allows it to offer higher wholesale prices.

Once we know how firms set their prices, we are able to investigate equilibrium allocations: Who buys what? Who buys at all? And how much will be sold? We can distinguish between welfare levels of varying customer types, and we can compare the consumer surplus under varying assumptions about the market structure. We are also able to assess the profitability of a firm's business, depending on the cost of production and the state of the industry. Above all, we can evaluate the impact of policy measures on each of these dimensions. Who gains? Who loses? Is there scope for a Pareto improvement? Are there unintended collateral effects?

Before addressing these questions formally, we briefly discuss the models of each chapter separately. Again, we focus on the following questions: How do firms exert market power by setting prices, even in the presence of competitors? What impact have prices on the selection of customers and trading partners? How does the selection process shape the equilibrium allocation? And what benefits and side effects do policy interventions entail?

In Chapter 1, we consider a two-firm model where each firm sells a high-quality and a low-quality product. Customers differ with respect to their brand preferences and their attitudes towards quality. The standard result in the literature (Armstrong and Vickers, 2001; Rochet and Stole, 2002) is that firms add identical markups on their marginal cost of production, independently of the quality of the product. That is, their ability to set prices above marginal cost is completely determined by the degree of horizontal differentiation.

We show that this result crucially depends on the assumption that the customers' valuation of quality is identical across firms. We argue that the disutility from consuming a low-quality product rather than a high-quality product can be firm-specific. For instance, a low-quality "no-name" substitute may differ from a trademark brand in various vertical dimensions, such as performance, durability, product safety, conditions of production, quality of ingredients, ease of operation, and eco-friendliness. Since consumers are heterogeneous as well, it is possible that a customer who prefers the high-quality product at one firm prefers the low-quality version at the other firm. Hence, even when the price differentials between qualitatively varying products are identical across firms, high-quality and low-quality goods are in direct competition.

We analyze the firms' incentives in such settings. If, for example, quality entails a firm-specific increase in utility, all consumers who buy a high-quality product obtain a firm-specific consumer rent. In equilibrium, this is true regarding all firms, but only regarding high-quality products. Accordingly, each firm has an incentive to increase the markup on its high-quality product, where it has relatively many captive (that is, infra-marginal) consumers. As a result, the self-selection of the customers will be biased towards low-quality products.

In theory, this admits a thought about policy intervention. However, dictating the terms of sale seems inappropriate. Nevertheless, we are able to identify situations where the firms' incentives to produce at lower cost are twisted. For instance, we show that in a setting that describes the retail industry, firms may actually benefit from (commonly) higher wholesale prices. Thus it can be necessary that the government enforces lower import prices.

In Chapter 2, we study the relation between prices and selection in greater detail. Here, prices appear as salary offers, which employers announce publicly. Although committing to the level of their remuneration, firms are not obliged to accept each candidate. Depending on the offered salary and the supposed composition of its applicants, a firm can adjust the standard its sets in a selection process like a job talk. The lower the promised salary, the more candidates a firm accepts.

By announcing salary offers, however, firms may also screen the pool of applicants. We show that, to attract productive candidates, the promised salary needs to be relatively high. Screening, in turn, has repercussions on a firm's hiring decision. If an employer knows that most of its applicants are highly productive, it tends to discount bad job-talk performances. Poor outcomes are conceived as the result of bad luck. Thus the hiring threshold is low, unless the announced salary is excessively high.

Unfortunately, this is what happens in equilibrium. Applicants know that firms screen them for productive types. They can therefore signal their ability by approaching employers with high salaries and correspondingly low employment probabilities. If, contrary to expectations, a candidate headed to a low-paying firm, he or she would be identified as being of the unproductive type.

We rationalize this formation of beliefs by means of a game-theoretic refinement criterion (D_1 by Banks and Sobel, 1987). The intuition behind it, however, is simple. No matter how employers interpret the signal a candidate conveys by approaching a particular firm: Whenever it is favorable for productive applicants to accept a lower salary in exchange for a higher employment probability, the same applies for unproductive applicants as well. This is not true the other way around.

The disregarding of low-paying firms on behalf of the candidates ignites a race for high salary offers at the outset. Thereupon, as a consequence of the pooling of productive and unproductive applicants at high-paying employers, hiring thresholds will be exceedingly high. As a consequence, employment probabilities are undesirably low, both in the eyes of employers (who end up with many vacancies) and the applicants (who stay with their outside opportunities).

One is drawn to the conclusion that salary ceilings may remedy this detrimental outcome. At least abstracting from international competition, employers would benefit from lower salaries, too. The fact that in practice firms oppose salary ceilings, however, indicates that globalized markets may indeed limit the benefits of such policies. Like the employers in our single-market model, governments trying to maximize national welfare might play a Prisoners' Dilemma as well by not stepping in.

In Chapter 3, we increase the number of instruments which a firm can use for screening. Specifically, we discuss a setting where retailers and suppliers compete for each other, not only by announcing wholesale prices and quantities, but also by employing exclusivity clauses. Although we discuss how retailers may select low-cost suppliers by offering to purchase higher quantities at lower wholesale prices, screening itself is not at the center of our attention. Rather, we focus on the effect exclusivity clauses have on competition.

In one of our main settings, retailers benefit from having suppliers which do not sell to competing retailers. The absence of alternative sales channels for the suppliers' products limits competition in the downstream market. This results in higher profits of the retailers. Thus there are incentives for retailers to impose exclusivity clauses.

However, exclusivity clauses also intensify the competition for suppliers. Once all retailers ask for exclusivity, each supplier has to choose a single retailer. Thus each retailer has to offer a transfer as high as possible just to be taken into consideration. The tough competition between the retailers forces them to extract as much consumer surplus as possible in the downstream market and redirect it to the suppliers. That is, exclusivity clauses strengthen competition, but competition for the suppliers, at the expense of final customers.

Unlike our models in the previous chapters, in the presence of exclusivity clauses, (wholesale) prices cannot be used to screen the market for productive trading partners. On the contrary, the competition-enhancing characteristic of exclusivity clauses completely deprives the retailers of their market power. Nevertheless, the market outcome is again inefficient. We show that policy makers are able to increase welfare by banning exclusivity clauses. (To be more precise, we identify circumstances where a ban on exclusivity clauses is beneficial, and compare them with others where banning exclusivity is counterproductive.)

By comparing the results of Chapters 1 and 2 with the ones of Chapter 3, it is interesting to note the impact of market imperfections on welfare. In the first two chapters, informational frictions are present. For instance, if, in contrast to our framework of the market-segmentation model (Chapter 1), firms were able to charge prices as functions of individual customers' preferences, the outcome would be completely competitive. Firm would not lose their inframarginal customers by reducing prices at the margin. In our setting, it is the informational asymmetry between customers and firms which results in higher profits, but also in a distorted allocation.

In our salary-posting model (Chapter 2), market imperfections have adverse welfare consequences, too. In the first place, employers do not observe their applicants' productivity types, but doing so would be sufficient to guarantee a first-best allocation. Furthermore, they lack instruments to internalize their candidates' tradeoff between salaries and employment probabilities. If the firms could tie their announced salaries to the performance required to pass a job talk, competition for productive and unproductive applicants would be decoupled. Productive candidates could still fail due to bad luck, but the resulting equilibrium would be second-best.

In our model on exclusivity clauses (Chapter 3), information is symmetric, and competitive instruments are available. Nevertheless, the supply chain (consisting of suppliers and retailers) turns out to behave like a vertically integrated monopolist. Thus here we reach an inefficient outcome in a competitive setting, without imposing informational asymmetries. This raises the question whether it is not necessarily imperfections which drive markets to fail. May there be inefficiencies even in the absence of frictions?

Drawing such a conclusion from Chapter 3 would be wrong. From the first fundamental theorem of welfare economics, we know that there must be a source of imperfection whenever market allocations are inefficient. In our exclusivity-clauses model, the mechanism which impairs efficiency is hidden in the imposed sequence of events. Once suppliers are not restricted to sell their product via the retail channel, competition for consumers is regained.

As a bottom line, we conclude that to evaluate the functioning of markets it is insufficient to merely focus on the sum of individual interactions. They cannot be analyzed in isolation. Rather, to adequately assess market outcomes and policy measures, it is important to take into account industrial structures as a whole.

Acknowledgements

I am very grateful to my supervisor Winand Emons for advice and encouragement throughout the process of writing my thesis. I also want to thank Natalia Fabra, who has kindly agreed to take part in the thesis committee.

I am thankful to my colleagues at the University of Bern for their help and encouragement during my PhD studies.

The chapter on market segmentation has benefited from helpful comments and suggestions from Marc Blatter, Jan Bouckaert, Antonio Cabrales, Fabrice Collard, Winand Emons, and Marc Möller.

The chapter on wage posting has benefited from useful comments from Marc Blatter, Lutz Dümbgen, Winand Emons, Marion Fischer, and Jürg Müller.

Marc Blatter was my co-author in the closing chapter on exclusivity clauses. I profited a lot from working and debating with him. We thank Winand Emons for helpful comments.

I greatly profited from presenting and discussing my papers at several workshops and conferences. I am grateful for comments from members of the PhD workshop at the University of Bern, from participants of the Swiss IO Day 2013 in Bern, the EARIE Conference 2013 in Évora, and the 1st Alumni Conference 2013 in Gerzensee.

Chapter 1

Competitive Market Segmentation

1.1 Introduction

One of the common findings of the literature on horizontal and vertical market segmentation are quality-independent markups: differences in costs are translated one-to-one into differences in prices.[1] This result stands in stark contrast to observed empirical patterns.

For example, Barsky et al. (2003) use scanner data from one of the largest Chicago area supermarket chains to estimate markup ratios between low-quality "no-name" products and high-quality "national brands".[2] Consistently across 19 product categories, they find higher markups on low-quality products than on high-quality products, with estimated ratios ranging from 1.14 (for canned tuna) up to 2.33 (for toothbrushes). This result is mirrored elsewhere, as in

[1] For comprehensive surveys on the topic, see Armstrong (2006) and Stole (2007).

[2] More precisely, they compute lower bounds on these markup ratios, as marginal cost is not observed.

the study of Scott Morton and Zettelmeyer (2004) who generically find that retailers "can earn greater net profit[s] from selling the store brands."[3]

In other cases, markups on high-quality products are systematically higher. For the automobile industry, Verboven (1999) provides empirical evidence that premium products have larger absolute markups than base products. Similarly, Barron, Taylor, and Umbeck (2000) estimate the average dealer margin for premium gasoline to be almost 60 percent higher than the average margin for regular gasoline. For the hardware industry, Deltas, Stengos, and Zacharias (2011) show that markups on flagship computers are substantially higher than markups on slower and older machines.

These contrasting examples provide reason for the following conjecture: When the firms' high-quality products are identical, and their respective lower-quality versions differ from each other (such as in retailing), markups on low-quality products tend to be higher. On the other hand, when competitors offer baseline products which are similar, but their premium versions stand out from each other (such as in the automobile industry), high-quality products are seemingly the better deal for firms.

In the present chapter, we provide a theory which is in line with this pattern. Contrary to existing theoretical work in this field, we do not assume that a buyer's difference in utility between consuming a high-quality product and a low-quality product is constant across firms.[4]

Existing theories on the subject usually draw on the sequential revelation of information in order to explain quality-related differences in markups. Verboven

[3]Analogous results appear, for instance, in Chintagunta, Bonfrer, and Song (2002) and Bonfrer and Chintagunta (2004), where markups on store brands are higher than markups on national brands in 17 of 18 categories. Regarding the music industry, Rabinovich, Maltz, and Sinha (2008) show (among other things) that markups are lower on more popular CDs.

[4]Stole (1995), who separately analyzes horizontal and vertical second-degree price differentiation in oligopolistic markets, states that "vertical preferences [. . .] are harmonious across firms – a customer with a high marginal valuation of quality for one firm will have similar preferences for other firms as well; all firms prefer these customers." While we agree that quality enters utility in a monotonous fashion, we oppose the idea of "harmonious" preferences. Marginal utility may differ from firm to firm.

(1999) and Ellison (2005), for instance, consider “add-on pricing games” where only prices of low-quality products are publicly accessible. The differences between low-quality and high-quality products are interpreted as “add-ons”, which provide additional utility if consumed together with the base good. Such a structure allows firms to sell add-ons at the monopoly price, a result which follows from Diamond’s (1971) influential paper on price adjustment under learning cost. This, in turn, explains higher markups on premium versions of a product. Such models are certainly relevant concerning questions revolving around two-part tariffs, hidden costs, and the like. In the above examples, however, it is hard to argue that there exists an informational asymmetry between prices of high-quality products and prices of low-quality products. We should, therefore, consider a static “standard pricing game”.

Another strand of literature (Armstrong and Vickers, 2001; Rochet and Stole, 2002; Yang and Ye, 2008) studies such “one-shot” models. Unfortunately, for general specifications of demand and cost, these models can only be solved numerically. Under similar assumptions as in the present chapter, however, closed-form solutions are available: In a symmetric equilibrium with fully covered markets, prices are cost-plus-fixed-fee, which implies that markups are quality-independent. Verboven (1999), who considers a discrete-choice version of this model, obtains the same result, which he calls “somewhat surprising”. In the following, we argue that this finding is actually the result of the fact that in all of these models, the firms basically play two separate competition-on-a-line games à la Hotelling (1929).[5] By considering alternative distributions of preferences, we investigate departures from this outcome.

[5] A notable exception is Bonatti (2011), who integrates brand-specific preferences in a model of competitive market segmentation. In a similar way as Armstrong and Vickers (2001), Rochet and Stole (2002), and Yang and Ye (2008), he allows firms to simultaneously pick quality levels and prices. He obtains a “no distortion at the top” outcome. In the present chapter, we consider quality levels as given. This enhances the tractability of the model. Accordingly, we can include horizontal differentiation and (more importantly) explicitly approximate customer prices in the case of uniformly distributed preferences.

In our model, we also describe horizontal differentiation by use of the conventional Hotelling framework. Our novelty concerns the characterization of vertical preferences. We assume the disutility from consuming a low-quality product rather than a high-quality product to be *firm-specific*.[6] As an illustration, consider the case of two retailing firms which both sell a popular brand of a product as well as their respective "no-name" substitutes. The no-name products may differ from the trademark brand in various dimensions, many of which may be called "vertical": technical sophistication, conditions of production, product safety, quality of ingredients, ease of operation, eco-friendliness, durability, and performance are just a few that come to mind. Similarly, consumers are heterogenous as well: Whereas a "gourmet" primarily cares about the sophistication of a meal, a "gourmand" puts more emphasis on its size. In a nutshell, it is possible that the difference between a customer's willingness-to-pay for the two qualities varies from firm to firm; but the sign and magnitude of this difference also varies among customers.

Taking the customers' preferences as given, we look at two firms, each selling a high-quality product and a low-quality product. If the customers' preferences for quality are identical across the firms – an instance which we consider as a *special case* – the firms essentially play two separate Hotelling games: The disutility which a costumer incurs when buying the low-quality product at firm A is the same as it would be at firm B. Thus, alongside the vertical dimension, the firms are in a Bertrand-like situation. In this case, horizontal differentiation remains as the single source of market power, and equilibrium markups are unaffected by vertical preferences.

By contrast, if the customers' vertical preferences are firm-specific, a fundamental asymmetry comes into play: On the one hand, a customer who buys the

[6]Instead of a firm-specific *disutility*, we could think of an *additional utility* which arises from consuming high-quality products. Think of the airline industry, where economy-class compartments are similar across many carriers. Regarding the business class or the first class, one airline focusses on wider seats, whereas another airline serves better food or offers a more up-to-date entertainment system. As we show in the final section of this chapter, such a setting tends to induce higher markups on high-quality products.

high-quality product has a constant willingness to pay, regardless of whether his or her vertical disutility (which *would* be incurred by consuming the low-quality product) assumes a low or a high value. On the other hand, a customer who buys the low-quality product has a higher willingness to pay the less the qualitative difference between the two varieties affects his or her utility. That is, the lower the realization of the vertical disutility at firm A, the likelier it becomes that even a B-affine customer starts buying the low-quality product at A. Thus, metaphorically speaking, we show that both firms are able to catch low-quality customers out of their competitor's pond. Accordingly, each firm's low-quality-product customer base becomes to some extent inflated. Usual marginal-versus-infra-marginal-consumers considerations then imply that the firms set higher prices for their lower-quality products.

We organize the remainder of this chapter as follows. In Section 1.2 we set up a model of spatial competition with two exogenously given qualities. We allow for general distributions, including firm-dependent vertical preferences. As we show in Section 1.3, the equilibrium relation between markups and quality depends on the distribution of customers. This contrasts with the earlier contributions, but is too general as a result to gain additional insights. Therefore, in the following sections we look at prototypical cases where the customers' taste parameters are uniformly distributed. In Section 1.4, we impose the standard assumption of perfectly correlated vertical preferences. This leads to quality-independent markups. In Section 1.5, we put this result into perspective by looking at independently and identically distributed taste parameters. In the case of homogenous high-quality products (abstracting form horizontal preferences), we find higher markups on low-quality products. This result is reversed once we assume that low-quality products are vertically the same, and additional benefits from consuming high-quality products are specific to the firm, as we exemplify in Section 1.6. Section 1.7 concludes. Parts of the proofs are relegated to the appendix.

1.2 Model

Following the standard procedure in the literature, we consider a Hotelling model of horizontal differentiation which we augment by a vertical component. Two profit-maximizing firms, A and B, both offer a low-quality product L and a high-quality product H.[7] For each firm $i \in \{A, B\}$, we denote i's prices for L and H by p_L^i and p_H^i. The constant marginal cost of production is c_L for L and c_H for H. Thus it is the same across firms but generally varies with quality. It is natural to assume that $c_H \geq c_L \geq 0$.

There is a continuous unit mass of consumers, each of which is described by a triplet $\mathbf{d} = (d, d_L^A, d_L^B) \in [0,1]^3$. d is the conventional (horizontal) taste parameter, whereas d_L^i reflects the reduction of a consumer's willingness to pay if he or she buys L instead of H at i. For the moment, we leave it open whether $d_L^A = d_L^B$ or not. Each consumer buys only one product at only one firm. If opting for firm i, the utility of consumer $\mathbf{d}$ is

$$u^i(\mathbf{d}) = \begin{cases} v - p_H^i - td^i & \text{if } q_H^i = 1 \\ v - p_L^i - td^i - t_L d_L^i & \text{if } q_L^i = 1, \end{cases} \tag{1}$$

where $d^i = d$ if $i = A$ and $d^i = 1 - d$ if $i = B$. t and t_L are weighting parameters which measure the relative importance of the horizontal and vertical differentiation.[8] We assume v, the base utility of both versions of the product, to be sufficiently high, such that each consumer ends up buying either L or H. Accordingly, we do not have to make any assumptions on a consumer's reservation value. Furthermore, we impose $c_H - c_L \leq t_L$, an assumption which ensures interior equilibria.

[7] In the following, everything which holds for A automatically translates to B, as we assume the firms to be symmetric.

[8] Hence horizontal preferences are continuously distributed in the compact set $[0, t]$, and vertical preferences are continuously distributed in the compact set $[0, t_L]$.

Based on (1), we can subdivide the decision of an individual consumer into two parts. First, the consumer chooses firm A if and only if

$$v - \min\{p_H^A, p_L^A + t_L d_L^A\} - td \geq v - \min\{p_H^B, p_L^B + t_L d_L^B\} - t(1-d). \qquad (2)$$

Given condition (2), the consumer prefers L over H if and only if

$$p_L^A + t_L d_L^A \leq p_H^A. \qquad (3)$$

In analyzing the model, we will look at symmetric subgame-perfect Nash-equilibria for the "pricing game" where each firm i chooses prices p_L^i and p_H^i and consumers subsequently buy their preferred product. These equilibria may or may not involve the selling of both qualities. However, whenever firm i sells both L *and* H in a (putative) equilibrium, we know from (3) that

$$0 \leq x^i := (p_H^i - p_L^i)/t_L < 1. \qquad (4)$$

That is, by defining x^i as the threshold value above which d_L^i must lie in order to buy H at i, we know that x^i is located between 0 and 1 in an interior equilibrium.

In the following sections, the structure of the firms' objective functions is generally subject to the relation between prices. Therefore, as long as a firm chooses to remain within an interior symmetric equilibrium, we impose (4) (as well as symmetric prices) in order to construct a firm's objective function. It is clear, however, that these relations can never be used strategically: each firm optimizes with respect to its own prices only.

In Section 1.3, we derive optimality conditions for arbitrary distributions of (d, d_L^A, d_L^B) and show that, in general symmetric interior equilibria, markups on H and L are not the same but depend on the particular distribution. To illustrate this, Section 1.4 and Section 1.5 compare the outcomes of the pricing game for two specific distributions of the consumers' preferences.

In Section 1.4, we assume that $d_L^A = d_L^B =: d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$. Such a specification is analogous to Verboven (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002), and describes the case where consumers perceive the difference between L and H to the same extent at both firms.
By contrast, in Section 1.5 we assume that $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3[0,1]$. There, vertical preferences with respect to the two firms are completely uncorrelated. It might be that at firm A a consumer barely notices any difference between L and H, whereas at firm B the willingness to pay is much higher for H than for L.

1.3 General Demand Function: Differing Markups on H and L

Before examining the competing firms' equilibrium behavior, we need to determine their demand as a function of all prices.

Demand Function Q_H^A, firm A's total demand for product H, is the (ex-ante) probability that a consumer prefers to buy H at A to buying H or L at B or L at A. Using (1), we have

$$\begin{aligned} Q_H^A = P[q_H^A = 1] = P\Big[& \underbrace{p_H^A + td \leq p_H^B + t(1-d)}_{d \leq \overline{HH}} \\ & \cap \underbrace{p_H^A + td \leq p_L^B + t(1-d) + t_L d_L^B}_{d \leq \overline{HL}} \\ & \cap \underbrace{p_H^A + td < p_L^A + td + t_L d_L^A}_{d_L^A > x^A} \Big]. \end{aligned} \tag{5}$$

In (5), $\overline{X^A X^B}$ refers to the "switching line", below which a consumer's value of d must lie such that he or she prefers to buy $X^A \in \{L, H\}$ at A as compared to buying $X^B \in \{L, H\}$ at B. As defined in (4), x^A is the critical value of d_L^A above which a consumer prefers to buy H at A instead of L at A. Analogically, we write Q_L^A as

$$\begin{aligned} Q_L^A = P[q_L^A = 1] = P \Big[& \underbrace{p_L^A + td + t_L d_L^A \leq p_H^B + t(1-d)}_{d \leq \overline{LH}} \\ & \cap \underbrace{p_L^A + td + t_L d_L^A \leq p_L^B + t(1-d) + t_L d_L^B}_{d \leq \overline{LL}} \\ & \cap \underbrace{p_L^A + td + t_L d_L^A \leq p_H^A + td}_{d_L^A \leq x^A} \Big]. \end{aligned} \tag{6}$$

After solving the first two inequalities in (5) for d, we write the conditional joint probability of $d \leq \overline{HH}$ and $d \leq \overline{HL}$ as

$$P\left[d \leq \overline{HH} \cap d \leq \overline{HL} | d_L^A, d_L^B\right] = \int_0^{\min\{\overline{HH}, \overline{HL}\}} f(d|d_L^A, d_L^B) \mathrm{d}d. \tag{7}$$

For this equality to be true, we need $\min\{\overline{HH}, \overline{HL}\} \in [0, 1]$. In an interior symmetric equilibrium, this condition holds by symmetry and condition (4), provided $0 \leq (p_H^A - p_L^B)/t \leq 1$.[9] That is, prices need to be sufficiently close; and therefore, for each combination of d_L^A and d_L^B, there are both consumers with small values of d who buy H at A and consumers with large values of d who buy any of the products at B. By incorporating the remaining inequality in

[9]In Section 1.4, this additional condition is redundant, as $d_L^A = d_L^B$ is sufficient for $\min\{\overline{HH}, \overline{HL}\} \in [0, 1]$ and $\min\{\overline{LH}, \overline{LL}\} \in [0, 1]$. In Section 1.5, we first assume the validity of the condition, and subsequently prove that $p_H^A - p_L^B > t$ cannot hold in any symmetric interior equilibrium.

(5), $d_L^A > x^A$, into (7), we obtain

$$Q_H^A = \int_0^1 \left(\int_{x^A}^1 \left(\int_0^{\min\{\overline{HH},\overline{HL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B. \tag{8}$$

In the upper part of Figure 1.1, we graphically represent Q_H^A, the region where $d \leq \overline{HH}$, $d \leq \overline{HL}$, and $d_L^A > x^A$. For given prices p_H^A and p_L^A, with $p_H^A > p_L^A$, consumers buy H at A if their value of d is low and their value of d_L^A is high. Furthermore, in order for a consumer to buy H, d_L^B must not be too small.

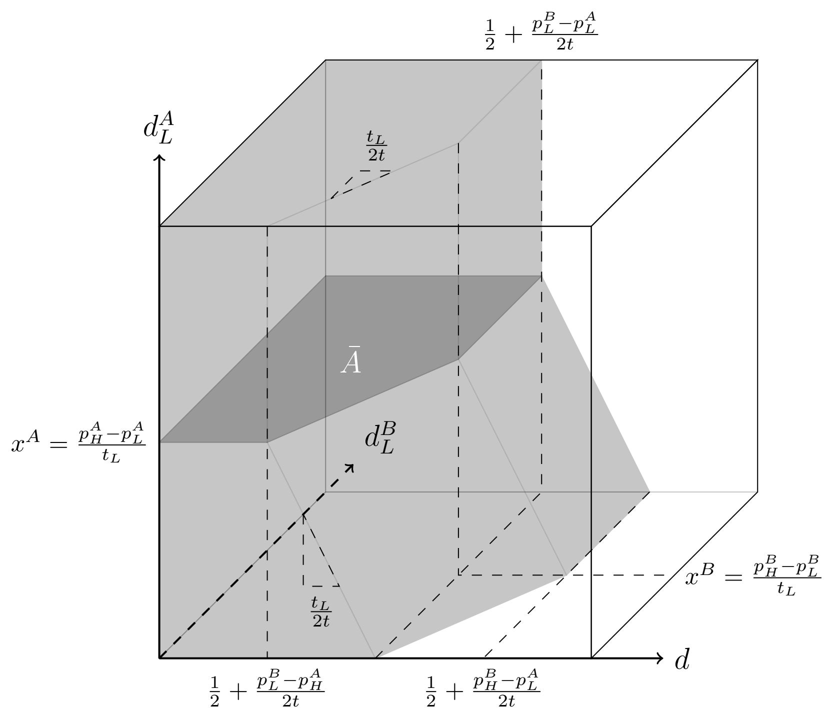


Figure 1.1: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $d_L^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $d_L^A \leq x^A$.

The derivation of Q_L^A is analogous to the derivation of Q_H^A. Consumers buy L at A if this is more favorable than either buying any of H or L at B or buying

H at A. Using (6), we obtain

$$Q_L^A = \int_0^1 \left(\int_0^{x^A} \left(\int_0^{\min\{\overline{LH},\overline{LL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B. \tag{9}$$

In the lower part of Figure 1.1, we graphically represent Q_L^A, the region where $d \leq \overline{LH}$, $d \leq \overline{LL}$, and $d_L^A \leq x^A$. By comparing Q_H^A and Q_L^A, the following asymmetry stands out. Regarding d, the support of Q_H^A is independent of d_L^A, whereas the support of Q_L^A increases for a decreasing d_L^A. In other words, regarding consumers of L, the utility is generally higher the lower d_L^A is. On the other hand, consumers of H all obtain the same level of utility, once we abstract from d. As it later turns out, this asymmetry plays a crucial role for the determination of markups whenever we assume that a change in d_L^A not necessarily implies a change in d_L^B.

Equilibrium In the following, we first determine the prices and profits which occur in symmetric interior equilibria. Later, for each distribution of $\mathbf{d}$ which we analyze, we also check whether firms have incentives to deviate to corner solutions, i.e., by only selling L or H. Furthermore, we demonstrate that symmetric corner solutions cannot constitute equilibria in each of the considered cases.

In an interior equilibrium, firm A maximizes

$$\pi^A = Q_H^A(p_H^A - c_H) + Q_L^A(p_L^A - c_L), \tag{10}$$

with Q_H^A and Q_L^A defined in (8) and (9). Since the integrands in Q_H^A and Q_L^A are continuous functions in p_H^A and p_L^A, (10) is differentiable, and the first-order conditions (FOCs) with respect to p_H^A and p_L^A are necessarily satisfied in an interior equilibrium. As we show in Appendix 1.A.1, after imposing $p_H^A = p_H^B =: p_H$ and $p_L^A = p_L^B =: p_L$ (and thus $x^A = x^B =: x$), we can express the FOC with respect to p_H^A as

$$\left(\int_0^1\int_x^1 m^H \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^H(d_L^A = x)}{t_L}\mathrm{d}d_L^B\right)(p_H - c_H)$$
$$= \underbrace{\int_0^1\int_x^1 A^H \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_H^A} + \int_0^1 \frac{A^H(d_L^A = x)}{t_L}\mathrm{d}d_L^B (p_L - c_L), \tag{11}$$

where A^H and m^H are defined as follows.

$$A^H := \int_0^{\min\{\overline{HH},\overline{HL}\}} f(\mathbf{d})\mathrm{d}d$$

denotes the amount of H-consumers at A for given values of d_L^A and d_L^B, and

$$m^H := (1/2t)f(\min\{\overline{HH},\overline{HL}\}, d_L^A, d_L^B)$$

is the "margin" of A^H, that is, the subset of A^H which leaves firm A towards firm B after a marginal increase of p_H^A.

How can we interpret equation (11)? Of course, in an interior optimum, the marginal benefits of increasing p_H^A equal the marginal costs. The left-hand side of (11) displays the latter: The first term, where $d_L^A > x$, relates to A's H-consumers which leave A towards B if A raises p_H^A. By doing so, the firm looses $p_H - c_H$ on each of these consumers. It also looses the same markup on consumers who stay at A but start to buy L instead of H. The measure of these intra-firm migrants is $\bar{A}/t_L$, where $\bar{A} := \int_0^1 A^H(d_L^A = x)\mathrm{d}d_L^B$ is shown in Figure 1.1. The same consumers, however, are also part of the marginal benefit of increasing p_H^A, which we display on the right-hand side of (11). On each such consumer, A gains $p_L - c_L$, the markup on L. The remaining part of the marginal benefit is Q_H^A, which refers to the infra-marginal consumers on whom A benefits by increasing p_H^A.

Analogically, and also in Appendix 1.A.1, we simplify the symmetric-version FOC with respect to p_L^A as

$$\begin{aligned}&\left(\int_0^1\int_0^x m^L \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^L(d_L^A = x)}{t_L}\mathrm{d}d_L^B\right)(p_L - c_L)\\ =&\underbrace{\int_0^1\int_0^x A^L \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_L^A} + \int_0^1 \frac{A^L(d_L^A = x)}{t_L}\mathrm{d}d_L^B(p_H - c_H),\end{aligned} \tag{12}$$

where $A^L := \int_0^{\min\{\overline{LH},\overline{LL}\}} f(\mathbf{d})\mathrm{d}d$, and $m^L := (1/2t)f(\min\{\overline{LH},\overline{LL}\}, d_L^A, d_L^B)$ are similarly interpreted as A^H and m^H. In Appendix 1.A.1, we show that $A^L(d_L^A = x) = A^H(d_L^A = x)$. That is, marginal consumers *within* firm A are the same for changes in p_H^A and changes in p_L^A. We highlight these consumers by $\bar{A}$ in Figure 1.1.

Beyond that, equation (12) again displays two considerations. First, we have the conventional tradeoff between gains on infra-marginal consumers and losses on marginal consumers who leave the firm. Second, regarding $\bar{A}/t_L$, the FOC displays an instance of "intra-firm competition" between H at A and L at A.

Before heading to applications of what we established so far, we state Proposition 1, which sheds some new light on the results of Verboven (1999), Armstrong and Vickers (2001), and Rochet and Stole (2002).

Proposition 1. *Suppose there is a symmetric interior equilibrium of the pricing game, where $p_H - p_L \leq t$. Then the firms' markups on H and L are generally different.*

Proof. Note that $\min\{\overline{HH},\overline{HL}\} = \overline{HH}$ if and only if $d_L^B > x$. Analogically, $\min\{\overline{LH},\overline{LL}\} = \overline{LH}$ if and only if $d_L^B > x$. Next, suppose that markups on H and L were identical. Define $\mathcal{M} = p_H - c_H = p_L - c_L$. In this case, adding up equations (11) and (12) yields

$$\begin{aligned}
&\frac{1}{2t}\left(\int_0^x \left(\int_0^x f(\overline{LL}, d_L^A, d_L^B)\mathrm{d}d_L^B + \int_x^1 f(\overline{LH}, d_L^A, d_L^B)\mathrm{d}d_L^B\right)\mathrm{d}d_L^A\right.\\
&+\left.\int_x^1 \left(\int_0^x f(\overline{HL}, d_L^A, d_L^B)\mathrm{d}d_L^B + \int_x^1 f(\overline{HH}, d_L^A, d_L^B)\mathrm{d}d_L^B\right)\mathrm{d}d_L^A\right)\mathcal{M}\\
&= Q_L^A + Q_H^A = 1/2,
\end{aligned}$$

where the second equality is based on the symmetry condition.
We see that, in general, markups on H and L are not identical: Even though a variation in the distribution of either d_L^A or d_L^B could be absorbed by a change of $\mathcal{M}$ (it necessarily holds that $x = (c_H - c_L)/2$), this is generally not possible for a change in both of these distributions. □

As a corollary, Proposition 1 implies that, if $f(\mathbf{d})$ is constant (and therefore equals 1) and there are identical markups on H and L, these markups are $\mathcal{M} = t$. More importantly, the same applies if

$$f(\mathbf{d}) = \begin{cases} 1 & \text{if } d_L^A = d_L^B, \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

which is the standard case in the literature.
In Section 1.4, we show that markups are indeed identical (and therefore equal $\mathcal{M}$) in this standard case, and that symmetric equilibria usually exist and are interior. Thereupon, in Sections 1.5 and 1.6, we discuss counterexamples where markups on H and L differ.

1.4 The Benchmark: Correlated Vertical Preferences

Here we consider the case where each consumer perceives the quality difference between L and H to the same extent at both firms. We show that, from a firm's point of view, the presence of a second (vertically differentiated) product is inconsequential, as compared to the standard Hotelling case.

Demand Function Specifically, we assume that $d_L^A = d_L^B =: d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$. The difference in utility between consuming H and L is the same at both firms. In this case, equations (8) and (9) simplify to

$$Q_H^A = \int_{x^A}^{1} \left(\int_0^{\min\{\overline{HH},\overline{HL}\}} \mathrm{d}d \right) \mathrm{d}d_L, \text{ and } Q_L^A = \int_0^{x^A} \left(\int_0^{\min\{\overline{LH},\overline{LL}\}} \mathrm{d}d \right) \mathrm{d}d_L.$$

We show graphical representations of Q_H^A and Q_L^A in Figures 1.2 and 1.3, respectively, which can be considered as cross-sections of Figure 1.1 for which $d_L^A = d_L^B$.

In Figure 1.2, each of three shaded areas represents consumers who prefer to buy H at A as compared buying either H at B ($d \leq \overline{HH}$), L at B ($d \leq \overline{HL}$) or L at A ($d_L^A > x^A$). The intersection of these areas is Q_H^A. In a symmetric equilibrium, $d \leq \overline{HL}$, which is bordered by the upward-sloping line, never binds: Whenever a consumer prefers buying H at A over buying L at A and H at B, it follows from symmetric prices and $d_L^A = d_L^B$ that this consumer also prefers buying H at A over buying L at B. In other words, there is no direct "inter-firm" competition between H and L.

In Figure 1.3, we represent Q_L^A. Here, the downward-sloping line borders $d \leq \overline{LH}$, a condition which also never binds, for exactly the same reason.

Equilibrium and Welfare In order determine prices and profits of a symmetric interior equilibrium, we start with the FOCs (11) and (12), which hold in a symmetric interior equilibrium. Thereupon, we show that firms do not have incentives to deviate to a corner solution, that is, by only selling L or H. Furthermore, we show that, in general, a symmetric corner solution is not an equilibrium either.

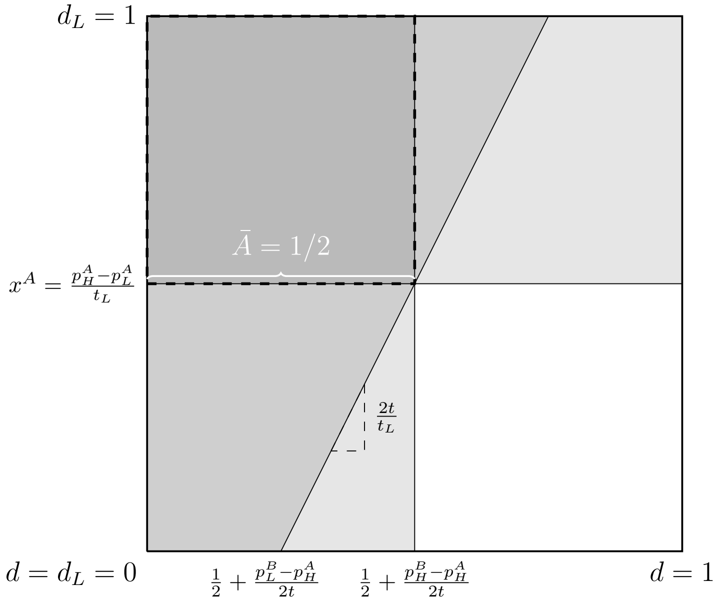


Figure 1.2: The fraction of consumers which buy firm A's high-quality product, Q_H^A, is given by the intersection of the three shaded areas.

By applying (13) to (11) and (12), we write the symmetric-version FOCs with respect to p_H^A and p_L^A as

$$\left(\frac{1-x}{2t} + \frac{1}{2t_L}\right)(p_H - c_H) = \frac{1-x}{2} + \frac{1}{2t_L}(p_L - c_L), \tag{14}$$

and

$$\left(\frac{x}{2t} + \frac{1}{2t_L}\right)(p_L - c_L) = \frac{x}{2} + \frac{1}{2t_L}(p_H - c_H). \tag{15}$$

Before discussing equations (14) and (15), we use them to establish Lemma 1.

Lemma 1. *If $d_L^A = d_L^B = d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$, symmetric interior equilibria of the pricing game are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$.*

Proof. Note that $x(p_H^*, p_L^*) = (c_H - c_L)/t_L$ and (p_H^*, p_L^*) solves (14) and (15). In the opposite direction, assume to the contrary that $p_H - c_H > p_L - c_L$. If this

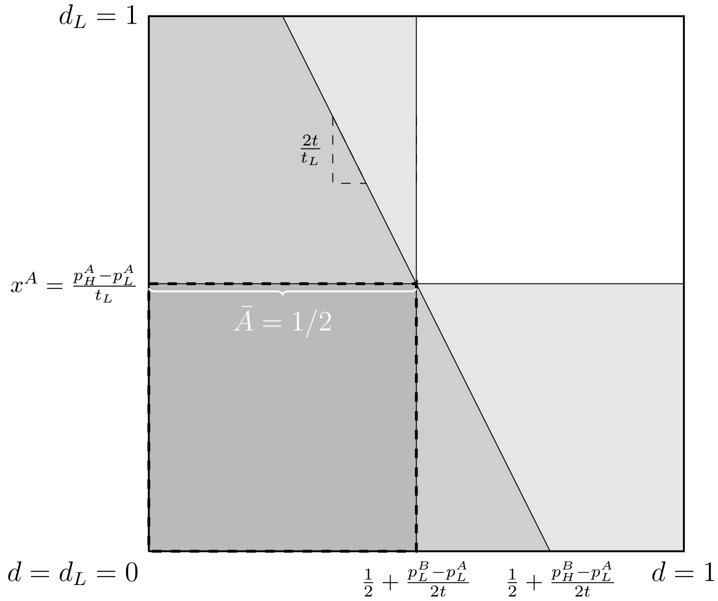


Figure 1.3: The fraction of consumers which buy firm A's low-quality product, Q_H^A, is given by the intersection of the three shaded areas.

is the case, (14) implies that $p_H - c_H < t$, while (15) implies that $p_L - c_L > t$. Therefore, $p_L - c_L > t > p_H - c_H$, which contradicts $p_H - c_H < p_L - c_L$. Since $p_H - c_H < p_L - c_L$ yields a similar contradiction, it must hold that $p_H - c_H = p_L - c_L =: \mathcal{M}$. Formulated this way, (14) reads $(1-x)/2t \times \mathcal{M} = (1-x)/2$, or $\mathcal{M} = p_H - c_H = p_L - c_L = t$. In Appendix 1.A.2, we show that there are no unilateral deviations towards corner solutions. $\square$

How can we interpret Lemma 1? In the symmetric equilibrium, each firm serves half the customer base, a fraction x of which buys L and a fraction $1-x$ buys H. Markups on H and L are identically equal to t, and profits are $\pi(p_H^*, p_L^*) = t/2$. The left-hand side of equation (14) displays the marginal cost of increasing p_H^A: $(1-x)/2t$ consumers (A's H-consumers) leave A towards B; and $1/2t_L$ consumers change the product but not the firm. The right-hand side of (14) shows the marginal benefit of increasing p_H^A: $(1-x)/2$ relates to A's infra-marginal consumers; and $1/2t_L$ start buying L instead of H at A. The interpretation of (15) is analogous. Thus, from identical markups on H and L, there is no need for A to care about customers who stay with A but only swap qualities. It solely remains to trade off between the marginal cost from consumers who leave A and the marginal benefit on infra-marginal ones. That is, regarding H, A and B play a standard Hotelling game with respect to these consumers for whom $d_L > x = (c_H - c_L)/t_L$. Concerning consumers with $d_L \leq x$, A and B play a Hotelling game regarding L. As we have seen earlier, in a symmetric equilibrium, A's version of H does not directly compete with B's version of L. That is, by marginally raising p_H^A, A affects the composition of its own customer base, and A looses some consumers who start buying H at B. From $d_L^A = d_L^B$, however, there is no first-order effect concerning consumers who both change the firm and the product. In Section 1.5, we drop the assumption of correlated vertical preferences. There, both H and L of B will pose a threat if A increases p_H^A or p_L^A. The alternative assumption $d_L^A \perp d_L^B$ will crucially change the result of Lemma 1.

Before concluding this section with a more general statement which takes into account the possibility of corner solutions, we compare the market outcome with first-best efficient allocations. In the present case, maximizing a utilitarian welfare function is equivalent to minimizing the sum of the firms' production cost and the consumers' disutility from both horizontal and vertical distance to the product which is actually bought. If and only if it is efficient that consumer $\bar{\mathbf{d}}$ buys L at A, it is efficient that consumers with lower values of d and d_L also buy L at A. Likewise, if and only if it is efficient that $\bar{\mathbf{d}}$ buys H at A, it is efficient that consumers with lower values of d and higher values of d_L also buy H at A. Hence, in order to determine a socially efficient allocation, it is sufficient to find cutoff values $\bar{d}^*$ and $\bar{d}_L^*$ which minimize

$$\begin{aligned}W(\bar{d}, \bar{d}_L) = &\int_0^{\bar{d}} \left(\int_0^{\bar{d}_L} c_L + td + t_L d_L \mathrm{d}d_L + \int_{\bar{d}_L}^1 c_H + td \mathrm{d}d_L \right) \mathrm{d}d \\ &+ \int_{\bar{d}}^1 \left(\int_0^{\bar{d}_L} c_L + t(1-d) + t_L d_L \mathrm{d}d_L + \int_{\bar{d}_L}^1 c_H + t(1-d) \mathrm{d}d_L \right) \mathrm{d}d.\end{aligned}$$

The FOCs of $W(\bar{d}, \bar{d}_L)$ with respect to $\bar{d}$ and $\bar{d}_L$ yield $\bar{d}^* = 1/2$ and $\bar{d}_L^* = (c_H - c_L)/t_L$. These cutoff levels comply with the market solutions described in the following proposition. Market allocations are efficient because, abstracting from horizontal preferences, $d_L^A = d_L^B$ implies that firms are in a Bertrand situation, which in turn leads to first-best efficiency.

Before relaxing $d_L^A = d_L^B$, we summarize the main findings of this section in Proposition 2. We present the remaining parts of the proof in Appendix 1.A.3.

Proposition 2. *If $d_L^A = d_L^B = d_L$, and $(d, d_L) \sim$ i.i.d. $\mathcal{U}_2[0,1]$, symmetric equilibria of the pricing game are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$. Consumers with $d \leq 1/2$ ($d > 1/2$) buy at firm A (B). Consumers with $d_L > (c_H - c_L)/t_L$ ($d_L \leq (c_H - c_L)/t_L$) buy H (L). Each firm's equilibrium profit is $\pi(p_H^*, p_L^*) = t/2$, and the market outcome is efficient.*

1.5 Uncorrelated Vertical Preferences (I): Markups on L May Be Higher

Here we consider the case where the consumer-specific perception of the quality difference between H and L depends on the firm. We show that, in contrast to the outcome of Section 1.4, the markup on L is higher than the markup on H, equilibrium profits increase, and the market outcome is inefficient.

Demand Function From $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3[0,1]$, we use the definitions in (8) and (9) with $f(\mathbf{d}) \equiv 1$ to describe Q_H^A and Q_L^A. The difference in utility between consuming H and L is now specific to the firm. For a graphical representation of Q_H^A and Q_L^A, recall Figure 1.1 in Section 1.3. There, we hinted at a fundamental asymmetry between Q_H^A and Q_L^A, which we study now in greater detail.

Abstracting from the horizontal characteristic d, consumers with high values of d_L^A (which buy H at A) all obtain the same utility, which equals the utility of consumers who are indifferent between buying L and H. To see this, consider

Figure 1.4, which reproduces Figure 1.1 and augments it by Q_H^B and Q_L^B (as they are allocated in a symmetric interior equilibrium). Consumer $\mathbf{d^{LH}}$ is indifferent

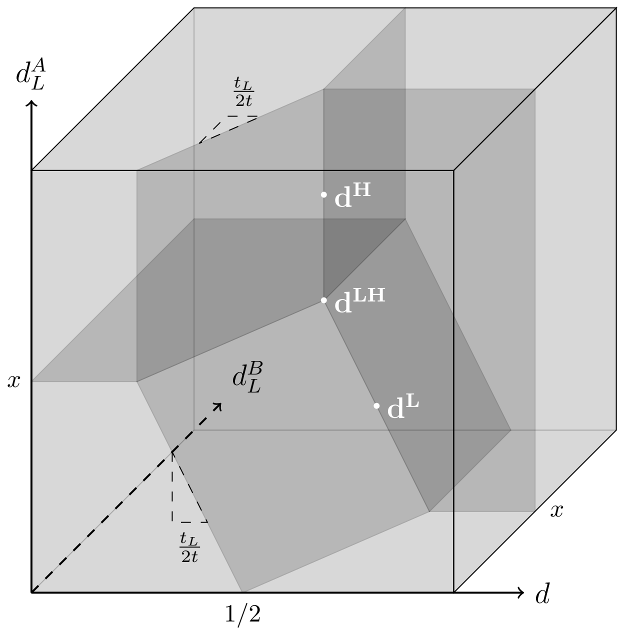


Figure 1.4: In a symmetric interior equilibrium allocation, consumers with high values of d_L^A and low values of d buy H at A; consumers with low values of d_L^A and low values of d buy L at A; consumers with high values of d_L^B and high values of d buy H at B; and consumers with low values of d_L^B and high values of d buy L at B.

between buying at A and buying at B, but also between buying L and buying H. Consumers with higher values of d_L^A, such as consumer $\mathbf{d^H}$, are indifferent between the firms, but prefer buying product H once they opt for firm A. The only difference between $\mathbf{d^{LH}}$ and $\mathbf{d^H}$ concerns the realization of d_L^A. For a H-consumer, however, d_L^A is irrelevant, as it only affects the utility of L-consumers. For these, the lower the value of d_L^A, the higher is the utility obtained from buying L. Therefore, consumers who resemble $\mathbf{d^{LH}}$, but exhibit lower values of d_L^A, are strictly better off than $\mathbf{d^{LH}}$ (by buying L at H). Consequentially, even consumers with higher values of d, such as $\mathbf{d^L}$, are equally well off as $\mathbf{d^{LH}}$.

Naturally, we can replicate this thought experiment for each consumer who is indifferent between either of the product versions at A. As we see in Figure 1.4, this results in an inflated quantity Q_L^A. More precisely, the ratio between A's infra-marginal L-consumers and A's marginal L-consumers (on the boundary towards B) exceeds the same ratio with respect to A's H-consumers. This, in turn, provides an incentive for A to raise p_L^A.

Equilibrium and Welfare As in Section 1.4, we start with the FOCs (11) and (12), which apply in a symmetric interior equilibrium. Thereupon, we show once more that firms do not have incentives to deviate to a corner solution, and we demonstrate that, in general, a symmetric corner solution is not an equilibrium either.

After imposing $f(\mathbf{d}) \equiv 1$ on (11) and (12), we write the symmetric-version FOCs with respect to p_H^A and p_L^A as

$$\left(\frac{1-x}{2t} + \frac{\bar{A}}{t_L}\right)(p_H - c_H) = (1-x)\bar{A} + \frac{\bar{A}}{t_L}(p_L - c_L), \tag{16}$$

and

$$\left(\frac{x}{2t} + \frac{\bar{A}}{t_L}\right)(p_L - c_L) = x\left(\bar{A} + \frac{p_H - p_L}{4t}\right) + \frac{\bar{A}}{t_L}(p_H - c_H), \tag{17}$$

where $\bar{A} := 1/2 - (t_L/4t)x^2$ represents the frontier between consumers of H and consumers of L at A (see Figure 1.1).

Equations (16) and (17) help us understand the firms' equilibrium behavior. The left-hand side of (16) displays the marginal cost of increasing p_H^A: A share of A's customers who are indifferent between the two firms, namely the fraction $1-x$ which buys H at A, leaves A towards B; an additional fraction $\bar{A}/t_L$ moves inside the firm to buy L. On both of these groups, A loses $p_H - c_H$ per customer. Regarding the customers which move within A, however, the marginal loss is offset by a marginal profit of $p_L - c_L$ on L. We see this on the right-hand side of (16), alongside with $(1-x)\bar{A}$, A's existing consumers of H, on whom A increases its profit by increasing p_H^A.

The interpretation of (17) is similar. An increase of p_L^A leads to a per-customer loss of $p_L - c_L$ on L, both on consumers leaving A towards B (x) and on the ones moving internally ($\bar{A}/t_L$). Again, part of this loss is retained by a gain on H. The marginal profit on infra-marginal consumers, however, looks different here ($x(\bar{A} + (p_H - p_L)/4t)$, instead of $x\bar{A}$). These additional $x(p_H - p_L)/4t$ consumers refer to a firm's inflated demand for L, as we elaborated in the previous paragraph.

If $p_H^A = p_L^A$, infra-marginal L-consumers, and with them the associated asymmetry, disappear. As we show next, $c_H = c_L$ is a sufficient condition for $p_H^A = p_L^A$ in a symmetric interior equilibrium. In this case, (16) and (17) have a simple solution, which is identical markups on H and L. This, however, only holds for $c_H = c_L$. In a next step, we will thus "perturb" c_H to examine equilibrium markups on H and L in a more general setting where $c_H \geq c_L$.

Lemma 2. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$, *and* $c_H = c_L$, *in a symmetric interior equilibrium of the pricing game it must hold that* $p_H = p_L$.

Proof. If $p_H < p_L$, nobody buys L. If $p_H > p_L$, we know from (16) that

$$(p_H - c_H)/2t < \bar{A}. \tag{18}$$

From (17), $p_H > p_L$ implies

$$(p_L - c_L)/2t > \bar{A} + (p_H - p_L)/4t. \tag{19}$$

By combining (18) and (19), we have $p_L - c_L > 2t\bar{A} + (p_H - p_L)/2 > 2t\bar{A} > p_H - c_H$, a contradiction to $c_H = c_L$ and $p_H > p_L$. $\square$

In words, in the case of identical costs $c_H = c_L$, it cannot be that markups on high-quality products are higher than markups on low-quality products. As we state in (18), if $p_H > p_L$, firms had an incentive to lower p_H and internally move consumers to H, unless there were relatively many infra-marginal consumers of H.[10] Meanwhile, as we state in (19), firms had an incentive to raise p_L, unless there were relatively little infra-marginal consumers of L. High gains on infra-marginal H-consumers and low gains on infra-marginal L-consumers, however, cannot occur simultaneously. The former implies a high amount H-consumers. The latter implies a high amount of L-consumers, as here the ratio between infra-marginal and marginal consumers (marginal with respect to B) decreases with the quantity of low-quality consumers.

Next, we show that, for the special case $c_H = c_L$, the equilibrium prices equal the ones in Section 1.4.

Lemma 3. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$, *and* $c_H = c_L$, *the only symmetric interior equilibrium of the pricing game is characterized by* $p_H^* = c_H + t$ *and* $p_L^* = c_L + t$.

Proof. Note that $x(p_H^*, p_L^*) = 0$, $A(p_H^*, p_L^*) = 1/2$, and therefore (p_H^*, p_L^*) solves (16) and (17). In the opposite direction, Lemma 2 requires $p_H - c_H = p_L - c_L =: \mathcal{M}$. Formulated this way, (17) reads $(1/2t + 1/2t_L)\mathcal{M} = 1/2 + (1/2t_L)\mathcal{M}$, implying $\mathcal{M} = p_H - c_H = p_L - c_L = t$. Later, that is, for the general case without restricting to $c_H = c_L$, we show that there are no unilateral deviations towards corner solutions. □

The intuition behind Lemma 3 is simple: Given $p_H = p_L$, the probability that a consumer buys H is 1. Therefore, vertical preferences do not play a role, and the firms essentially play a standard Hotelling game, the familiar outcome of which is $p_H^* = c_H + t$. The according profits are $\pi(p_H^*, p_L^*) = t/2$.

Once we drop the assumption $c_H = c_L$, matters become more difficult. In particular, the analytic solution to (16) and (17) is generally intricate and barely interpretable. Since Lemma 3 reveals a simple solution for $c_H = c_L$, however, we can use "perturbation methods" (see, for instance, Judd, 1996) in order to locally approximate p_H^* and p_L^* for $c_H > c_L$ near $c_H = c_L$. In our case, the

[10] "Relatively" refers to the relation between A's $(1-x)A$ infra-marginal H-consumers and the marginal gain $(1-x)(p_H - c_H)$ on consumers who start buying at A if p_H^A decreases.

appropriate perturbation technique consists of using Taylor's theorem alongside with the implicit function theorem for $\mathbb{R}^2$. In Appendix 1.A.4, we derive second-degree Taylor approximations for p_H^* and p_L^* which we present in Lemma 4.

Lemma 4. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H \geq c_L$ *sufficiently close, the only symmetric interior equilibrium of the pricing game is characterized by*

$$p_H^* = c_H + t + \mathcal{O}\left((c_H - c_L)^3\right)$$

and

$$p_L^* = c_L + t + \frac{(c_H - c_L)^2}{2t} + \mathcal{O}\left((c_H - c_L)^3\right).$$

Sketch of Proof. After defining $\mathbf{p} := (p_H, p_L)$, we write (16) and (17) as $f(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 0$. Next, we define $\mathbf{F}(\mathbf{p}) := (f(\mathbf{p}), g(\mathbf{p}))$. By the implicit function theorem, it holds that

$$[\mathbf{F}_{\mathbf{p}}]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_c]_{c_H=c_L} = 0, \tag{20}$$

and

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_{\mathbf{p}}]_{c_H=c_L}\mathbf{p}'' + 2[\mathbf{F}_{\mathbf{p}c}]_{c_H=c_L}\mathbf{p}' + [\mathbf{F}_{cc}]_{c_H=c_L} = 0, \tag{21}$$

where $\mathbf{p}' := \mathrm{d}\mathbf{p}/\mathrm{d}c_H$, $\mathbf{p}'' := \mathrm{d}^2\mathbf{p}/(\mathrm{d}c_H)^2$, $\mathbf{F}_{\mathbf{p}(c)}$ is the Jacobian of $\mathbf{F}$ with respect to $\mathbf{p}$ (c_H), $\mathbf{F}_{\mathbf{p}c}$ ($\mathbf{F}_{cc}$) is the derivative of $\mathbf{F}_{\mathbf{p}}$ ($\mathbf{F}_c$) with respect to c_H, and $\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')$ is a multiplicative operation of $\mathbf{p}'$ and the Hessian tensor of $\mathbf{F}$ with respect to $\mathbf{p}$, which we explicitly formulate in Appendix 1.A.4. Also in Appendix 1.A.4, we compute $\mathbf{F}_{\mathbf{p}}$ and $\mathbf{F}_c$, and solve (20) for $\mathbf{p}'$. This yields $\mathbf{p}' = (1, 0)$.[11] After plugging this first-order approximation into (21), and computing $\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')$, $\mathbf{F}_{\mathbf{p}c}$, and $\mathbf{F}_{cc}$, we obtain $\mathbf{p}'' = (0, 1/t)$. By Taylor's theorem, we yield the proposed second-order approximation of $\mathbf{p}$. In Appendix 1.A.4, we also show that there are no incentives to unilaterally deviate to selling only one product. □

[11] This first-order approximation perfectly mirrors Lemma 1 in Section 1.4. There, however, we have $\mathbf{p}'' = 0$, an invalid result in the case of uncorrelated vertical preferences.

Lemma 4 states that, once we assume that vertical preferences are uncorrelated, markups on L are higher than on H, at least in a neighborhood of $c_H = c_L$.[12] We discussed the reason for this in the previous paragraph (see Figure 1.4): In contrast to H-buyers, the consumer rent of L-buyers generally increases the further consumers are from $\bar{A}$, the "switching line" between buying H and L. This leads to proportionally more infra-marginal L-consumers, and acts as an incentive to narrow the gap between p_H^A and p_L^A.[13]

Before concluding this section, we make a remark about welfare. As in Section 1.4, from a utilitarian point of view, we are only interested in the allocation of consumers but not in prices. However, if prices were such that both a social planner and a consumer $\bar{\mathbf{d}} = (\bar{d}, \bar{d}_L^A, \bar{d}_L^B)$ were indifferent between $\bar{\mathbf{d}}$ buying any of the four product versions, these prices were socially optimal regarding all other consumers as well. From identical marginal costs of A and B, it follows that a planner chooses $\bar{d}^* = 1/2$. From (1), the planner's allocation further satisfies

$$v - \bar{d}/2 - c_H = v - \bar{d}/2 - t_L \bar{d}_L^A - c_L \quad \Leftrightarrow \quad \bar{d}_L^{A*} = (c_H - c_L)/t_L.$$

On the other hand, regarding the market solution, we have seen in Lemma 4 that for the vertically indifferent consumer x it holds that

$$x(p_H^*, p_L^*) = \frac{p_H^* - p_L^*}{t_L} \simeq \frac{c_H - c_L}{t_L} - \frac{(c_H - c_L)^2}{2tt_L}.$$

Accordingly, too many consumers buy H. That is, the markup on L is not higher for efficiency reasons, but only because, on top of horizontal competition softening, firm-specific vertical preferences additionally cushion competition.

[12] This might be different for slightly modified assumptions, as we show in the following section.

[13] An interesting corollary to Lemma 4 is that firms generally profit from higher (!) marginal costs on H. Only with costs (and prices) apart, the above asymmetry comes into effect. The markup on L becomes strictly higher than t, and profits reach values above $t/2$.

We summarize the central findings of this section in Proposition 3, which also rules out corner equilibria, except for $c_H = c_L$. We discuss the remaining parts of the proof of Proposition 3 in 1.A.5.[14]

Proposition 3. *If* $\mathbf{d} \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H \geq c_L$*, with* c_H *and* c_L *sufficiently close, symmetric equilibria of the pricing game are characterized by* $p_H^* \simeq c_H + t$ *and* $p_L^* \simeq c_L + t + (c_H - c_L)^2/2t$*. A's (B's) consumers buy* H *if and only if* d_L^A $(d_L^B) > (p_H^* - p_L^*)/t_L$*, and otherwise buy* L*. Each firm's equilibrium profit is* $\pi(p_H^*, p_L^*) = t/2$ *at* $c_H = c_L$*, and locally increases in* c_H*. From a welfare point of view, too many consumers buy* H*.*

1.6 Uncorrelated Vertical Preferences (II): Markups on H May Be Higher

In the previous section, we described vertical preferences by the firm-specific disutility a buyer receives if he or she consumes L instead of H. This implies that, abstracting from horizontal characteristics, the high-quality product H is perceived in exactly the same manner at the two firms. In this final section, we suppose that the relation between H and L is in the opposed direction. Consuming H instead of L now generates additional utility, which we assume to be firm-specific.

To do so, we could stay with the previous utility function, and assume for the distribution of consumers that $\mathbf{d} \sim$ i.i.d. $\mathcal{U}_3\left[(0,1) \times (-1,0) \times (-1,0)\right]$. This would both alter the formulation of Q_H^A and Q_L^A and result in a counterintuitive interpretation. In particular, L would be the high-quality product, and H would be the low-quality product, which seems somewhat odd.

[14]The proof also includes an argument why $p_H^i - p_L^j \leq t$, $\{i,j\} \in \{A,B\}$, a condition we took for granted so far. A rough intuition is as follows. With prices for H and L far apart, firm A wishes to relocate consumers from L to H by lowering p_H^A, unless there is a large amount of infra-marginal H-consumers. The latter, however, cannot be the case with $p_H^A - p_L^A > t$, as in such a (putative) equilibrium most consumers prefer to buy L.

For the sake of exposition, we thus modify the utility function (1) to

$$u^i(\mathbf{d}') := u^i(d, u_H^A, u_H^B) = \begin{cases} v - p_H^i - td^i + t_H u_H^i & \text{if } q_H^i = 1, \\ v - p_L^i - td^i & \text{if } q_L^i = 1. \end{cases} \tag{22}$$

We remain assuming unit demand, $t > 0$, $t_H > 0$, and $\mathbf{d}' \sim$ i.i.d. $\mathcal{U}_3[0,1]$. Intuitively, instead of seeing L as a (firm-specific) inferior version of H, we interpret the difference between H and L as a firm-specific "add-on" in the sense of Verboven (1999) and Ellison (2005).

Apart from that, our analysis remains the same as in Section 1.5. For this reason, we do not repeate the above arguments one by one but focus on the fundamental intuition behind the analysis.

From (22), we specify A's demand for H as

$$Q_H^A = \int_0^1 \left(\int_{x^A}^1 \left(\int_0^{\min\{\overline{HH}, \overline{HL}\}} \mathrm{d}d \right) \mathrm{d}u_H^A \right) \mathrm{d}u_H^B,$$
$$Q_L^A = \int_0^1 \left(\int_0^{x^A} \left(\int_0^{\min\{\overline{LH}, \overline{LL}\}} \mathrm{d}d \right) \mathrm{d}u_H^A \right) \mathrm{d}u_H^B.$$

Except re-labeling (writing u_H^A and u_H^B instead of d_L^A and d_L^B), Q_H^A and Q_L^A are the same as in (8) and (9). What differs, however, are the expressions for $\overline{HH}$, $\overline{HL}$, $\overline{LH}$, and $\overline{LL}$. Recall that we defined $\overline{X^A X^B}$ as the value of d such that a consumer is indifferent between buying X^A at A and X^B at B. As we see in Figure 1.5, it is now $\min\{\overline{LH}, \overline{LL}\}$ which is constant in u_H^A, whereas $\min\{\overline{HH}, \overline{HL}\}$ increases in u_H^A. This asymmetry results in an inflated amount of infra-marginal H-consumers, and we expect $p_H^* - c_H$ to exceed $p_L^* - c_L$.

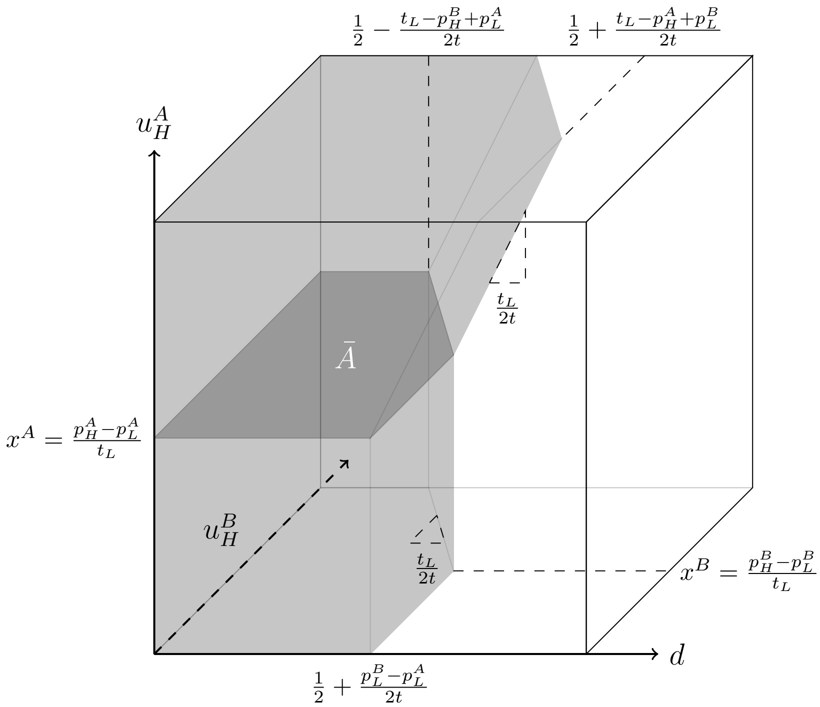


Figure 1.5: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $u_H^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $u_H^A \leq x^A$.

Indeed, by approximating equilibrium prices around $c_H = c_L + t_H$[15] in the same fashion as we did in the previous section, for $c_H \leq c_L + t_H$, we obtain

$$p_H^* = c_H + t + \frac{(c_H - c_L - t_H)^2}{2t} + \mathcal{O}\left(c_H - c_L - t_H)^3\right)$$

and

$$p_L^* = c_L + t + \mathcal{O}\left((c_H - c_L - t_H)^3\right).$$

[15]We need to approximate around $c_H = c_L + t_L$ instead of $c_H = c_L$ in order to avoid non-continuities at $c_H = c_L$. At $c_H = c_L + t_H$, we obtain $Q_H^A = 0$, which is analogous to $Q_L^A = 0$ at $c_H = c_L$ in Section 1.5. In both instances, departing from this limiting cases, the first- and second-order effects come just into effect, and we can compare the results.

Hence, markups are higher for H, and too many consumers buy L, as compared to what would socially be efficient.

After checking for unilateral deviations and considering putative corner equilibria, we can state Proposition 4, which is analogous to Proposition 3 in Section 1.5.

Proposition 4. *If the consumers' preferences are as in* (22)*,* $\mathbf{d}' \sim$ *i.i.d.* $\mathcal{U}_3[0,1]$*, and* $c_H \leq c_L + t_H$*, with* c_H *and* $c_L + t_H$ *sufficiently close, symmetric equilibria of the pricing game are characterized by* $p_H^* \simeq c_H + t + (c_H - c_L - t_H)^2/2t$ *and* $p_L^* \simeq c_L + t$*. A's (B's) consumers buy* H *if and only if* u_H^A $(u_H^B) > (p_H^* - p_L^*)/t_L$*, and otherwise buy* L*. Each firm's equilibrium profit is* $\pi(p_H^*, p_L^*) = t/2$ *at* $c_H = c_L + t_H$*, and locally increases for a decreasing* c_H*. From a welfare point of view, too many consumers buy* L*.*

1.7 Conclusion

General models of horizontal and vertical market segmentation find that, in oligopolistic contexts, markups do not vary with quality.

We qualify this somewhat counterintuitive result by relaxing the assumption that vertical preferences are perfectly harmonious across firms: a high reduction in utility from consuming the low-quality product instead of the high-quality product at one firm not necessarily implies the same at the firm's competitor. In such a setting, we find that markups on low-quality products exceed markups on high-quality products in a symmetric interior equilibrium. To the contrary, if low-quality products are only distinguished by their horizontal characteristics, and supplementary utility from consuming a high-quality product is specific to the firm, we obtain the opposite result.

From a welfare perspective of view, we want horizontal competition to be weak. Although this softens competition, the lack of horizontal competitiveness does not affect horizontal allocative efficiency.[16] In addition, if firms are horizon-

[16]This, of course, crucially depends on our assumption of fully covered markets.

tal substitutes, they differentiate their customers vertically. A social planner wants to prevent this. Hence, regarding horizontal differentiation, the firms' objective is perfectly in line with the objective of the planner. On the other hand, regarding vertical preferences, the objectives of firms and planner are diametrically opposed. If the prices of low-quality products are distorted, we want the marginal valuation of quality to be high, such that more consumers buy the high-quality product. If the prices of high-quality products are distorted, we want the marginal valuation of quality to be low, such that more consumers buy the low-quality product. On the other hand, as we have seen, firms' profits unanimously increase in the extent of price distortions.

In order to test our opposed results of Sections 1.5 and 1.6, we could ask for what species of products it is the high-quality version for which the differentiation is specific to the firm, and for which it is the low-quality version. According to our theory, it is firm-specific differentiation which opens the door for higher markups.

An interesting theoretical exercise would be to endogenize the firms' choice of the type and degree of differentiation. This, however, brings along intricacies regarding asymmetric departure points in the second stage. As often in the field, simulations could be a viable backdoor strategy.

Finally, our theory could be improved by allowing consumers to buy multiple (or zero) units; by allowing firms to sell more than two product versions; by considering asymmetric equilibria; or by including the entry decision of (additional) firms. These and related considerations indicate that the scope for augmenting and adjusting our model is almost unlimited.

1.A Appendices to Chapter 1

1.A.1 First-order Conditions

The FOC with respect to p_H^A is

$$\begin{aligned}\frac{\partial \pi^A}{\partial p_H^A} = \int_0^1 \Bigg(& -\int_{x^A}^1 \underbrace{\frac{1}{2t} f\left(\min\{\overline{HH},\overline{HL}\}, d_L^A, d_L^B\right)}_{=:m^H} \mathrm{d}d_L^A \\ & - \frac{1}{t_L}\left(\underbrace{\int_0^{\min\{\overline{HH}(d_L^A=x^A),\overline{HL}(d_L^A=x^A)\}} f(d,x^A,d_L^B)\mathrm{d}d}_{=:A^H(d_L^A=x^A)}\right)\Bigg)\mathrm{d}d_L^B(p_H^A-c_H) \\ & + \int_0^1\left(\int_{x^A}^1\left(\underbrace{\int_0^{\min\{\overline{HH},\overline{HL}\}} f(d,d_L^A,d_L^B)\mathrm{d}d}_{=:A^H}\right)\mathrm{d}d_L^A\right)\mathrm{d}d_L^B \\ & + \int_0^1\left(\frac{1}{t_L}\underbrace{\int_0^{\min\{\overline{LH}(d_L^A=x^A),\overline{LL}(d_L^A=x^A)\}} f(d,x^A,d_L^B)\mathrm{d}d}_{=:A^L(d_L^A=x^A)}\right)\mathrm{d}d_L^B(p_L^A-c_L) \stackrel{!}{=} 0, \end{aligned} \tag{A.1}$$

with

$$\begin{aligned}\overline{HH} &:= \frac{1}{2}+\frac{p_H^B-p_H^A}{2t}, \\ \overline{HL} &:= \frac{1}{2}+\frac{p_L^B-p_H^A}{2t}+\frac{t_L}{2t}d_L^B, \\ \overline{LH} &:= \frac{1}{2}+\frac{p_H^B-p_L^A}{2t}-\frac{t_L}{2t}d_L^A, \\ \overline{LL} &:= \frac{1}{2}+\frac{p_L^B-p_L^A}{2t}+\frac{t_L}{2t}(d_L^B-d_L^A).\end{aligned}$$

In a symmetric equilibrium with $p_L^A = p_L^B =: p_L$, $p_H^A = p_H^B =: p_H$ (and thus $x^A = x^B =: x$), it holds that

$$\min\{\overline{HH},\overline{HL}\} = \begin{cases}\overline{HH} = 1/2 & \text{if } d_L^B \in (x,1] \\ \overline{HL} = (1/2t)(t+p_L-p_H+t_L d_L^B) & \text{if } d_L^B \in [0,x],\end{cases} \tag{A.2}$$

and

$$\min\{\overline{LH},\overline{LL}\} = \begin{cases} \overline{LH} = (1/2t)(t + p_H - p_L - t_L d_L^i\ ) & \text{if } d_L^B \in (x,1] \\ \overline{LL} = (1/2t)(t + t_L(d_L^B - d_L^A)) & \text{if } d_L^B \in [0,x]. \end{cases} \tag{A.3}$$

Therefore, and from evaluating (A.2) and (A.3) at $d_L^A = x$, we have

$$A^H(d_L^A = x | d_L^B > x) = A^L(d_L^A = x | d_L^B > x) = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d,$$

since

$$A^H(d_L^A = x) = \int_0^{\overline{HH}(d_L^A = x)} f(d, x, d_L^B)\mathrm{d}d = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d,$$

and

$$A^L(d_L^A = x) = \int_0^{\overline{LH}(d_L^A = x)} f(d, x, d_L^B)\mathrm{d}d = \int_0^{1/2} f(d, x, d_L^B)\mathrm{d}d.$$

Analogically, for $d^B \leq x$, we obtain

$$A^H(d_L^A = x | d_L^A \leq x) = A^L(d_L^A = x | d_L^A \leq x) = \int_0^{\frac{1}{2} + \frac{p_L - p_H}{2t} + \frac{t_L}{2t} d_L^B} f(d, x, d_L^B)\mathrm{d}d.$$

Consequently, we can simplify the FOC (A.1) to

$$\begin{aligned} &\left(\int_0^1 \int_x^1 m^H \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^H(d_L^A = x)}{t_L} \mathrm{d}d_L^B\right)(p_H - c_H) \\ = &\underbrace{\int_0^1 \int_x^1 A^H \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_H^A} + \int_0^1 \frac{A^H(d_L^A = x)}{t_L} \mathrm{d}d_L^B (p_L - c_L) \end{aligned} \tag{A.4}$$

The FOC with respect to p_L^A is

$$\frac{\partial \pi^A}{\partial p_L^A} = \int_0^1 \left(- \int_0^{x^A} \underbrace{\frac{1}{2t} f\left(\min\{\overline{LH}, \overline{LL}\}, d_L^A, d_L^B\right)}_{=:m^L} \mathrm{d}d_L^A \right.$$
$$- \frac{1}{t_L} \left(\underbrace{\int_0^{\min\{\overline{LH}(d_L^A = x^A), \overline{LL}(d_L^A = x^A)\}} f(d, x^A, d_L^B) \mathrm{d}d}_{=A^L(d_L^A = x^A)} \right) \Bigg) \mathrm{d}d_L^B (p_L^A - c_L)$$
$$+ \int_0^1 \left(\int_0^{x^A} \left(\underbrace{\int_0^{\min\{\overline{LH}, \overline{LL}\}} f(d, d_L^A, d_L^B) \mathrm{d}d}_{=A^L} \right) \mathrm{d}d_L^A \right) \mathrm{d}d_L^B$$
$$+ \int_0^1 \left(\frac{1}{t_L} \underbrace{\int_0^{\min\{\overline{HH}(d_L^A = x^A), \overline{HL}(d_L^A = x^A)\}} f(d, x^A, d_L^B) \mathrm{d}d}_{=A^H(d_L^A = x^A)} \right) \mathrm{d}d_L^B (p_H^A - c_H) \stackrel{!}{=} 0. \quad \text{(A.5)}$$

For the version of (A.5) which applies in a symmetric interior equilibrium, we use $A^L(d_L^A = x) = A^H(d_L^A = x)$ from above. Accordingly, we have

$$\left(\int_0^1 \int_0^x m^L \mathrm{d}d_L^A \mathrm{d}d_L^B + \int_0^1 \frac{A^L(d_L^A = x)}{t_L} \mathrm{d}d_L^B \right) (p_L - c_L)$$
$$= \underbrace{\int_0^1 \int_0^x A^L \mathrm{d}d_L^A \mathrm{d}d_L^B}_{=Q_L^A} + \int_0^1 \frac{A^L(d_L^A = x)}{t_L} \mathrm{d}d_L^B (p_H - c_H). \quad \text{(A.6)}$$

1.A.2 Proof of Lemma 1 (Completion)

In absence of corner solutions, the proof is given in the main body of the text. Alternatively, firm A may only sell either H or L. By only selling H, A's objective function is

$$\pi^A = \int_0^1 \min \left\{ \frac{1}{2} + \frac{p_H^B - p_H^A}{2t}, \frac{1}{2} + \frac{p_L^B - p_H^A}{2t} + \frac{t_L}{2t} d_L \right\} \mathrm{d}d_L (p_H^A - c_H),$$

the FOC of which is

$$p_H^A - c_H = \int_0^1 \min \left\{ t + p_H^B - p_H^A, t + p_L^B - p_H^A + t_L d_L \right\} \mathrm{d}d_L.$$

After plugging in $p_H^B = c_H + t$ and $p_L^B = c_L + t$, we obtain $\tilde{p}_H^A = c_H + t - (c_H - c_L)^2/4t_L$. A's markup on H thus is smaller than or equal to t. As A's equilibrium profit is $t/2$, it remains to be shown $Q_H^A(\tilde{p}_H^A) \leq 1/2$. Formally,

$$\begin{aligned} Q_H^A(\tilde{p}_H^A) = &\int_{\frac{c_H - c_L}{t_L}}^1 \frac{1}{2} + \frac{(c_H + t) - \left(c_H + t - (c_H - c_L)^2/4t_L\right)}{2t} \mathrm{d}d_L \\ &+ \int_0^{\frac{c_H - c_L}{t_L}} \frac{1}{2} + \frac{(c_L + t) - \left(c_H + t - (c_H - c_L)^2/4t_L\right)}{2t} + \frac{t_L}{2t} d_L \mathrm{d}d_L \leq \frac{1}{2}, \end{aligned}$$

from the definition of Q_H^A. This holds for all c_H and c_L, because

$$\begin{aligned} &\Leftrightarrow \frac{(c_H - c_L)^2}{4t_L} + \int_0^{\frac{c_H - c_L}{t_L}} t_L d_L \mathrm{d}d_L \leq \int_0^{\frac{c_H - c_L}{t_L}} c_H - c_L \mathrm{d}d_L \\ &\Leftrightarrow \frac{(c_H - c_L)^2}{4t_L} + \frac{(c_H - c_L)^2}{2t_L} \leq \frac{(c_H - c_L)^2}{t_L}. \end{aligned}$$

By only selling L, A's objective function is

$$\pi^A = \int_0^1 \min\left\{\frac{1}{2} + \frac{p_H^B - p_L^A}{2t} - \frac{t_L}{2t} d_L, \frac{1}{2} + \frac{p_L^B - p_L^A}{2t}\right\} \mathrm{d}d_L (p_L^A - c_L).$$

From the associated FOC, we obtain $\tilde{p}_L^A = c_L + t - (t_L - c_H + c_L)^2/4t_L$. Again, it is sufficient to show that $Q_L^A(\tilde{p}_L^A) \leq 1/2$. Formally,

$$\begin{aligned} Q_L^A(\tilde{p}_H^A) = &\int_{\frac{c_H - c_L}{t_L}}^1 \frac{1}{2} + \frac{(c_H + t) - \left(c_L + t - (t_L - c_H + c_L)^2/4t_L\right)}{2t} - \frac{t_L}{2t} d_L \mathrm{d}d_L \\ &+ \int_0^{\frac{c_H - c_L}{t_L}} \frac{1}{2} + \frac{(c_L + t) - \left(c_L + t - (t_L - c_H + c_L)^2/4t_L\right)}{2t} \mathrm{d}d_L \leq \frac{1}{2}, \end{aligned}$$

from the definition of Q_L^A. This holds for all considered values of c_H and c_L, because

$$\begin{aligned} &\Leftrightarrow \frac{(t_L - c_H + c_L)^2}{4t_L} + \int_{\frac{c_H - c_L}{t_L}}^1 (c_H - c_L) \mathrm{d}d_L \leq \int_{\frac{c_H - c_L}{t_L}}^1 t_L d_L \mathrm{d}d_L \\ &\Leftrightarrow \frac{t_L - c_H + c_L}{4} + (c_H - c_L) \leq \frac{t_L}{2}\left(1 + \frac{c_H - c_L}{t_L}\right) \\ &\Leftrightarrow c_H - c_L \leq t_L, \end{aligned}$$

which is true by assumption.

1.A.3 Proof of Proposition 2 (Completion)

From Lemma 1 and condition (4), it follows that symmetric interior equilibria exist if and only if $0 \leq c_H - c_L < t_L$, and that they are characterized by $p_H^* = c_H + t$ and $p_L^* = c_L + t$. Regarding symmetric corner equilibria, first assume that both firms only sell H. In this case, firm A maximizes

$$\pi^A = \left(\frac{1}{2} + \frac{p_H^B - p_H^A}{2t}\right)(p_H^A - c_H), \tag{A.7}$$

with respect to p_H^A, as in the standard Hotelling case. The (symmetric) solution of (A.7) is ${p^A}^*_H = c_H + t$, and A's profit is ${\pi^A}^* = t/2$. Whenever A finds a price $\tilde{p}_L^A$ such that its total demand increases without decreasing the markup on either of its products, only selling H cannot be an equilibrium. From (2), A steals customers from B by offering L if and only if

$$v - (\tilde{p}_L^A + t_L \times 0) - t/2 > v - (c_H + t) - t/2 \;\;\Leftrightarrow\;\; \tilde{p}_L^A < c_H + t. \tag{A.8}$$

Meanwhile, A's markup on either product is not lowered if and only if

$$\tilde{p}_L^A - c_L \geq p_H^A - c_H = t \;\;\Leftrightarrow\;\; p_L^A \geq c_L + t. \tag{A.9}$$

(A.8) and (A.9) are simultaneously feasible if and only if

$$c_L + t < c_H + t \;\;\Leftrightarrow\;\; c_L < c_H.$$

For $c_H = c_L$, check that $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$ solve (14) and (15) for $p_H^B = c_H + t$ and $p_L^B = \infty$. The resulting profit is $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = t/2$, hence there is no incentive to offer both H and L in this case.

If both firms only sell L, A objective function is also the one of a standard Hotelling game, because $d_L^A = d_L^B = d_L$. Therefore, ${p^A}^*_L = c_L + t$, and ${\pi^A}^* = 1/2$. In this case, the equivalents to (A.9) and (A.8) yield that A has an incentive to offer H if and only if $c_H - c_L < t_L$. For $c_H = c_L + t_L$, $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$ solve (A.4) and (A.6) for $p_H^B = \infty$ and $p_L^B = c_L + t$. Again, the resulting profit is $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = 1/2$, and there is no incentive to offer both H and L.

1.A.4 Proof of Lemma 4 (Completion)

We write (16) and (17) as

$$\tilde{f}(\mathbf{p}) := \frac{\bar{A}}{t_L}\left(p_H - c_H - p_L + c_L - t_L(1-x)\right) + \frac{1-x}{2t}(p_H - c_H) = 0,$$

and

$$\tilde{g}(\mathbf{p}) := \frac{\bar{A}}{t_L}\left(p_L - c_L - p_H + c_H - t_L x\right) + \frac{x}{2t}\left(p_L - c_L - \frac{p_H - p_L}{2}\right) = 0.$$

In the following, we use $f(\mathbf{p}) = 2tf(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 2tg(\mathbf{p}) = 0$, which simplifies fractions, as we see later on. Furthermore, by using $\bar{A} = 1/2 - (t_L/4t)x^2$ and $x = (p_H - p_L)/t_L$, we write $f(\mathbf{p}) = 0$ and $g(\mathbf{p}) = 0$ as

$$\begin{aligned} 2t_L f(\mathbf{p}) := &\left(2tt_L - (p_H - p_L)^2\right)(2(p_H - p_L) - c_H + c_L - t_L) \\ &+ 2t_L(t_L - p_H + p_L)(p_H - c_H) = 0, \end{aligned} \tag{A.10}$$

and

$$\begin{aligned} 2t_L g(\mathbf{p}) := &\left(2tt_L - (p_H - p_L)^2\right)(2(p_L - p_H) + c_H - c_L) \\ &+ 2t_L(p_H - p_L)(p_L - c_L) - t_L(p_H - p_L)^2 = 0. \end{aligned} \tag{A.11}$$

The partial derivatives of (A.10) and (A.11) with respect to p_H, p_L, and c_H are

$$\begin{aligned} 2t_L \frac{\partial f(\mathbf{p})}{\partial p_H} =& - 2(p_H - p_L)(2(p_H - p_L) - c_H + c_L - t_L) \\ &+ \left(2tt_L - (p_H - p_L)^2\right)2 - 2t_L\left(2p_H - p_L - c_H - t_L\right), \end{aligned} \tag{A.12}$$

$$\begin{aligned} 2t_L \frac{\partial f(\mathbf{p})}{\partial p_L} =& \ 2(p_H - p_L)(2(p_H - p_L) - c_H + c_L - t_L) \\ &- \left(2tt_L - (p_H - p_L)^2\right)2 + 2t_L\left(p_H - c_H\right), \end{aligned} \tag{A.13}$$

$$2t_L \frac{\partial f(\mathbf{p})}{\partial c_H} = - 2tt_L + (p_H - p_L)^2 - 2t_L\left(t_L - p_H + p_L\right), \tag{A.14}$$

$$\begin{aligned} 2t_L \frac{\partial g(\mathbf{p})}{\partial p_H} =& - 2(p_H - p_L)(2(p_L - p_H) + c_H - c_L) \\ &- \left(2tt_L - (p_H - p_L)^2\right)2 + 2t_L(2p_L - p_H - c_L), \end{aligned} \tag{A.15}$$

$$\begin{aligned} 2t_L \frac{\partial g(\mathbf{p})}{\partial p_L} =& \ 2(p_H - p_L)(2(p_L - p_H) + c_H - c_L) \\ &+ \left(2tt_L - (p_H - p_L)^2\right)2 - 2t_L\left(3p_L - 2p_H - c_L\right). \end{aligned} \tag{A.16}$$

$$2t_L \frac{\partial g(\mathbf{p})}{\partial c_H} = \ 2tt_L - (p_H - p_L)^2. \tag{A.17}$$

After substituting $p_H = c_H + t$ and $p_L = c_L + t$ into (A.12) through (A.17), we write (20) as

$$\begin{pmatrix} t + t_L & -t \\ -t & t \end{pmatrix} \begin{pmatrix} p'_H \\ p'_L \end{pmatrix} + \begin{pmatrix} -(t + t_L) \\ t \end{pmatrix} = 0,$$

the solution of which is $(p'_H, p'_L) = (1, 0)$. After taking the derivatives of (A.12) through (A.17) with respect to p_H, p_L, and c_H, and after plugging in $p_H = c_H + t$ and $p_L = c_L + t$ once more, we use $(p'_H, p'_L) = (1, 0)$ in order to write (21) as

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H = c_L}\mathbf{p}' + \begin{pmatrix} t + t_L & -t \\ -t & t \end{pmatrix} \begin{pmatrix} p''_H \\ p''_L \end{pmatrix} + 2\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0. \tag{A.18}$$

Further, we write $[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}'$ as $\mathbf{F}_{\mathbf{pp}(1)}\text{vec}(\mathbf{p}'\mathbf{p}'^T)$, where $\mathbf{F}_{\mathbf{pp}(1)}$ is the 1-mode flattening matrix of the Hessian tensor of $\mathbf{F}$ with respect to $\mathbf{p}$. That is,

$$[\mathcal{T}_{\mathbf{pp}}(\mathbf{p}')]_{c_H=c_L}\mathbf{p}' = \begin{pmatrix} f_{HH} & f_{HL} & f_{LH} & f_{LL} \\ g_{HH} & g_{HL} & g_{LH} & g_{LL} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} f_{HH} \\ g_{HH} \end{pmatrix},$$

where $f_{ij} := \left[\partial^2 f(\cdot)/\partial p_i \partial p_j\right]_{c_H=c_L}$ and $g_{ij} := \left[\partial^2 g(\cdot)/\partial p_i \partial p_j\right]_{c_H=c_L}$. From (A.12) and (A.15), we have $f_{HH} = g_{HH} = -1$, and we write (A.18) as

$$\begin{pmatrix} t + t_L & -t \\ -t & t \end{pmatrix} \begin{pmatrix} p''_H \\ p''_L \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix},$$

the solution of which is $(p''_H, p''_L) = (0, 1/t)$.
Alternatively, firm A may only sell either H or L. In order to analyze such unilateral deviations for the general case $c_H \geq c_L$, it is useful to first show that the objective function is concave at $c_H = c_L$. From (A.1) and (A.5) with $f(\mathbf{d}) \equiv 1$, and from the fact that the $\pi^A(p_H^A, p_L^A)$ is twice differentiable, it follows that

$$\left[\frac{\partial^2 \pi^A}{(\partial p_H^A)^2}\right]_{c_H=c_L} = -\frac{1}{t} - \frac{1}{2t_L}, \quad \left[\frac{\partial^2 \pi^A}{(\partial p_L^A)^2}\right]_{c_H=c_L} = -\frac{1}{2t_L},$$

and

$$\left[\frac{\partial^2 \pi^A}{\partial p_H^A \partial p_L^A}\right]_{c_H=c_L} = \left[\frac{\partial^2 \pi^A}{\partial p_L^A \partial p_H^A}\right]_{c_H=c_L} = \frac{1}{2t_L}.$$

π^A is locally strictly concave at $c_H = c_L$ if the Hessian matrix $\mathbf{H}$ of its second derivatives is negative definite. This is the case here, since the leading principal minors of

$$-\mathbf{H} = \begin{pmatrix} \frac{1}{t} + \frac{1}{2t_L} & -\frac{1}{2t_L} \\ -\frac{1}{2t_L} & \frac{1}{2t_L} \end{pmatrix}$$

are $1/t + 1/2t_L$ and $1/2tt_L$, that is, positive.
Next, for the case that $c_H = c_L$, consider firm A's deviation to only selling H. In this case, its objective function is

$$\tilde{\pi}^A = \int_0^1 \min\left\{\frac{1}{2} + \frac{p_H^B - p_H^A}{2t}, \frac{1}{2} + \frac{p_L^B - p_H^A}{2t} + \frac{t_L}{2t} d_L^B\right\} \mathrm{d}d_L^B.$$

We state the according FOC with respect to p_H^A as

$$\frac{\partial \tilde{\pi}^A}{\partial p_H^A} = \int_0^1 \underbrace{\min\left\{t + p_H^B - p_H^A, t + p_L^B - p_H^A + t_L d_L^B\right\}}_{t+p_H^B - p_H^A \leq t + p_L^B - p_H^A + t_L d_L^B \;\Leftrightarrow\; d_L^B \geq (p_H^B - p_L^B)/t_L} \mathrm{d}d_L^B - (p_H^A - c_H) \overset{!}{=} 0.$$

Using $p_H^B = c_H + t$ and $p_L^B = c_L + t$, this is

$$\begin{aligned} &\int_0^{\frac{c_H - c_L}{t_L}} (t + c_L + t - p_H^A + t_L d_L^B)\mathrm{d}d_L^B \\ &+ \int_{\frac{c_H - c_L}{t_L}}^1 (t + c_H + t - p_H^A)\mathrm{d}d_L^B = p_A^A - c_H. \end{aligned}$$

From $c_H = c_L$, the first term cancels out, which implies $\tilde{p}_H^A = c_H + t$. Since L is bought with probability 0 at $c_H = c_L$, we have $\pi^A(\tilde{p}_H^A, \infty) = t/2$, and there is no incentive to unilaterally deviate. Furthermore, at $c_H = c_L$, we can equivalently set $(\tilde{p}_H^A, \tilde{p}_L^A)$ to $(c_H + t, c_L + t)$, since there the "no-selling constraint" with respect to L is not binding. From strict concavity and continuity of π^A at $c_H = c_L$, we know that the prices in Lemma 4 are uniquely best answers for $c_H > c_L$ sufficiently close. This makes the no-selling constraint concerning $(\tilde{p}_H^A, \tilde{p}_L^A)$ binding, which proves that deviating to only selling H is unilaterally not beneficial.
Next, consider firm A's deviation to only selling L. Here, the objective function is

$$\begin{aligned} \tilde{\pi}^A = \int_0^1 \int_0^1 \min\Bigg\{ &\frac{1}{2} + \frac{p_H^B - p_L^A}{2t} - \frac{t_L}{2t} d_L^A, \\ &\frac{1}{2} + \frac{p_L^B - p_L^A}{2t} + \frac{t_L}{2t}(d_L^B - d_L^A)\Bigg\} \mathrm{d}d_L^A \mathrm{d}d_L^B (p_L^A - c_L). \end{aligned} \tag{A.19}$$

For $c_H = c_L$, we write the according FOC with respect to p_L^A as

$$\int_0^1 \int_0^1 (t + p_H^B - p_L^A - t_L d_L^A)\mathrm{d}d_L^A \mathrm{d}d_L^B \overset{!}{=} p_L^A - c_L,$$

which implies $\tilde{p}_L^A = c_L + t - t_L/4$. By plugging $\tilde{p}_L^A$, $p_H^B = c_H + t$, and $p_L^B = c_L + t$ into (A.19), and by imposing $c_H = c_L$, we yield

$$\pi^A(\infty, \tilde{p}_L^A) = \left(\frac{1}{2} - \frac{t_L}{8t}\right)\left(t - \frac{t_L}{4}\right) < \frac{1}{2}t = \pi^A(p_H^*, p_L^*).$$

Since both (p_H^*, p_L^*) and $\tilde{p}_L^A$ are continuous functions in c_H and c_L, unilaterally deviating to only selling L is also not beneficial in the case of $c_H > c_L$ with c_H and c_L sufficiently close.

1.A.5 Proof of Proposition 3 (Completion)

First, assume that both firms only sell H. In this case, they play a standard Hotelling game, the solution of which is $p_H^* = c_H + t$ and $\pi^A(p_H^*) = t/2$. Consider A's potential deviation to selling both products. The according FOCs are given by (A.1) and (A.5) with $p_L^B = \infty$. By simplifying these we yield (16) and (17), with $\bar{A} := 1/2 + (p_H^B - p_H^A)2t$. For $c_H = c_L$, the according solution is $\tilde{p}_H^A = c_H + t$ and $\tilde{p}_L^A = c_L + t$. The resulting allocation of consumers is the same as in the standard Hotelling case, and $\pi^A(\tilde{p}_H^A, \tilde{p}_L^A) = t/2$. Therefore, both firms only offering H is another equilibrium if $c_H = c_L$. If $c_H > c_L$, we apply the same approximation method as in the proof of Lemma 4. We obtain

$$\tilde{p}_H^A = c_H + t + \frac{(c_H - c_L)^2}{4t_L} + \mathcal{O}\left((c_H - c_L)^3\right)$$

and

$$\tilde{p}_L^A = c_L + t + \left(\frac{1}{t} + \frac{1}{2t_L}\right)\frac{(c_H - c_L)^2}{2} + \mathcal{O}\left((c_H - c_L)^3\right).$$

Since $\tilde{p}_H^A - \tilde{p}_L^A > 0$, firm A seeks to sell L, which in turn eliminates the putative equilibrium. Consider the opposite case where both firms only sell L. Then, A's objective function is

$$\pi^A = \int_0^1 \int_0^1 \left(\frac{1}{2} + \frac{p_L^B - p_L^A}{2t}\right) \mathrm{d}d_L^A \mathrm{d}d_L^B (p_L^A - c_L).$$

From the associated FOC, and after imposing symmetry, we obtain $p_L^* = c_L + t$ and $\pi^A(p_L^*) = t/2$. In this case, A can increase its profit by only selling H at $\tilde{p}_H^A = c_L + t$. By doing so, A covers the whole market, and its profit is $\pi^A(\tilde{p}_H^A) = t - (c_H - c_L)$. Since $\pi^A(\tilde{p}_H^A) > \pi^A(p_L^*)$ if and only if $t/2 > c_H - c_L$, firm A prefers to sell H if $c_H \geq c_L$ are sufficiently close. This eliminates the second putative equilibrium.

So far, we assumed that $p_H^i - p_L^j \leq t$, $\{i, j\} \in \{A, B\}$. Suppose now, to the opposite, that $p_H^A - p_L^A > t$. From (4), there is is no interior equilibrium if $t > t_L$. Therefore, it remains for us to demonstrate that $t < p_H^* - p_L^* \leq t_L$ cannot be an equilibrium. From $p_H^A - p_L^B > t$ it follows that (8) and (9) no longer represent Q_H^A and Q_L^A, since $\min\{\overline{HH}, \overline{HL}\}$ and $\min\{\overline{LH}, \overline{LL}\}$ lie outside $[0, 1]$ for some combinations of $d_L^A \in [0, 1]$ and $d_L^B \in [0, 1]$. This is illustrated in Figure 1.A.1. Therefore, FOCs (16) and (17) need to be adjusted accordingly. The adapted analogue of (16) is

$$\left(\frac{\bar{F}}{2t} + \frac{\bar{A}}{t_L}\right)(p_H - c_H) = \bar{F}\bar{A} + \frac{\bar{A}}{t_L}(p_L - c_L), \tag{A.20}$$

where

$$\bar{A} := \frac{1}{2t_L}\left(t_L - p_H + p_L + \frac{t}{2}\right)$$

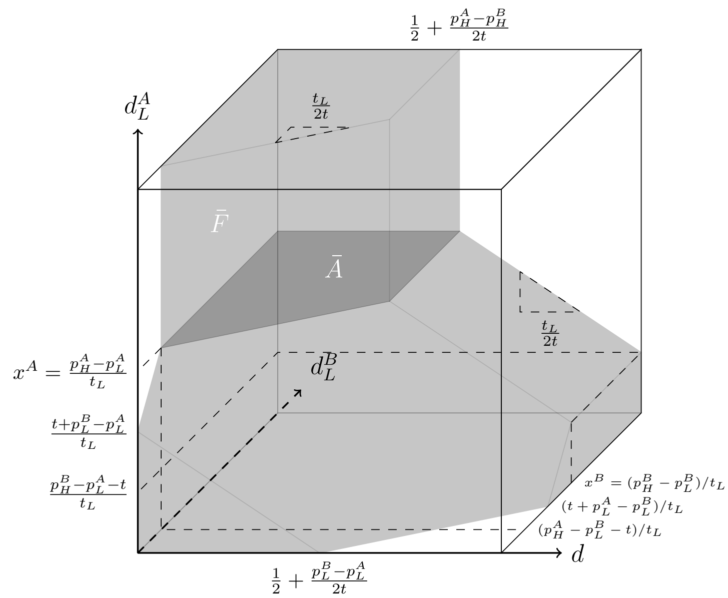


Figure 1.A.1: Consumers who buy A's high-quality product, Q_H^A, are displayed in the upper region, where $d_L^A > x^A$. Consumers who buy A's low-quality product, Q_L^A, are displayed in the lower region, where $d_L^A \leq x^A$.

represents the frontier between consumers of H and consumers of L at A, and

$$\bar{F} := \left(1 - \frac{p_H - p_L - t}{t_L}\right)\left(1 - \frac{p_H - p_L}{t_L}\right)$$

represents the "inter-firm" marginal consumers (see Figure 1.A.1). For $c_H \geq c_L$ with c_H and c_L sufficiently close, (A.20) is necessarily violated if

$$\frac{\bar{A}}{t_L}(p_H - p_L) > \bar{F}\bar{A} - \frac{\bar{F}}{2t}(p_H - c_H). \tag{A.21}$$

From $p_H - p_L \leq t_L$, we have $\bar{A}/t_L \geq t/4t_L^2$; and from $p_H - p_L > t$, we have $\bar{F} < (t_L - t)/t_L$. Using (A.21), it is thus sufficient to show that

$$\frac{t}{4t_L^2} \times t > \frac{t_L - t}{t_L}\left(\bar{A} - \frac{p_H - c_H}{2t}\right) \tag{A.22}$$

Also from $p_H - p_L > t$, we have $p_H - c_H > t + p_L - c_H$. For c_H sufficiently close to c_L, this implies $p_H - c_H > t$, since $p_L \geq c_L$ in any interior equilibrium. Further, $p_H - p_L > t$ implies that $\bar{A} < (1/2t_L)(t_L - t/2) = 1/2 - t/4t_L$. Using (A.22), it consequently suffices to show that

$$\left(\frac{t}{2t_L}\right)^2 > \frac{t_L - t}{t_L}\left(\frac{1}{2} - \frac{t}{4t_L} - \frac{1}{2}\right) \tag{A.23}$$

Since the left-hand side of (A.23) is positive, and the right-hand side of (A.23) is negative, we have eliminated the third putative equilibrium.

Chapter 2

Selection upon Wage Posting

2.1 Introduction

Why do most job interviews end up without an agreement? In a recent study on the Swiss market for skilled labor, Blatter, Muehlemann, and Schenker (2012) establish that firms interview an average of 4.8 applicants to fill a single vacancy. Similar results are available for other countries.[1] In areas such as finance and consulting, the percentage of applicants that are actually offered positions is even considerably smaller. As Rivera (2011) points out, "elite professional service firms often receive thousands or even tens of thousands of applications for fewer than two hundred spots, yielding admissions ratios at the most prestigious firms that are more competitive than that of any Ivy League College." Related to this, Moen (2003) summarizes that it is a "shared belief in the labor market segmentation literature [...] that too few high wage (primary sector) jobs are created in the market."

Observed high-end salaries, on the other hand, suggest that "high-end labor" is a very scarce resource. Within a simple Walrasian framework, the salaries

[1]In an earlier study on the US labor market, Barron, Bishop, and Dunkelberg (1985) report an average of 6.3 interviewed applicants to fill a position.

prevailing in the mentioned industries indicate a steep supply curve, that is, a small number of job candidates.

How can these two facts, small acceptance rates and high salaries, fit together? We introduce a model where firms first announce their salaries. Thereupon, candidates of differing productivities apply. Finally, each firm determines a threshold regarding productivity signals, above which it is willing to hire an applicant. These signals or "scores" can be interpreted as bits of information which emerge, for instance, in the course of an entry talk.

Using parametric assumptions, we simulate the firms' selection process, which in turn determines the applicants' employment probabilities. We show that, compared to a Pareto optimal allocation, applicants approach firms with salaries too high and employment probabilities too low. In particular, we find an explanation why most encounters between employers and applicants end up without agreements, even though—or rather because—salaries are high.

What are the key ingredients of a job market of which the equilibrium features the afore-mentioned attributes?

Most of the modern labor-market literature is along the lines of Mortensen and Pissarides (1994). The matching approach which is applied there stands in a rich tradition of models that include bargaining as a mean of wage determination.[2] In contrast to simplifying supply-and-demand considerations, this branch of search theory is suitable to address issues such as the extent of unemployment. However, it does not provide a sufficiently explicit characterization of the matching and wage formation process itself. Furthermore, it lacks to determine the division of the surplus endogenously.

In addition to these theoretical concerns, models with ex-post bargaining are repeatedly contested by empirical work. Hall and Krueger (2008), for instance, estimate that between a quarter and a half of the workers in the United States are employed in jobs where wages are posted. Similar results can also be found

[2] An overview of these models is given, for instance, in Rogerson, Shimer, and Randall (2005).

in earlier literature.[3] Hence, although models with announced wages exhibit the disputable feature of firms being committed to ignore counteroffers, empirical evidence suggests that wage posting takes place to a significant extent.

Theoretical work uses the term “directed search” to describe labor markets with posted wages. In directed-search models, workers typically face trade-offs between higher wages and higher probabilities of getting a job. Thereby, credible announcing of wages generally ensures constrained-efficient market outcomes, whereas the “constraints” arise from the fact that workers cannot coordinate their application behavior and, as a result, are forced to randomly pick some employer. By introducing such coordination failures, frictions arise in these models as well, but workers are directed towards efficient applying behavior, since firms implicitly reveal their hiring probabilities by announcing wages.[4]

There exists, however, a branch within this literature which stresses potential market failure, arising from the existence of heterogenous applicants. A series of papers within the framework of Lang and Manove (2003) works out discretely separated employment rates which are due to negligible differences in productivity. Similarily, Moen (2003) mentions unconditional wage announcements and worker-firm specific productivities as a source of inefficiently low aggregate

[3]By showing that higher wages attract longer queues of applicants, the study of Holzer, Katz, and Krueger (1991) implicitly implies that there is at least partial commitment on behalf of the firms to refuse any kind of bargaining.

Along the same lines is the finding in Wial (1991) where it is described that chronic excess supply of adequate candidates often coincides with jobs that are ex-ante considered to be good ones, whereas the notion of “good” includes pay and is often passed on interpersonally.

[4]By assuming that firms are price-takers, Montgomery (1991) shows for the case of a large labor market with identical applicants that employers endogenize the workers’ trade-off between search intensity and wage, which ensures efficiency. Going one step further, but omitting welfare considerations, Peters (1991) takes into account the effect of unilaterally deviating firms by considering the limit case of an atomic economy. Based on these two papers, Moen (1997) and Shimer (2001) introduce heterogeneity concerning the productivity of vacant jobs and workers, respectively, whereby the equilibrium efficiency features of the earlier developed models are maintained.

Analogous results, also concerning multiple worker types, are obtained in Coles and Eeckhout (2000) and Michelacci and Suarez (2006), where firms endogenously opt for posted wages as opposed to auctions and bargaining.

production. More specifically, Moen shows that low-productivity workers jeopardize high-productivity workers to such an extent that firms are forced to raise their posted wages in order to screen the market for superior productivities. Although they are capable of doing so, in a separating equilibrium, the high-type candidates' employment probabilities turn out to be well below efficiency.

We adopt a similar idea in the following. But in contrast to the afore-mentioned work, we introduce two innovations. First, we do not assume that firms can only hire one applicant. Second, and more importantly, we take into consideration that, upon being contacted, employers are not fully informed about their candidates' productivity. As opposed to existing work on directed search, we proceed on the assumption that, once a firm is contacted, the hiring process is not yet at its end. However, unlike work within the matching tradition, we do not assume that the remaining interaction consists of negotiating on the payment. Instead, we find it a more convincing assumption that job talks are meant to evaluate the applicants' abilities.

Imagine the following scenario. Firms post their salaries, say on the Internet or in a newspaper. These binding offers are noticed by a large number of potential employees who reflect on their relative prospects at the different firms. Their productivities are private knowledge, but the candidates know that part of this information becomes revealed once job interviews take place. Hence, after the candidates choose a firm to submit their applications, employers obtain additional information about a candidate's type. A job interview may be interpreted as a series of tasks, where a firm specifies a performance level which the candidate needs to pass. Only successful applicants are hired. They receive the salary which was promised at the outset.

Before firms use job talks as a sorting device, applicants on their part potentially reveal their type by selecting a particular employer. Thus to determine the beliefs of firms prior to job interviews, we have to examine a signaling game. Accordingly, the inclusion of incomplete knowledge comes at the cost of off-the-equilibrium-path beliefs which we need to construct. If firms deduce that lower salaries are rather selected by unproductive candidates, no applicant would head to a low-paying job. But why should a firm—in our case off the equilibrium path—make a conclusion of that kind? In the course of this chapter, we provide an argument which rationalizes such a structure of beliefs.

Once employers expect that applicants uniformly approach the best-paying firms, they are willing to outbid their opponents as long as they will find a mixture of applicants which justifies such sizable salaries. But the higher the promised payment is, the more thoroughly a firm needs to scrutinize its applicants. As only the best are worth their price, most applicants are left without an employment. Thus we find an explanation for the low hiring rates we referred to at the beginning.

We organize the rest of the chapter as follows. In Section 2.2, we describe the assumptions of the model. In Section 2.3, we look at a single firm (a "monopsonist") to illustrate the relation between salaries and the employment rate. In Section 2.4, we present the results for the competitive case. We discuss a more general specification of the interviewing process in Section 2.5. In Section 2.6, we provide a comparative statics analysis, contrast the market outcomes with Pareto optimal allocations, and briefly address policy measures. In Section 2.7, we endogenize the firms' off-the-equilibrium-path beliefs, which we impose up to there. Section 2.8 concludes. All proofs are relegated to the appendix.

2.2 Model

We look at a continuum of applicants (candidates) with a total mass of 1. $\theta_i \in \{0, 1\}$ denotes the value of the marginal product of applicant i. Thus each candidate is either productive ($\theta_i = 1$) or completely unproductive ($\theta_i = 0$). Both types have an outside option of 0. There are J identical firms. To

circumvent issues of market power, we let $J \to \infty$.[5] We assume both applicants and firms to be risk-neutral maximizers of their expected monetary payoffs.
The timing of the game is as follows. In period 1, nature chooses each applicant's productivity $\theta_i \in \{0, 1\}$. With probability $q_1 = 1 - q_0$, an individual applicant's productivity is 1. In period 2, each firm posts an unconditional salary w_j at which it obliges itself to pay accepted candidates. In period 3, candidates simultaneously select firms. If the expected payoff is the same at several firms, we assume that one of the most promising offers is picked randomly. In period 4, each applicant's score value s_i is realized. In period 5, the contacted firms decide on which applicants to employ.
Regarding period 4, we differentiate between two specifications:

Scenario 1. $s_i = \theta_i + \varepsilon_i$, whereas $\varepsilon_i \sim$ i.i.d. $\mathcal{N}(0, \sigma^2)$.

Scenario 2. $s_i = \sum_{k=1}^{n} t_k$, whereas $n \in \mathbb{N}^+$ and $t_k \in \{0, 1\}$. We define $p_i := P[t_k = 1|\theta_i] \in (0, 1)$, and assume that $p_1 > p_0$.

We discuss Scenario 1 in Sections 2.3 and 2.4. Normally distributed score values enable the firms to accurately control their composition of the workforce. In fact, from the law of large numbers, this can be done to an arbitrarily precise extent. In order to overcome this artificiality, in Section 2.5 we refer to Scenario 2, where we consider binomially distributed score values. By doing so, we interpret $\{t_k\}_{k=1}^{n}$ as a series of tasks which are examined in the course of a job talk. From the number of successfully mastered tasks, a firm draws conclusions about an applicant's type. As the binomial distribution converges to a normal distribution, Scenario 1 constitutes a special case of Scenario 2 where $n \to \infty$. Hence Scenario 2 is the more general framework.
By $\mu(w_j)$, we denote the firms' posterior belief that a candidate who approaches firms with payment w_j is of type 1. If candidates behave homogeneously, posterior beliefs of the firms equal their prior beliefs, thus $\mu(w_j) = q_1$. When

[5] As previously mentioned, we also look at the case of a single firm in the preliminary Section 2.3, so as to separately study the role of job interviews.

candidates behave heterogeneously, we assume firms to conclude that the highest accepted offer is only chosen by type-1 applicants, while any other offer is only chosen by type-0 applicants. For the moment, we take this (obviously strong) assumption as given. In Section 2.7, however, we show that it endogenously arises as the consequence of a standard refinement criterion ("D1"). By denoting by I the set of firms which are chosen by either applicant type with positive probability, we summarize the structure of beliefs in Assumption 1.

Assumption 1.

$$\begin{aligned} \forall j, l \in I : w_j = w_l \quad &\Rightarrow \quad \mu(w_j) = \mu(w_l) = q_1, \\ \exists l \in I : w_l < \max_{j \in I}\{w_j\} \quad &\Rightarrow \quad \mu(\max_{j \in I}\{w_j\}) = 1 \text{ and } \mu(w_l) = 0. \end{aligned}$$

2.3 The Monopsony Case

We first consider Scenario 1 with a single employer. In this case the analysis of the applicants' behavior becomes redundant since they have no alternative to approaching the "monopsonist". Accordingly, all applicants apply at the monopsonist, as long as it offers $w^m \geq 0$. In a subgame-perfect equilibrium, the monopsonist sets $w^m = 0$, and all candidates end up being employed. Although this result follows straightforwardly, we proceed by conducting a step-by-step analysis of the game. The central aim of the following backwards induction is to focus on the employer's selection process, and to illustrate the relation between salaries and employment probabilities.

The Hiring Decision At the final stage, the monopsonist decides on which applicants to accept. Given it hires an applicant with a score value $\underline{s}^m$, it also hires applicants with higher score values, because $\partial P[\theta_i = 1|s_i]/\partial s_i > 0$. Therefore, the monopsonist's hiring problem can be reduced to finding the profit-maximizing $\underline{s}^m$.

Independently of $w^m \geq 0$, the employer expects an applicant to be of type 1 with probability q_1 since, prior to the realization of the score value, nothing about an applicant's type has transpired. Hence, we can state the firm's ex-post profit maximization problem as

$$\begin{aligned}\underline{s}^{m*} = &\ \arg\max_{\underline{s}^m}\{\pi(\underline{s}^m)\} \\ := &\ \arg\max_{\underline{s}^m}\left\{(1-\Phi_1[\underline{s}^m])\,q_1 - w_j\left[(1-\Phi_1[\underline{s}^m])\,q_1\right.\right. \\ &\left.\left. +\ (1-\Phi_0[\underline{s}^m])\,q_0\right]\right\}, \end{aligned} \tag{1}$$

where $\Phi_{\theta_i}[\cdot]$ denotes the cumulative distribution function of $s_i(\theta_i)$. The first-order condition of problem (1) is

$$q_1\phi_1(\underline{s}^m) \stackrel{!}{=} w^m(q_1\phi_1[\underline{s}^m] + q_0\phi_0[\underline{s}^m]), \tag{2}$$

where $\phi_{\theta_i}[\cdot]$ denotes the density function of $s_i(\theta_i)$.
The left-hand side of equation (2) is the marginal benefit from increasing $\underline{s}^m$: the productive candidate's density at $\underline{s}^m$ times the ex-ante probability q_1 of such an applicant. The right-hand side of (2) is the marginal cost of increasing $\underline{s}^m$: the marginal probability of hiring *any* applicant, $q_1\phi_1[\underline{s}^m]+q_0\phi_0[\underline{s}^m]$, multiplied by the salary w^m.
By applying $s_i \sim$ i.i.d. $\mathcal{N}(\theta_i, \sigma^2)$ on (2), we obtain the monopsony's optimal threshold $\underline{s}^{m*}$.

Lemma 1.

$$\underline{s}^{m*} = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{w^m}{(1-w^m)}\right)\sigma^2 \tag{3}$$

Suppose, for instance, that $q_1 = q_0 = 0.5 = w^m = 0.5$. In this case, the monopsony obtains $\theta_i - w^m = 0.5$ per type-1 applicant and -0.5 per type-0 applicant. Therefore, as long as the marginal probability of hiring a type-1 applicant is above the marginal probability of hiring a type-0 applicant, the firm wishes to decrease its lower bound $\underline{s}^m$ to accept more candidates. The monopsonist reaches its optimum by setting the marginal probabilities equal to each other. This is reflected in (3), which for the current example reads $\underline{s}^{m*} = 1/2 + \ln(1)\sigma^2 = 1/2$.

The Choice of the Payment When choosing its payment offer, the monopsonist takes two effects into account. Ceteris paribus, an increasing payment lowers the monopsonist's profit. Through $\underline{s}^{m*}(w^m)$, however, it also forces the firm to adjust its composition of the workforce. By scaling up $\underline{s}^m$, the monopsonist increases its proportion of type-1 applicants. Equivalently,

$$\lim_{\underline{s}^m \to \infty} q_1\phi_1[\underline{s}^m] + q_0\phi_0[\underline{s}^m] = q_1\phi_1[\underline{s}^m] \tag{4}$$

because $\lim_{\underline{s}^m} \phi_1[\underline{s}^m]/\phi_0[\underline{s}^m] = \lim_{\underline{s}^m} e^{(2\underline{s}^m-1)/2\sigma^2} = \infty$. Equation (4) and an analogous statement for $\underline{s}^m \to -\infty$ imply that the monopsonist's first-order condition (2) has an interior solution whenever $0 < w^m < 1$. In Corollary 1 of Lemma 1, we rephrase this condition by turning our attention to corner solutions. Furthermore, we state an analogous result regarding the impact of the ex-ante distribution of types.

Corollary 1. *In (3),* $\lim_{q_1 \searrow 0} \{\underline{s}^{m*}(\cdot)\} = \lim_{w^m \nearrow 1} \{\underline{s}^{m*}(\cdot)\} = \infty$, *as there are either only type-0 candidates applying or* w^m *is prohibitively high. Likewise,* $\lim_{q_1 \nearrow 1} \{\underline{s}^{m*}(\cdot)\} = \lim_{w^m \searrow 0} \{\underline{s}^{m*}(\cdot)\} = -\infty$.

Of course, if the firm raises $\underline{s}^m$, it does not only affect the mixture of its workforce but also its size. If the monopsonist sets $w^m = 0$, its marginal cost, the right-hand side of equation (2), equals 0. This allows it to set $\underline{s}^m$ to $-\infty$, thus it accepts every applicant. As a result, the monopsonist realizes $\pi^{m*} = q_1$. We state this result in Corollary 2 of Lemma 1.

Corollary 2. *The monopsony sets* $w^{m*} = \arg\max_{w^m}\{\pi^m(w^m)\} = 0$.

Welfare Analysis and Policy From a Utilitarian point of view, welfare increases in the number of employed type-1 applicants, which in turn decreases with $\underline{s}^m$.[6] By reconsidering equation (3), we see that

$$\frac{\partial \underline{s}^{m*}}{\partial w^m} = \frac{\sigma^2}{w^m(1-w^m)} > 0. \tag{5}$$

Hence the monopsonist's behavior is in line with the one of a social planner. The total surplus is q_1.
Before shifting the focus of the analysis to a market with many employers, we briefly look at potential government interventions in the monopsony case.

Minimum Salary From equation (5), a minimum salary lowers the amount of hired type-1 applicants. Utilitarian welfare is reduced.

Maximum Salary Since the monopsonist chooses $w^{m*} = 0$, any restriction from above is without consequences.

2.4 The Competitive Case

As opposed to the analysis of the previous section, we now consider the case with $J \to \infty$ employers. As a consequence, we have to deal with a combination of a screening game and a signaling game. Screening occurs at the outset: firms try to influence the composition of their workforce by offering their respective salaries. Signaling occurs subsequently: each applicant faces an array of salary offers; thus by approaching a particular firm, it becomes possible to convey a signal. Respecting this setting of incomplete information, we use the notion of a Perfect Bayesian Equilibrium (henceforth PBE): at each stage of the game, agents maximize their payoffs given their beliefs, which are consistent in equilibrium. Regarding off-the-equilibrium-path beliefs, we impose Assumption 1, which we endogenize in Section 2.7. As before, we proceed by solving the game backwards.

[6] The cumulative distribution function of a normally distributed variable is strictly increasing in its argument.

The Hiring Decision At the final stage, each firm deliberates on which applicants it accepts to employ. Firm j's decision depends on its salary offer, w_j, and its belief on its applicants type, $\mu(w)$. Thereby, $w := (w_j, w_{-j})$ denotes the vector of all salaries offered in the first period. That is, upon observing the entire array of salary offers and receiving a certain amount of applicants, firm j assesses its applicants to be of type 1 with probability $\mu(w)$ and of type 2 with probability $1 - \mu(w)$.

Apart from this, the analysis of the hiring decision in the competitive case mirrors the one in case of a monopsonistic employer, discussed in Section 2.3. As a result, we obtain an individual firm's optimal threshold $\underline{s}_j^*$.

Lemma 2.

$$\underline{s}_j^* = \frac{1}{2} + \ln\left(\frac{(1-\mu(w))}{\mu(w)}\frac{w_j}{(1-w_j)}\right)\sigma^2 \tag{6}$$

According to Lemma 2, $\underline{s}_j^*$ increases in the fraction of type-0 applicants, in firm j's offered salary, as well as in the variance of the signal distribution.[7] Also referring to Section 2.3, we summarize the limit behavior of $\underline{s}_j^*$ as follows.

Corollary 3. *In (6),* $\lim_{\mu(w)\searrow 0}\left\{\underline{s}_j^*(\cdot)\right\} = \lim_{w_j\nearrow 1}\left\{\underline{s}_j^*(\cdot)\right\} = \infty$, *as either j expects only type-0 candidates applying or w_j is prohibitively high. Likewise,* $\lim_{\mu(w)\nearrow 1}\left\{\underline{s}_j^*(\cdot)\right\} = \lim_{w_j\searrow 0}\left\{\underline{s}_j^*(\cdot)\right\} = -\infty$.

The Selection of a Firm To determine $\mu(w)$, we have to consider the applicants' selection of employers in period 3. Each candidate i selects firm j^* which maximizes his or her expected payoff. That is,

$$(j^*|\theta_i) = \arg\max_j \left\{w_j P[s_i \geq \underline{s}_j^*(\cdot)|\theta_i]\right\}. \tag{7}$$

In (7), the probability that firm j accepts applicant i, $P[s_i \geq \underline{s}_j^*(\cdot)|\theta_i]$, depends on the employers belief $\mu(w)$ (through $\underline{s}_j^*$). From Assumption 1, we have that

[7] As an exception, $\underline{s}_j^*$ is independent of σ^2 if $w_j(1-\mu(w)) = (1-w_j)\mu(w)$. In this case, $\underline{s}_j^* = 1/2$.

i is believed to be a type-0 applicant whenever i approaches an employer with an offer below the maximum offer among all selected firms. Furthermore, from Corollary 3, such a firm will never accept a positive amount of applicants whenever it offers a positive salary. As a result, the applicants only choose among the employers with the highest salary offers.

Lemma 3. *For $\theta_i \in \{0, 1\}$,*

$$(j^*|\theta_i) \in K : \forall k \in K : w_k = \max_j \{w_j\},$$

and each candidate randomly picks one of the firms in K.

Given the other applicants go for a firm with the highest salary offer, no individual applicant wants to be exposed as being of the low productivity type 0. Similarly, if some candidates select a firm which does not belong to the set of firms with the highest offer, there is an incentive for each applicant to go for the highest offer and to be declared as being of type 1.

The Choice of the Offered Salary Being aware of the subsequent equilibrium behavior, in the initial state each firm j offers salary w_j in order to maximize its expected profit. Proposition 1 states that in equilibrium the maximum offered salary cannot be smaller than 1.

Proposition 1. *In the PBE, it holds that*

$$\nexists \hat{w} < 1 : \forall j \in J : w_j \leq \hat{w}.$$

The proof of Proposition 1 is somewhat cumbersome.[8] In order to understand its message, it suffices to consider a counterfactual example where the highest

[8] Whereas this proof and its counterpart regarding Proposition 2 treat firms as only being able to choose among pure strategies, extensions to mixed strategies such as done in the proof of Lemma 3 would be straightforward. We impose the restriction to pure strategies to ease notation, and because the results are unaffected.

offered salary is, say, 0.99. For the sake of the argument, assume that all firms offer this salary. As a result, and since we focus on $J \to \infty$, an individual employer obtains 0. By increasing its offer, for example, to 0.991, an individual firm l can achieve to become the only employer contacted by the candidates (see Lemma 3). Does such a deviation pay off? In order to see why this is true, be aware that once the workers have contacted firm l, according to (6), l sets

$$\underline{s}_l^* = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{0.991}{0.009}\right)\sigma^2,$$

which is approximately 5.2 for $q_0 = q_1$ and $\sigma^2 = 1$. As a consequence,

$$P[s_i \geq \underline{5}.2|\theta_i] \simeq \begin{cases} 0.0000001 & \text{for } \theta_i = 0, \\ 0.0000134 & \text{for } \theta_i = 1. \end{cases}$$

Thus, by choosing $\underline{s}_l^* = 5.2$, firm l ensures that its ratio between productive and unproductive workers is roughly 134:1. That is, for each type-0 worker which leaves the firm with a loss of 0.991, it hires 134 type-1 applicants, on each of which it obtains a margin of 0.009. As a result, l's profit per 135 employed candidates is $134 \times 0.009 - 0.901 = 0.215$.
That is, for $w_j < 1$, it is always feasible to steer the ratio between accepted type-1 and type-0 applicants towards the desired direction. This has, however, detrimental effects on the employment rate, as we point out in the following corollary.

Corollary 4. *In the PBE, we have* $\max_j \{w_j\} = 1$, *and both the employment rate and profits are 0.*

Of course, in its present form, this result is too crude to match the observed pattern. In the following, we show that the reason for this lies in the continuos space of the score values. With discrete score values, any employment rate is possible, although we approach the result of Corollary 4 the more we refine the domain of possible score values.

2.5 Generalization of the Competitive Case

In contrast to above, we now assume that only coarse signals about productivity are transmitted. We accordingly modify all so-far conducted steps.

The Hiring Decision Once a firm expects to face $\mu(w)$ type-1 candidates and $1-\mu(w)$ type-0 ones, it is confronted with the same trade-off as in Section 2.4. If the minimum-requirement level $\underline{s}_j$ is excessively high, a lot of productivity is lost because both applicant types are rejected. If it is modest, the firm employs too many type-0 candidates on which it loses. Correspondingly, the higher is w_j, the higher $\underline{s}_j$ has to be. In fact, in the following we show that in a PBE each firm's salary offer w_j is insomuch high that only the fulfillment of the highest possible threshold $\underline{s}_j = n$ gives rise to an employment.

As a first step to seeing that, consider that, in contrast to the above case, the distinguishing assumption of Scenario 2 implies that the signal of each applicant can only take on integer values within a bounded set. Specifically, signal $s_i(\theta_i)$ follows a binomial distribution and thus assumes the probability mass function $P[s_i = k|\theta_i] = \binom{n}{k} p_{\theta_i}(1-p_{\theta_i})^{n-k}$, where p_{θ_i} denotes the constant probability that an single of n assessed tasks is successfully handled by candidate i with $\theta_i \in \{0,1\}$. In analogy to (1), we state the ex-post profit maximization problem of firm j as

$$\underline{s}_j^* = \arg\max_{\underline{s}_j \in \mathbb{N}_0} \left\{ \sum_{k=\underline{s}_j}^{n} \binom{n}{k} \Big[(1-w_j)\mu(w) p_1^k (1-p_1)^{n-k} \right. \\ \left. - w_j \left(1-\mu(w)\right) p_0^k (1-p_0)^{n-k} \Big] \right\}. \tag{8}$$

Due to the discreteness of problem (8), first-order conditions are of no help here. Nevertheless, Lemma 4 provides a sufficient condition for $\underline{s}_j^* = n$, whereas in the course of the proof of Proposition 2 we point out that the requested condition always holds.

Lemma 4. *Assume there exists a unique solution for $\underline{s}_j^*$. Then, for $\mu(w) \in (0,1)$, we have*

$$\forall j \in J : \frac{w_j}{1-w_j} \geq \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n \quad \Rightarrow \quad \underline{s}_j^* = n. \tag{9}$$

Thus the obligation of paying a high salary w_j forces firm j to investigate more thoroughly whether an applicant is of type 1. Furthermore, if the values of p_1 and p_0 are close together, and if there are relatively few type-1 applicants expected, it is particularly important to let a candidate accomplish as many assessment tasks as possible.

The Selection of a Firm Regarding the choice of a firm, the former analysis involving an unbounded and continuous signal space carries over to the present setting with bounded and discrete test scores. A replicated proof of Lemma 3 would merely adjust the probability of being accepted at firm j in the case of homogenous behavior of the applicants to

$$P[s_i \geq n | s_i \sim \text{Binomial}(n, p_{\theta_i})] = p_{\theta_i}^n \in (0,1),$$

leaving the remainder of the discussion unaffected. Therefore we omit a repetition of the proof.

The Choice of the Offered Salary Again, by choosing a salary w_j, firm j has to take the sequential consequences on $\mu(w)$ and $\underline{s}_j^*(\cdot)$ into account. Proposition 2 provides a simple decision rule for the firms in period 1. It states that, in contrast to the case of a continuous signal space, the maximum salary chosen in the initial period can be, and generally is, below 1, even though it approaches 1 for $n \to \infty$. Since the proof of Proposition 2 makes use of Lemma 4, we need to scrutinize the condition stated in (9) each time we apply the lemma, irrespective of whether it is on or off the equilibrium path.[9]

[9]Otherwise, we would risk running into a circular argument.

Proposition 2. *In the PBE, we have*

$$\max_j \{w_j\} = \frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} =: w^*. \tag{10}$$

In the proof of Proposition 2, we show that in any constellation where the highest offered salary is not equal to w^* there are incentives for at least one firm to either increase or decrease its offer. Here, we only sketch out that there actually exists a PBE which involves a maximum salary as described in (10). To see this, note that

$$\frac{w^*}{1 - w^*} = \frac{q_1 p_1^n}{q_0 p_0^n} = \frac{\mu(w)}{1 - \mu(w)} \left(\frac{p_1}{p_0}\right)^n,$$

whereas the later equality follows from the random assignment of candidates to jobs that offer w^* and Assumption 1 about the structure of beliefs. It follows from condition (9) in Lemma 4 that, upon offering w^*, firm j requires its candidates to pass the maximum number of tasks, n. From the binomial distribution of the score values, the odds of passing n tasks and becoming employed is $p_{\theta_i}^n$, $\theta_i \in \{0, 1\}$. Accordingly, each firm's profit is

$$(1 - w^*) q_1 p_1^n - w^* q_0 p_0^n = 0.$$

Given all competing firms offer w^*, there is no incentive to deviate by announcing a higher salary: all applicants would accept the deviating firm's offer, and the latter would end up with a loss. Similarly, cutting the offered salary is useless, as Assumption 1 guarantees that all applicants approach the best-paying firm in order to not reveal themselves as type-0 candidates.

As Proposition 2 indicates, not all firms necessarily make identical offers. As long as the highest offered salary equals w^*, each firm obtains zero profit, regardless of whether it attracts applicants or not. However, for instance under the restriction of symmetric firm behavior, w^* is the unique offered salary in a

PBE. Hence, for simplicity, in the remainder of this chapter we assume that in equilibrium $w_j = w^*$ for all firms $j \in J$.
We close this section with Corollary 5 of Proposition 2 regarding the employment rate and expected payoffs.

Corollary 5. *In the PBE, the probability of an applicant of type θ_i to become employed is $p^n_{\theta_i}$. Accordingly, the population-wide employment rate is $(q_1 p^n_1 + q_0 p^n_0)/(q_1 + q_0)$, and the expected payoff of an applicant with productivity θ_i is $(p^n_{\theta_i} q_1 p^n_1)/(q_1 p^n_1 + q_0 p^n_0)$.*

2.6 Welfare and Policy

Regarding total surplus, it solely matters how many workers are employed in equilibrium. Irrespective of informational issues, full employment of productive candidates can be achieved by enforcing $\underline{s}_j = \min_i \{s_i\}$, alongside $w_j \in [0, 1]$, for each firm j. Such constrained-efficient allocations are also Pareto optimal, as is, among others, the monopsonist's solution.
In Table 2.1, we compare values of the model's key variables for both considered scenarios with the outcome of the monopsonist's problem. In both scenarios a

	Outcomes		
	Scenario 1	Scenario 2	Monopsony
Maximum salary	1	$\in \left(\frac{q_1}{q_1+q_0}, 1\right)$	0
Employment rate	0	$\in (0, 1)$	1
Fraction of type-1 employees	1 (limit)	$\in \left(\frac{q_1}{q_1+q_0}, 1\right)$	$\frac{q_1}{q_1+q_0}$
Producer surplus (applicants)	0	$\in (0, q_1)$	0
Consumer surplus (firms)	0	0	q_1

Table 2.1: Comparative statics

monopsonist j chooses a salary as low as possible, and at the same time it hires as many applicants i with $\theta_i = 1$ as possible. Respecting the candidates' individual-rationality constraints which impose $w_j \geq 0$, these two objectives are not competing. With $w_j = 0$, all applicants apply at j. Thereupon, due to $\underline{s}_j = \min_i \{s_i\}$, all candidates, including the productive ones, are employed. Regarding the profit maximizing threshold, we have $\underline{s}_j^* = -\infty$ in Scenario 1 with a continuous and unbounded signal space, and $\underline{s}_j^* = 0$ in Scenario 2 with an atomic, bounded distribution. Ignoring equity considerations due to transfer payments, the monopsony maximizes overall welfare.

Under competition, the main difference between the two scenarios is that in Scenario 2 the firms do not have illimitable means to sort out type-0 applicants by raising $\underline{s}_j$. Anticipating this, they are not willing to set their salary offers arbitrarily close to 1 in the initial period. Therefore, the odds of becoming employed are non-zero for both types of candidates. With discrete score values, the applicants' surplus is

$$\begin{aligned} & w^* \left(q_1 P \left[s_i \geq n | \theta_i = 1 \right] + q_0 P \left[s_i \geq n | \theta_i = 0 \right] \right) \\ = \; & \frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} \left(q_1 p_1^n + q_0 p_0^n \right) = q_1 p_1^n. \end{aligned}$$

Accordingly, overall welfare is higher for values of p_1 closer to 1 and a lower number of testing criteria n. That is, the coarser the firms' framework for the assessment of the candidates, the more likely it is that candidates withstand even the highest classification requirement. This is reflected in a higher employment rate.

Policy Since n and, through the difficulty level of the assessment tasks, p_1 are parameters which are generically determined within firms, they are no adequate means of regulation. It is, however, plausible that policy-makers restrict the range of w_j. In fact, it is easy to see that by upwards restricting $w_j \leq \overline{w}$, any measure of unemployment can be eliminated by appropriately choosing $\overline{w}$. Thus in our stylized model, lowering $\overline{w}$ from its equilibrium value leads to a

Pareto improvement, because firms become enabled to extract a positive profit, whereas an applicant of type θ_i obtains $\overline{w}P[s_i \geq \underline{s}_j^*(\overline{w})|\theta_i]$.
The preference ordering of firms and applicants over possible values of $\overline{w}$, however, is not aligned. Consider Scenario 1. Regarding firm j's profit,

$$\pi_j(\overline{w}) \;= \left(1 - \Phi_1[\underline{s}_j(\overline{w})]\right) \frac{q_1}{J} - \overline{w} \left[\left(1 - \Phi_1[\underline{s}_j(\overline{w})]\right) \frac{q_1}{J} + \left(1 - \Phi_0[\underline{s}_j(\overline{w})]\right) \frac{q_0}{J}\right],$$

we apply the regular envelope theorem to see that the impact of an increase in $\overline{w}$ on j's payoff is

$$\left.\frac{\partial \pi_j(\cdot)}{\partial \overline{w}}\right|_{s_j(\overline{w})=s_j^*(\overline{w})} = -\left[\left(1 - \Phi_1[\underline{s}_j^*(\overline{w})]\right) \frac{q_1}{J} + \left(1 - \Phi_0[\underline{s}_j^*(\overline{w})]\right) \frac{q_0}{J}\right],$$

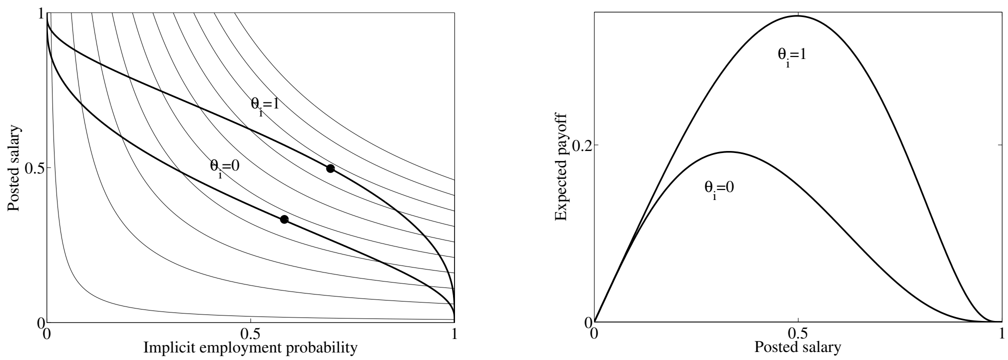
thus strictly negative for all $\overline{w} \in [0,1)$. As opposed to this, for both candidate types there exists a clear-cut and positive optimal upper bound on salaries.

Proposition 3. *When salaries are restricted by $w_j \leq \overline{w}$, for each applicant type $\theta_i \in \{0,1\}$ there exists a unique $\overline{w}_{\theta_i}$ which satisfies*

$$\overline{w}_{\theta_i} = \arg\max_{\overline{w}} \left\{\overline{w}P[s_i \geq \underline{s}_j^*(\overline{w})|\theta_i]\right\}. \tag{11}$$

Furthermore, $\overline{w}_1 > \overline{w}_0$; and type-$\theta_i$ applicants' payoffs are strictly increasing in $\overline{w}$ if and only if $0 \leq \overline{w} < \overline{w}_{\theta_i}$.

Consider Figure 2.1. Each value of $\overline{w}$ implicitly defines a candidate's employment probability. The thick lines in panel (a) thus constitute "budget lines" for both candidate types. Crucially, the implicit employment probability decreases in the posted salary, but faster so for type-0 applicants. Therefore, with any $\overline{w}$, type-1 applicants have higher expected payoffs than type-0 applicants, as we show in panel (b). In addition, the highest payoff is achieved at a higher value of $\overline{w}$ for type-1 applicants, as can be seen in both panels.
Somewhat surprisingly, when it comes to determining $\overline{w}$, the preference order of type-0 workers is better aligned with the preference order of firms. The only



(a) Posted salaries, implicit employment probabilities, and common indifference curves

(b) Expected payoffs as functions of posted salaries

Figure 2.1: Determination of $\overline{w}_1$ and $\overline{w}_0$

party which remotely benefits from not having a salary ceiling is the type-1 applicants.

2.7 Endogenization of Beliefs

In both Scenario 1 and Scenario 2, the signaling behavior of the applicants clearly drives the main results. Specifically, from Assumption 1 it follows that all candidates necessarily approach the best-paying firm, because otherwise they would—correctly or mistakenly—reveal themselves as being of the unproductive type 0. Obviously, this is a strong assumption, which we imposed axiomatically up to now. Thus we owe it to the reader to provide an appropriate motivation. In the following, we consider Scenario 1. We examine to what extent two established refinements for signaling games apply. To begin with, consider the equilibrium outcome where both candidate types choose a firm k with $w_k = \max_j \{w_j\} = 1$.

Refinement 1. The "Intuitive Criterion" by Cho and Kreps (1987) eliminates equilibria where firms asses the probability that an off-the-equilibrium-path message is sent by an applicant for whom sending it is dominated by the equilibrium strategy to be non-zero. Accordingly, the equilibrium profile survives the Intuitive Criterion if and only if the remaining type of applicant does not strictly benefit from sending the off-the-equilibrium-path message.

In the present model, it only pays off for an applicant to deviate from the equilibrium strategy (that is, heading for $w_l < w_k$ instead of $w_k = 1$) if this induces a higher probability of becoming employed. From Lemma 2, the employment probability of a type-θ_i applicant is

$$1 - \Phi_{\theta_i}\left[\frac{1}{2} + \ln\left(\frac{(1-\mu(w))}{\mu(w)}\frac{w_l}{(1-w_l)}\right)\sigma^2\right], \tag{12}$$

which is positive for $\mu(w) > 0$ and $w_l < 1$. Since in equilibrium both candidate types have a payoff of 0, the firms cannot exclude either of them upon receiving such a message. Hence the proposed equilibrium does not violate the Intuitive Criterion.

Refinement 2. The '"D_1 Criterion" by Banks and Sobel (1987) is more restrictive in as much as it constrains the firms to eliminate types which are "less likely" than others to send particular off-the-equilibrium-path messages. That is, upon observing an applicant to approach a firm offering $w_l < w_k$, the firms deduce that the deviating candidate is of the type for whom deviating is profitable in a higher number of cases, characterized by $\mu(w)$.

For $w_l = 0$, irrespectively of $\mu(w)$, it does never pay off to deviate for either type. For $0 < w_l < 1$, (12) implies that, for both types, the set of $\mu(w)$ which rewards departure from $w_k = 1$ is given by $\mu(w) \in (0, 1)$. Thus $M_1 = M_0$, and neither is a proper superset of the other. Therefore, we cannot further restrict off-the-equilibrium-path beliefs, which leaves the D_1 Criterion—as well as the Intuitive Criterion—without bite.

So far we focussed on the case where the highest offered salary is 1 and thus no applicant is employed in equilibrium. However, there are many more subgames to consider, each of them being a signaling game with a fixed distribution of

$\{w_j\}_{j \in J}$. For each of these subgames, we now show that the beliefs imposed in Assumption 5 necessarily arise under the D_1 Criterion.[10]

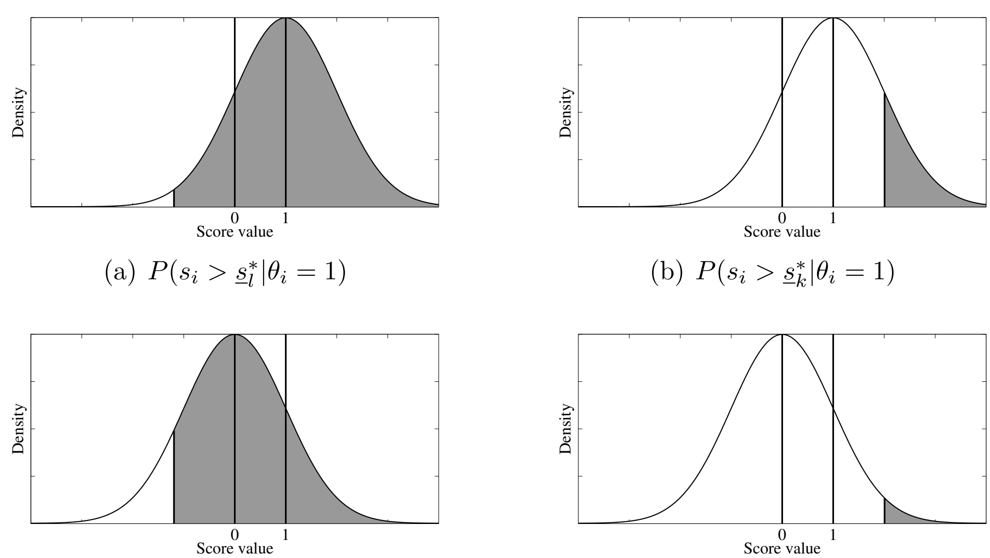
Lemma 5. *Denote by k a firm with $w_k = \max_j \{w_j\}$ and by l a firm with $w_l < w_k$ in any subgame. Let M_θ be the set of beliefs $\mu(w)$ for which*

$$P\left[s_i \geq \underline{s}_l^*(\mu(w), w_l) | \theta_i = \theta\right] w_l > P\left[s_i \geq \underline{s}_k^*(w_k) | \theta_i = \theta\right] w_k, \tag{13}$$

where $\underline{s}_k^*(w_k)$ is the equilibrium value complying with Lemma 3 and the further course of the game. Then $M_1 \subset M_0$.

That is, each $\mu(w)$ which gives type-1 applicants an incentive to deviate from their putative equilibrium strategies also induces type-0 applicants to do so. The same, however, does not apply the other way around.

Consider Figure 2.1. In order for $\mu(w) \in M_1$, w_l must be associated with a



(a) $P(s_i > \underline{s}_l^* | \theta_i = 1)$

(b) $P(s_i > \underline{s}_k^* | \theta_i = 1)$

(c) $P(s_i > \underline{s}_l^* | \theta_i = 0)$

(d) $P(s_i > \underline{s}_k^* | \theta_i = 0)$

Figure 2.1: Relative probabilities of becoming employed

[10] Accordingly, they satisfy the Intuitive Criterion as well.

sufficient increase in the likelihood of a type-1 applicant to become employed. Such a candidate compares the relation of hiring probabilities, indicated in panels (a) and (b), with the relative wage w_k/w_l. Specifically, in the proof of Lemma 5 we show that if the proportion $P[s_i \geq \underline{s}_l^*|\theta_i = 1]/P[s_i \geq \underline{s}_k^*|\theta_i = 1]$ exceeds w_k/w_l, the same must be true for $P[s_i \geq \underline{s}_l^*|\theta_i = 0]/P[s_i \geq \underline{s}_k^*|\theta_i = 0]$. In terms of Figure 2.1, this means that whenever the shaded area in panel (a) is sufficiently large as compared with the shaded area in panel (b), the same necessarily holds with regard to the relation between the indicated areas in panels (c) and (d). Therefore, each $\mu(w)$ which satisfies (13) with respect to $\theta = 1$ also satisfies (13) with respect to $\theta = 0$, but the opposite is not true. As a result, firm l which offers the lower salary w_l infers that its applicants are likelier to be of type 0. The D_1 Criterion then requires firm l to rule out the possibility of being approached by a type-1 candidate. From Corollary 3 of Lemma 2, it eventually follows that the probability of being hired at firm l is 0. Accordingly, there are no incentives for either candidate to apply at lower-paying firms. This justifies the structure of the off-the-equilibrium-path beliefs which we impose in Assumption 1.

2.8 Conclusion

In contrast to the existing literature on directed job search, the equilibria we presented in this chapter exhibit tremendous inefficiencies. Due to "D_1 considerations", applicants only approach the best-paying firms. Therefore, firms incur Bertrand-style outbidding. In retrospect, this forces them to excessively raise their hiring thresholds, which leads to unemployment.

To be clear, our results are far too extreme for the purpose of interpreting them in a "literal" manner. For the sake of highlighting a subtle but potentially crucial emergence, we made a number of simplifications. Regarding future research, it thus would be of interest to address some of the following issues in greater detail.

First, we assume homogenous firms. This may not pose a problem concerning the analysis of a specific market. However, within a more comprehensive framework, applicants would differ with respect to their outside opportunities including alternative job offers and unemployment benefits. Once we endogenized their participation constraints, the mechanism which deteriorates expected employment probabilities might be mitigated.
Secondly, if we abandoned the imposed dichotomy regarding the applicants' productivity, we would also have to adjust the firms' hiring rules and the specification of their beliefs. Discrete jumps in the value of the marginal product, which are essential to the discussed phenomena, are more probable to arise at the less densely populated upper part of the distribution. Therefore we may suppose that the examined effect plays a more important role within this range.
Of course, there exists a wide array of other possible extensions, such as on-the-job search (Delacroix and Shi, 2006), endogenized waging-rule determination (Michelacci and Suarez, 2006), job destruction, multiple applications, multi-period problems, learning processes, and many more. Incorporating these in a model of incomplete knowledge issues serious challenges. Replacing the normally distributed score values by some easier tractable random variables could be an option to counteract formal difficulties.
The above points make clear that our model is intended to serve as a starting point, indicating a potential direction of research, and not as a tool with predictive power. In its current state, it is neither appropriate for policy advice. In particular, implications of measures such as a salary ceiling strongly depend on institutional factors such as the international environment.
We showed, however, that a job-market model with imperfect observability of type is analytically tractable, and the arising PBE exhibits some properties which are well worth noting. In contrast to most of the existing literature, we pointed out that a directed-search equilibrium can be far from efficient, and that both job-seekers and—even more—firms may be keen on taking some action.

2.A Appendices to Chapter 2

2.A.1 Proof of Lemma 1

From

$$\phi_1(\underline{s}^m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\underline{s}^m-1)^2}{2\sigma^2}} \text{ and } \phi_0(\underline{s}^m) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\underline{s}^m)^2}{2\sigma^2}},$$

we write (2) as

$$q_1(1-w_j)e^{-\frac{(\underline{s}^m-1)^2}{2\sigma^2}} = w_j q_0 e^{-\frac{-(\underline{s}^m)^2}{2\sigma^2}}.$$

Upon taking logarithms and rearranging, we obtain (3). For $w^m \in (0,1)$ and $q_1 \in (0,1)$, $\underline{s}^m(\cdot)$ is well-defined and continuous in its arguments. Furthermore, the expected profit is positive at $\underline{s}^{m*}$, zero for $\underline{s}^m(\cdot) \to \infty$, and negative for $\underline{s}^m(\cdot) \to -\infty$. Therefore condition (3) is not only necessary but also sufficient for $\underline{s}^{m*}$ being a global maximizer.

2.A.2 Proof of Corollary 2

The monopsonist maximizes

$$\pi^m(w^m) = (1-\Phi_1[\underline{s}^m(w^m)])q_1 - w_j\left[(1-\Phi_1[\underline{s}^m(w^m)])q_1 + \;\; (1-\Phi_0[\underline{s}^m(w^m)])q_0\right].$$

From $w^m \geq 0$, it is sufficient to show that $\pi^m(w^m)$ decreases in w^m once we impose $\underline{s}^m(w^m) = \underline{s}^{m*}(w^m)$ (from subgame perfection). By applying the regular envelope theorem, we observe that, as w^m increases, the change in the monopsonist's profit is

$$\left.\frac{\partial \pi^m(w^m, \underline{s}^m(w^m))}{\partial w^m}\right|_{\underline{s}^m(w^m)=\underline{s}^{m*}(w^m)} = -\left(1-\Phi_1[\underline{s}^{m*}(w^m)]\right)q_1 - \left(1-\Phi_0[\underline{s}^{m*}(w^m)]\right)q_0,$$

which is strictly negative for any $w^m \in [0,1)$.

2.A.3 Proof of Lemma 3

Suppose to the contrary that for either $\theta_i = 0$ or $\theta_i = 1$ (or both) it holds that $(j^*|\theta_i) \notin K$ with some positive probability p. If a firm $l \in L = J\backslash K$ gets contacted by such a candidate, it infers from Assumption 1 that $\mu(w) = 0$. From (6), l sets $\underline{s}_l^* = \infty$. Thus it holds for the deviating applicant's expected payoff that

$$p \times w_l P[s_i > \infty|\theta_i] + (1-p) \times w_k = (1-p) \times w_k < w_k.$$

On the other hand, if an individual candidate opts for $k \in K$, the off-the-equilibrium-path beliefs of firm k ensure that $\underline{s}_k^* = -\infty$ and the expected payoff of the applicant is w_k. Alternatively, suppose that all candidates approach firms within L. In this case, applicant i's expected payoff is

$$w_l P[s_i > \underline{s}_l^*(\cdot)|\theta_i] \leq w_l < w_k.$$

If applicant i deviates to firm k, he or she would (wrongly or rightly) be identified as being of type 1, thus earning w_k for sure.

Finally, it does not pay to deviate from the suggested equilibrium, as only downwards deviations are possible. By doing so, a candidate would be interpreted as being of type 0, thus earning nothing.

2.A.4 Proof of Proposition 1

Case 1. Suppose to the contrary that $\forall j \in J : w_j = \hat{w} < 1$. For a finite number J of firms, it holds from (6) that each firm's profit is

$$\begin{aligned}\pi_j(\hat{w}) = & \ (1-\hat{w})\left(1-\Phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\hat{w}}{1-\hat{w}}\right)\sigma^2\right]\right)\frac{q_1}{J} \\ - & \ \hat{w}\left(1-\Phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\hat{w}}{1-\hat{w}}\right)\sigma^2\right]\right)\frac{q_0}{J}.\end{aligned}$$

As $J \to \infty$, we have $\pi_j(\hat{w}) \to 0$ for any $\hat{w} \in (0,1)$. Hence, with a continuum of firms, we solely remain to show that it is possible for a single firm to get a positive profit by deviating regarding the offered salary. Since $\hat{w} < 1$, there exists an $\varepsilon > 0$ such that $1-\varepsilon > \hat{w}$. Therefore, it is possible for a single firm l to set $w_l = 1-\varepsilon$ and thereby achieve a profit of

$$\begin{aligned}\pi_l(w_l) \ &= \ \varepsilon\left(1-\Phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]\right)q_1 \\ &- \ (1-\varepsilon)\left(1-\Phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]\right)q_0\Bigg]. \\ &=: \ \varepsilon(1-\Phi_1[\cdot])q_1 - (1-\varepsilon)(1-\Phi_0[\cdot])q_0. \end{aligned} \tag{A.1}$$

We next show that $\pi_l(w_l) > 0$, or, from (A.1),

$$\frac{q_1}{q_0}\frac{\varepsilon}{1-\varepsilon} > \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}, \tag{A.2}$$

for $\varepsilon > 0$ sufficiently small. Since $\lim_{\varepsilon\searrow 0}\{q_1\varepsilon/q_0(1-\varepsilon)\} = 0$, it is *necessary* for (A.2) that

$$\lim_{\varepsilon\searrow 0}\left\{\frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}\right\} = 0. \tag{A.3}$$

In order to show (A.3), we apply l'Hôpital's rule, as in the limit both the numerator and the denominator of the fraction at hand are 0, because for $\theta \in \{0,1\}$ it holds that $\lim_{\varepsilon\searrow 0}\Phi_\theta[\cdot] = 1$. According to l'Hôpital,

$$\lim_{\varepsilon\searrow 0}\left\{\frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}\right\} = \lim_{\varepsilon\searrow 0}\left\{\frac{\frac{\partial(1-\Phi_0[\cdot])}{\partial\varepsilon}}{\frac{\partial(1-\Phi_1[\cdot])}{\partial\varepsilon}}\right\} = \lim_{\varepsilon\searrow 0}\left\{\frac{\phi_0\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]}{\phi_1\left[\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{1-\varepsilon}{\varepsilon}\right)\sigma^2\right]}\right\}, \tag{A.4}$$

By employing $s_i \sim$ i.i.d. $\mathcal{N}(\theta_i, \sigma^2)$, we rewrite (A.4) as

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\exp\left(- \frac{\left[\ln\left(\frac{q_0}{q_1} \frac{(1-\varepsilon)}{\varepsilon} \right) \sigma^2 + \frac{1}{2} \right]^2}{2\sigma^2} \right)}{\exp\left(- \frac{\left[\ln\left(\frac{q_0}{q_1} \frac{(1-\varepsilon)}{\varepsilon} \right) \sigma^2 - \frac{1}{2} \right]^2}{2\sigma^2} \right)} \right\}$$

$$= \lim_{\varepsilon \searrow 0} \left\{ \exp\left[-\ln\left(\frac{q_0}{q_1} \frac{(1-\varepsilon)}{\varepsilon} \right) \right] \right\} = \lim_{\varepsilon \searrow 0} \left\{ \frac{q_1}{q_0} \frac{\varepsilon}{(1-\varepsilon)} \right\} = 0,$$

which is the required necessary condition. In order to find a sufficient condition for (A.2), we use l'Hôpital's rule a second time. Specifically, we need

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\partial \frac{q_1}{q_0} \frac{\varepsilon}{1-\varepsilon}}{\partial \varepsilon} \right\} > \lim_{\varepsilon \searrow 0} \left\{ \frac{\partial \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]}}{\partial \varepsilon} \right\}. \tag{A.5}$$

The left-hand side of (A.5),

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{q_1}{q_0} \frac{1}{(1-\varepsilon)^2} \right\},$$

is positive. Therefore, it is sufficient to show that this does not hold for its right-hand side, which we rewrite as

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{[(-\phi_0(\cdot))(1-\Phi_1[\cdot]) + \phi_1(\cdot)(1-\Phi_0[\cdot])] \frac{1}{\varepsilon(1-\varepsilon)}}{(1-\Phi_1[\cdot])^2} \right\}. \tag{A.6}$$

(A.6) is negative if $\phi_0(\cdot)(1-\Phi_1[\cdot]) > \phi_1(\cdot)(1-\Phi_0[\cdot])$. De novo, we use l'Hôpital's rule to show that

$$\lim_{\varepsilon \searrow 0} \left\{ \frac{\phi_0(\cdot)}{\phi_1(\cdot)} \right\} > \lim_{\varepsilon \searrow 0} \left\{ \frac{1-\Phi_0[\cdot]}{1-\Phi_1[\cdot]} \right\} \quad \Leftrightarrow \quad \lim_{\varepsilon \searrow 0} \left\{ \frac{1}{1-\varepsilon} \right\} > \lim_{\varepsilon \searrow 0} \{\varepsilon\},$$

which completes the proof for the first case.

Case 2. Suppose now that there are multiple offered wages, whereas $\hat{w} := \max_j\{w_j\} < 1$. Consider firm l which offers $\tilde{w}_l$ such that $0 \leq \tilde{w}_l < \hat{w}$. From Lemma 3 it follows that no worker applies at l, leaving l's profit to be zero. However, as it was argued in the course of Case 1, there exists a wage $w_l \in (\hat{w}, 1)$, which yields a positive profit. Therefore, $\tilde{w}_l$ cannot be a best answer from the beginning.

2.A.5 Proof of Corollary 4

The first fact directly follows from (1). Then, from Lemma 3, applicants of both types $\theta_i \in \{0, 1\}$ choose firm j^* such that

$$(j^*|\theta_i) \in K : \forall k \in K : w_k = 1.$$

In this case, equilibrium belief formation as stated in Assumption 1 ensures that $(1-\mu(w))/\mu(w) = q_0/q_1$. Furthermore,

$$(\underline{s}_j^*|w_j = 1) = \lim_{w_j \nearrow 1} \left\{ \frac{1}{2} + \ln\left(\frac{q_0}{q_1} \frac{w_j}{(1-w_j)} \right) \sigma^2 \right\} = \infty.$$

Hence in equilibrium no candidate will be hired.

2.A.6 Proof of Lemma 4

From (8), the difference in firm j's profit between the two cases $\underline{s}_j = s \in \mathbb{N}$ and $\underline{s}_j = s-1$ is

$$\begin{aligned} &\Delta\pi_j(s,w) := \pi_j(s,w) - \pi_j(s-1,w) \\ = \; &\binom{n}{s} \left[w_j \left(1-\mu(w)\right) p_0^s (1-p_0)^{n-s} \right. \\ - \; &\left. (1-w_j)\mu(w) p_1^s (1-p_1)^{n-s} \right]. \end{aligned} \tag{A.7}$$

By rearranging (A.7), we obtain that $\Delta\pi_j(s,w)$ is non-negative as long as

$$\frac{w_j}{1-w_j} \geq \underbrace{\frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^s \left(\frac{1-p_1}{1-p_0}\right)^{n-s}}_{=:\kappa(\cdot)>0}. \tag{A.8}$$

The derivative of the right-hand side of (A.8), $\kappa(\cdot)$ with respect to s is

$$\frac{\partial\kappa(\cdot)}{\partial s} = \ln\left(\frac{p_1}{(1-p_1)} \frac{(1-p_0)}{p_0} \right) \kappa(\cdot),$$

which is positive, since $p_1 > p_0$. Therefore, if (A.8) holds for the highest possible value of s, n, it implicitly holds for lower values of s as well. That is,

$$\frac{w_j}{1-w_j} \geq \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n \;\; \Rightarrow \;\; \forall s \in \mathbb{N} \leq n : \Delta\pi_j(s,w) \geq 0. \tag{A.9}$$

From the uniqueness assumption with regard to $\underline{s}_j^*$, (A.9) completes the proof.

2.A.7 Proof of Proposition 2

Case 1. Suppose to the contrary that $\forall j \in J : w_j = \hat{w} < w^*$. Independently from the choice of $\underline{s}_j^*$, with $J \to \infty$ we have $\pi_j(\hat{w}) \to 0$, since applicants choose among all firms with equal probability.
Since $\hat{w} < w^*$, there exists an $\varepsilon > 0$ such that $w^* - \varepsilon := \tilde{w}_l > \hat{w}$ can be offered by a deviating firm l. From Assumption 1, all applicants select l and thus $\mu(w) = q_1$ and $(1-\mu(w)) = q_0$

(see Lemma 3). From the profit function (8) with $\underline{s}_j = n$, it then follows that

$$\begin{aligned}
\pi_l(\tilde{w}_l) &= (1-\tilde{w}_l)q_1 p_1^n - \tilde{w}_l q_0 p_0^n \\
&= (1-w^*+\varepsilon)q_1 p_1^n - (w^*-\varepsilon)q_0 p_0^n \\
&= \left(\frac{q_0 p_0^n}{q_1 p_1^n + q_0 p_0^n} + \varepsilon\right) q_1 p_1^n - \left(\frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} - \varepsilon\right) q_0 p_0^n \\
&= \varepsilon(q_1 p_1^n + q_0 p_0^n) > 0.
\end{aligned}$$

As it is sufficient to show that l obtains a profit by choosing $\underline{s}_j = n$, the result does not hinge on condition (9) in Lemma 4.

Case 2. Suppose now that there are multiple offered salaries with $\hat{w} := \max_j\{w_j\} < w^*$. Consider firm l which offers $w_l < \hat{w}$. As before, it follows Assumption 1 (and Lemma 3) that no applicant chooses firm l, leaving l without a profit. By referring to Case 1, it holds that there exists an offer $\tilde{w}_l \in (\hat{w}, w^*)$ which yields a positive profit. Therefore, w_l cannot be a best answer.

Case 3. Finally suppose that the highest offer is above w^*. By denoting that highest salary by $w_l := w^* + \varepsilon$, it follows that

$$\frac{w_l}{1-w_l} > \frac{w^*}{1-w^*} = \frac{q_1 p_1^n}{q_0 p_0^n} = \frac{\mu(w)}{1-\mu(w)} \left(\frac{p_1}{p_0}\right)^n, \tag{A.10}$$

whereas $\frac{q_1}{q_0} = \frac{\mu(w)}{1-\mu(w)}$ results from the random assignment of workers to firms offering w_l. According to Lemma 4, the inequality in (A.10) ensures that $\underline{s}_l^* = n$ at the final stage. Let the number of firms which offer w_l be L. Since for each firm $l \in L$ it applies that $\mu(w_l) = \frac{q_1}{L}$ and $(1-\mu(w_l)) = \frac{q_0}{L}$, their equilibrium profit reads

$$\begin{aligned}
\pi_l(w_l) &= (1-w^*-\varepsilon)\frac{q_1 p_1^n}{L} - (w^*+\varepsilon)\frac{q_0 p_0^n}{L} \\
&= \left(\frac{q_0 p_0^n}{q_1 p_1^n + q_0 p_0^n} - \varepsilon\right) \frac{q_1 p_1^n}{L} - \left(\frac{q_1 p_1^n}{q_1 p_1^n + q_0 p_0^n} + \varepsilon\right) \frac{q_0 p_0^n}{L} \\
&= -\frac{\varepsilon}{L}(q_1 p_1^n + q_0 p_0^n) < 0.
\end{aligned}$$

Since each $l \in L$ could avoid losses by choosing $\tilde{w}_l < w_l$, w_l cannot be a best answer.

2.A.8 Proof of Proposition 3

With a salary ceiling $\overline{w}$, the threshold for an applicant's score value at any firm j is

$$\underline{s}_j(\overline{w}) = \frac{1}{2} + \ln\left(\frac{q_0}{q_1}\frac{\overline{w}}{(1-\overline{w})}\right)\sigma^2. \tag{A.11}$$

By incorporating the subsequent course of the game, we write (11) as

$$\begin{aligned}\overline{w}_{\theta_i} &= \arg\max_{\overline{w}} \left\{\overline{w}\left(1-\Phi_{\theta_i}\left[\underline{s}_j(\overline{w})\right]\right)\right\} \\ &= \arg\max_{\overline{w}} \left\{\overline{w}\left(1-\Phi_0\left[\underline{s}_j(\overline{w})-\theta_i\right]\right)\right\}. \end{aligned} \tag{A.12}$$

From (A.11), it follows that the first-order condition to problem (A.12) is

$$1-\Phi_0\left[\underline{s}_j-\theta_i\right] = \phi_0(\underline{s}_j-\theta_i)\frac{\sigma^2}{1-\overline{w}},$$

which we write as

$$\begin{aligned}\frac{\sigma^2}{1-\overline{w}} &= \int_{\underline{s}_j-\theta_i}^{\infty} \frac{\phi_0(t)}{\phi_0(\underline{s}_j-\theta_i)}\mathrm{d}t \\ &= \int_0^{\infty} \frac{\phi_0(\underline{s}_j-\theta_i+u)}{\phi_0(\underline{s}_j-\theta_i)}\mathrm{d}u \\ &= \int_0^{\infty} e^{-\frac{u^2}{2\sigma^2}-\left(\frac{1}{2}+\ln\left(\frac{q_0}{q_1}\frac{\overline{w}}{(1-\overline{w})}\right)\sigma^2-\theta_i\right)\frac{u}{\sigma^2}}\mathrm{d}u. \end{aligned} \tag{A.13}$$

The left-hand side of (A.13) equals σ^2 for $\overline{w}=0$, then continuously and strictly increases, and approaches infinity for $\overline{w}\nearrow 1$. Meanwhile, its right-hand side approaches infinity for $\overline{w}\searrow 0$, then strictly and continuously decreases, and converges to 0 as $\overline{w}\nearrow 1$. Since the objective function is positive at an interior solution, (A.13) uniquely determines $\overline{w}_{\theta_i}$. Since the expected payoff is 0 at both $\overline{w}=0$ and $\overline{w}=1$ and its derivative is 0 only at $\overline{w}_{\theta_i}$, it also holds that the objective function increases for values below $\overline{w}_{\theta_i}$ and decreases for values above $\overline{w}_{\theta_i}$. Finally, since the right-hand side of (A.13) increases in θ_i, we have $\overline{w}_1>\overline{w}_0$.

2.A.9 Proof of Lemma 5

We express (13) as

$$\frac{P\left[s_i\geq \underline{s}_l^*(\mu(w),w_l)|\theta_i=\theta\right]}{P\left[s_i\geq \underline{s}_k^*(w_k)|\theta_i=\theta\right]} =: \frac{1-\Phi_\theta[\kappa]}{1-\Phi_\theta[\kappa+\Delta]} > \frac{w_k}{w_l},$$

where it follows from $w_k>w_l$ that $\Delta>0$. Thereby,

$$\frac{1-\Phi_0[\kappa]}{1-\Phi_0[\kappa+\Delta]} > \frac{1-\Phi_1[\kappa]}{1-\Phi_1[\kappa+\Delta]} = \frac{1-\Phi_0[\kappa-1]}{1-\Phi_0[\kappa+\Delta-1]} \tag{A.14}$$

implies that

$$\mu(w)\in M_1 \;\;\Rightarrow\;\; \mu(w)\in M_0.$$

For (A.14) to be true for arbitrary κ and $\Delta>0$, it is sufficient to show that

$$\frac{\partial\frac{1-\Phi_0[\kappa]}{1-\Phi_0[\kappa+\Delta]}}{\partial\kappa}>0 \;\;\Leftrightarrow\;\; \frac{\partial\left\{\ln\left(1-\Phi_0[\kappa]\right)-\ln\left(1-\Phi_0[\kappa+\Delta]\right)\right\}}{\partial\kappa}>0.$$

Otherwise put,

$$\frac{\partial \int_{\kappa}^{\kappa+\Delta} \frac{\partial \ln(1-\Phi_0[t])}{\partial t}\mathrm{d}t}{\partial\kappa} = \frac{\partial \int_{0}^{\Delta} \frac{\partial \ln(1-\Phi_0[\kappa+u])}{\partial u}\mathrm{d}u}{\partial\kappa} < 0. \tag{A.15}$$

(A.15) necessarily holds if

$$\frac{\partial \frac{\partial \ln(1-\Phi_0[\kappa+u])}{\partial u}}{\partial\kappa} = \frac{\partial \frac{-\phi_0[\kappa+u]}{1-\Phi_0[\kappa+u]}}{\partial\kappa} < 0. \tag{A.16}$$

(A.16) needs to be true for any κ, thus equivalently we can show that

$$\frac{\partial \frac{-\phi_0[\kappa]}{1-\Phi_0[\kappa]}}{\partial\kappa} < 0. \tag{A.17}$$

Since in

$$\begin{aligned}
\frac{-\phi_0[\kappa]}{1-\Phi_0[\kappa]} &= -\left(\int_{\kappa}^{\infty} \frac{\phi_0(t)}{\phi_0(\kappa)}\mathrm{d}t\right)^{-1} \\
&= -\left(\int_{0}^{\infty} \frac{\phi_0(u+\kappa)}{\phi_0(\kappa)}\mathrm{d}u\right)^{-1} \\
&= -\left(\int_{0}^{\infty} e^{-\frac{u^2}{2\sigma^2}-\frac{\kappa u}{\sigma^2}}\mathrm{d}u\right)^{-1},
\end{aligned}$$

for a fixed $u > 0$, each integrand is strictly decreasing in κ, the same holds for the negative of the inverse of the integral.

Chapter 3

Exclusivity Clauses: Enhancing Competition, Raising Prices

3.1 Introduction

If a firm accepts a contract with an exclusivity clause, it may not deal with competitors of the firm that issued the contract. The debate on such exclusivity agreements divides policy makers and scholars into two fractions. Critics argue that exclusivity clauses are anticompetitive because they foreclose entry.[1] Advocates invoke efficiency motives, claiming that exclusivity protects value-generating investments from free-riding.[2]

In this chapter, we compare arguments for and against exclusivity clauses. To do so, it is crucial to specify which type of exclusivity we are looking at. According

[1]Proponents of the foreclosure argument include Aghion and Bolton (1987), Rasmusen, Ramseyer, and Wiley (1991), and Segal and Whinston (2000b). Upon Fumagalli and Motta's (2006) critique that the former authors' findings rely on buyers being final customers, Simpson and Wickelgren (2007) and Wright (2009) extend the argument to a setting more similar to ours, including buyers competing in a downstream market.

[2]Proponents of the incentives argument include Williamson (1979) and Marvel (1982). Segal and Whinston (2000a) and Bernheim and Whinston (1998) specify that the efficiency-enhancing property of exclusivity clauses only holds in cases where investment has external effects on third parties.

to Segal and Whinston (2000a), "a contract between a buyer and a seller is said to be exclusive if it prohibits one party to the contract from dealing with other agents." In our model, the character of exclusivity depends on which side of the market offers contracts. Most of the existing literature deals with what is usually called "exclusive dealing": Suppliers write contracts with exclusivity clauses, which prohibit retailers to purchase from other suppliers. In addition to this, we also discuss what we refer to as "exclusive provision": Retailers offer contracts with exclusivity clauses, which stipulate that contracting suppliers are not allowed to additionally sell to other retailers.[3]

A prominent example for exclusive provision is the distribution of Apple's *iPhone* via selected service providers.[4] For instance, in the United States, the *iPhone* was sold exclusively through AT&T between 2007 and 2011.[5] That is, every customer who wanted to buy an *iPhone* had to accept AT&T's monopoly price. Obviously, this generates a correspondingly high willingness-to-pay for exclusivity on behalf of AT&T. As financial data suggests, it was indeed Apple who ended up reaping the bulk of the benefits from this exclusivity arrangement.[6]

In the following, we show that this straightforward mechanism carries over to more competitive settings. This is remarkable, as even with several retailers reselling a homogeneous product obtained from multiple suppliers, the resulting allocation might be the same as in the monopoly case.[7]

[3]In addition to exclusive provision and exclusive dealing, one could also look at mutual exclusivity (as is the case of marriage arrangements). We refrain from discussing such agreements in this chapter.

[4]The arrangements between Apple and the service providers are usually referred to as "tying" agreements. In the present chapter, however, we ignore the complementary between the primary good and potential services provided by retailers. In fact, we consider retailers as pure reselling entities.

[5]Similar agreements were established in numerous other countries, such as Germany (T-Mobile), UK (O2), Japan (SoftBank Mobile), and Spain (Movistar).

[6]See, for instance, Kuittinen (2012).

[7]For the sake of brevity, we do not show the equivalence between the monopolistic and the competitive setting here. The results are available on request.

Imagine a setting with two suppliers and two retailers. Exclusivity clauses require that the suppliers sell their product to only one retailer.[8] In return, a retailer could offer more favorable quantities and wholesale prices. Alternatively, the retailer could offer less favorable terms, but allow the suppliers to contract with its competitor as well. If both retailers choose this second option, they effectively eliminate competition between them *in the upstream market*: as long as the suppliers' production cost is covered, both offers are accepted. In this case, the retailers purchase Cournot quantities and both make positive profits. However, if a retailer found a way to also eliminate competition *in the downstream market*, its margin would be even higher. By means of the exclusivity clause, it is equipped with an instrument which facilitates this purpose. An arbitrarily small compensation is sufficient to induce both suppliers to accept such a clause. Hence, both of them provide the same retailer, which in turn obtains a monopoly position in the downstream market.

Of course, one retailer obtaining the monopoly profit and the other getting nothing cannot be an equilibrium. Facing exclusivity clauses, suppliers have to select a single retailer. Hence they only head for the most favorable deal. This in turn induces tough competition between the retailers. In fact, they are forced to extract as much consumer surplus as possible and redirect it to the suppliers. Thus exclusivity clauses strengthen competition, but competition for the suppliers instead of competition for the customers.

Our efficiency argument is quite in contrast with a variety of papers that support the traditional Chicago School argument that exclusivity clauses mainly arise to economize on transaction costs.[9] In Marvel (1982), exclusive dealing allows suppliers to prevent retailers from opportunistic behavior. Once the retailers refrain from buying no-frills substitutes, suppliers are willing to invest into value-generating pre-sale information. Thus exclusive dealing provides a

[8] Accordingly, we consider exclusive provision as defined above. We formally discuss this case in Section 3.3. The conclusions drawn here are only partially applicable to exclusive dealing, as we show in Section 3.4.

[9] Classical examples are Bork (1978) and Williamson (1979).

supplier with a property right on promotional expenses. In a similar vein, McAfee and Schwartz (1994) show that a (monopolistic) supplier is confronted with its own incentive to renegotiate terms with competing retailers. Doing so impairs the rewards for incumbent retailers and decreases their ex-ante willingness to pay. Exclusivity clauses are an easily observable and verifiable approach to curb such opportunism. As in Marvel (1982), they increase the joint profit by inducing a higher level of investment. Accordingly, demand might increase as well. This, in turn, leaves the potential for positive welfare implications.

In contrast, welfare is clearly negatively affected if exclusivity clauses are strategically deployed to deter entry which otherwise would have taken place. In Rasmusen, Ramseyer, and Wiley (1991) and Segal and Whinston (2000b), suppliers need to reach a certain scale to produce at minimum average cost. By locking in sufficiently many buyers, an incumbent supplier can thus prevent entry of competitors, and buyers agree to sign contracts including exclusivity clauses. In an earlier article, Aghion and Bolton (1987) show that entry prevention is also possible without a positive minimum efficient scale if the incumbent can draw on more sophisticated contracts including "liquidated damages" clauses. Such contingent transfers to the incumbent supplier serve as an entry fee for outside suppliers, and enable the incumbent supplier–retailer pair to extract monopoly rents.[10]

Our model builds upon a similar idea, although we drop the asymmetry between incumbents and entrants and thus investigate a more competitive framework. Consequently, contracting parties use exclusivity clauses not for strategic purposes but because they are forced to use them to withstand competition. This turns out to be self-destructive. Whenever a contract-issuing firm employs an exclusivity clause, it has to promise the most favorable prices and quantities

[10]In line with the existing theory, the empirical literature on exclusivity clauses is inconclusive. Evidence on customer price effects is scarce and mainly centered around the beer brewing industry. For an overview, see Lafontaine and Slade (2008). Furthermore, authors such as Slade (2000) and Asker (2004) find that exclusivity constraints lead to higher markups of the vertical supply chain and increased customer prices. These observations are consistent with both lines of reasoning.

to its counterparties. Thus it ends up with zero profit. Altogether, we obtain the same quantities and prices as in a setting with a single vertically integrated firm.[11]

We organize this chapter as follows. In Section 3.2 we introduce the model. Next we discuss how the outcome depends on the sequence of play. In Section 3.3 we study the case of exclusive provision. We distinguish between two scenarios: fist we study homogeneous suppliers; then we consider a situation in which their costs of production differ. In the latter instance, screening becomes possible, along with an outcome where only the efficient supplier prevails. In Section 3.4 we contrast exclusive provision with exclusive dealing, the standard setting in the literature. Exclusivity clauses now force *suppliers* to maximize the retailers' individual profits. This induces Cournot rather than monopoly quantities, resulting in a lower customer price. While a ban on exclusive provision may increase welfare, prohibiting exclusive dealing has adverse effects: Absent exclusivity clauses, suppliers set their wholesale prices above marginal cost. In turn they have to reduce their quantities, as otherwise the retailers buy from one supplier only. This increases the customer price in the downstream market. In Section 3.5, we take account of the conflicting implications of exclusive provision and exclusive dealing. Which type of contract arises endogenously? We conjecture that retailers could confront exclusive-dealing contracts with more profitable counteroffers. Exclusive-provision contracts are less vulnerable, thus we expect the retailers to make offers in equilibrium. In Section 3.6, we conclude by discussing our results in light of their policy implications. All proofs are relegated to the appendix.

[11] To be precise, this conclusion is restricted to the exclusive-provision framework. With exclusive dealing, the maximization of individual retailers' surpluses entails Cournot quantities and prices.

3.2 Model

We consider the following vertical relation. Suppliers produce a good, which they sell to retailers. These, in turn, resell it to final customers. Demand is given by $p(Q) = a - bQ$, where p is the output price and Q is the total quantity sold in the downstream market. Demand is positive for a positive range of p, and the law of demand applies; that is, $a > 0$ and $b > 0$.

In the upstream market there are two groups of strategic players: two suppliers and two retailers.[12] Each supplier $s \in \{H, L\}$ maximizes the expected value of

$$\pi_s = \sum_{r \in \{1,2\}} (w_{sr} - c_s) q_{sr}, \tag{1}$$

where q_{sr} is the quantity of the output that supplier s sells to retailer $r \in \{1, 2\}$, and w_{sr} is the (per-unit) wholesale price. c_s is the marginal cost of production of supplier s.[13] We assume supplier H to be (weakly) less efficient than supplier L. Specifically, $c_H = c + \Delta$, and $c_L = c - \Delta$ with $\Delta \geq 0$. Suppliers cannot sell their output directly to customers. Instead, supplier s sells it at a wholesale price w_{sr} to retailer r.

The retailers buy the input in the upstream market and sell it to final customers. Each retailer $r \in \{1, 2\}$ maximizes the expected value of

$$\pi_r = \sum_{s \in \{H,L\}} (p(Q) - w_{sr}) q_{sr}, \tag{2}$$

with $Q = \sum_{s \in \{H,L\}} \sum_{r \in \{1,2\}} q_{sr}$. r resells the product to final customers. For simplicity, we assume that this can be done without incurring any additional (reselling) cost.

[12] Most of the results which follow are readily applicable to settings with more firms, because Bertrand-related mechanisms lead to perfect competition already with two firms.

[13] From $p(Q) = a - bQ$, we see that $c_s \geq a$ implies that there are no gains from trade. Therefore, without any loss of generality, we impose that $c_s < a$ for $s \in \{1, 2\}$.

The determination of w_{sr} and q_{sr} crucially depends on the sequence of moves. In Section 3.3, we consider the case where the retailers fix wholesale prices and quantities which they are willing to buy. In Section 3.4, the suppliers announce terms of sale. We specify the contracts below, along with the exclusivity clauses which the firms may impose. For now, a general description of the contracting game suffices: In a first stage, either the retailers (Section 3.3) or the suppliers (Section 3.4) make binding take-it-or-leave-it offers. In the second stage, the recipients of these offers may accept one, both, or none of these offers. Their choice might be restricted, as they potentially have to abide by exclusivity clauses.
We focus on symmetric, subgame-perfect Nash equilibria in pure strategies, according to the following assumptions.

Assumption 2 (Symmetry). Identical players choose identical strategies. As the suppliers differ in their production efficiency, Assumption 2 only applies to the retailers.

Assumption 3 (Pure Strategies). We analyze Nash equilibria in pure strategies. If a retailer faces two identical offers from suppliers H and L, the offer from (the more efficient) supplier L is chosen. If a supplier faces two identical offers from (identical) retailers, each of these offers is selected with probability 0.5.

Assumption 4 (Subgame Perfection). The contract-submitting parties do not make offers which generate negative payoffs if accepted. Accordingly, we rule out equilibria in weakly dominated strategies.

3.3 Exclusive Provision

In this section, we consider the case where the retailers $r \in \{1, 2\}$ make binding offers, and the suppliers $s \in \{H, L\}$ select among these. Offers from the retailers are of the type $\Phi_r := (q_r, w_r, e_r)$, where $q_r = q_{sr}$ for $s = \{H, L\}$, and $w_r = w_{sr}$ for $s = \{H, L\}$. Furthermore, e_r is a binary choice variable, which indicates

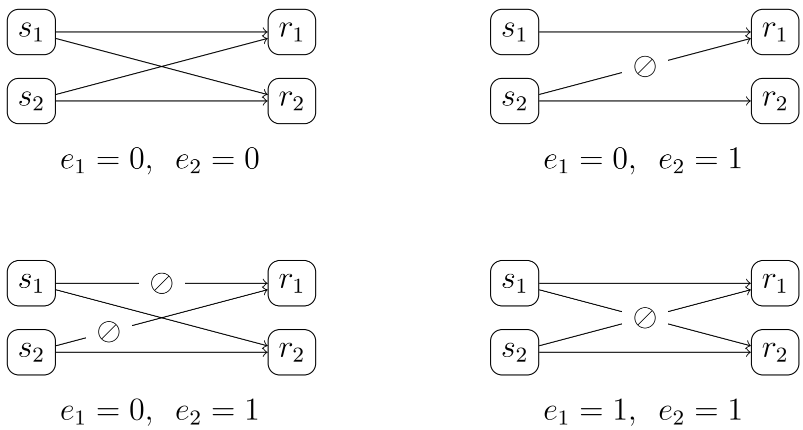


Figure 3.1: Exclusive Provision

whether retailer r stipulates an exclusivity clause. If $e_r = 1$, each supplier which sells to r is not allowed to sell the product to the other retailer. If $e_r = 0$ for $r = \{1, 2\}$, the suppliers are allowed to accept both offers.[14]

In Figure 3.1, we illustrate exclusive provision. In the upper-left diagram, we depict a situation without exclusivity clauses. Thus both suppliers, s_1 and s_2, can provide both retailers, r_1 and r_2. In the upper-right diagram, a supplier selling its product to r_2 is not allowed to additionally provide r_1. Meanwhile, there is no contractual restriction which prevents r_1's suppliers to also provide r_2. Nevertheless, s_1 cannot serve both retailers either. If s_1 approaches r_2, it forgoes the possibility to provide r_1. We depict this in the lower-left panel. In fact, the suppliers' options are equally restricted, regardless of whether one or both retailers impose exclusivity clauses. Accordingly, in both instances we obtain the same set of possible outcomes (compare the upper-right panel with the lower-right panel).

[14]Note that, say, $e_1 = 1$ and $e_2 = 0$ implies that the suppliers cannot deal with both retailers. Thus a retailer can single-handedly install overall exclusivity, and no retailer can unilaterally remove an overall exclusivity regime if $e_r = 1$ for $r = \{1, 2\}$.

In order to derive equilibria Φ_r^* (and eliminate putative equilibria $\hat{\Phi}_r$), we study properties of q_r, w_r, and e_r which necessarily hold whenever no retailer has a unilateral incentive to offer an alternative bundle $\tilde{\Phi}_r$.[15] At the end of this section, we show that equilibria Φ_r^* necessarily feature exclusivity clauses. For the moment, to facilitate the analysis, we simply assume that $e_r^* = 1$ for $r = \{1, 2\}$.

From (2), we can write the expected profit of retailer r as[16]

$$E[\pi_r] = \sum_{x \in \{1,2\}} P[r \text{ has } x \text{ suppliers}](p(Q) - w_r)xq_r. \tag{3}$$

From (1), it further follows that supplier s's profit upon accepting r's offer is

$$\pi_s(r) = (w_r - c_s)q_r. \tag{4}$$

Provided that both suppliers' participation constraints are satisfied ($\pi_s(r)$ is non-negative for either r), we infer from (4) that supplier $s \in \{H, L\}$ approaches retailer r whenever $q_r(w_r - c_s) > q_{-r}(w_{-r} - c_s)$. If $q_r(w_r - c_s) = q_{-r}(w_{-r} - c_s)$, each supplier is indifferent between the two retailers, and Assumption 3 implies that the suppliers accept the offer of each retailer with probability 1/2. In this case, r obtains $2q_r$ with probability 1/4, q_r with probability 1/2, and 0 with probability 1/4. Accordingly, (3) equals

$$\pi_r = \begin{cases} (a - 2bq_r - \S_r)2q_r & \text{if } (w_r - c_s)q_r > (w_{-r} - c_s)q_{-r}, \\ (a - bq_r - bq_{-r} - w_r)q_r & \text{if } (w_r - c_s)q_r = (w_{-r} - c_s)q_{-r}, \\ 0 & \text{if } (w_r - c_s)q_r < (w_{-r} - c_s)q_{-r}. \end{cases} \tag{5}$$

[15] In this section, we interchangeably use Φ_r^* to denote equilibria and equilibrium contracts (analogous with putative equilibria $\hat{\Phi}_r$). Implicitly, equilibria further require that each supplier accepts the most profitable contract. (Regarding ties, see Assumption 3.)

[16] For ease of notation, we omit the expectation operator in the following.

By means of the following lemmas, we show that for $c_H = c_L$ the retailers strongly compete for the two suppliers. For sufficiently high levels of Δ, they only compete for the efficient supplier L, and the less efficient supplier H is driven out of the market.

Note that Lemmas 1 trough 4 throughout make reference to equilibria where the retailers make offers. Furthermore, we consistently suppress the implicit assumption that $e_r^* = 1$ for $r \in \{1, 2\}$.

Lemma 1. *Both retailers obtain zero profit. Thus, if both suppliers' participation constraints are satisfied, we have*

$$w_r^* = a - b2q_r^*. \tag{6}$$

If only the efficient supplier's participation constraint is satisfied, we have

$$w_r^* = a - bq_r^*. \tag{7}$$

Lemma 1 implies zero profits on behalf of the retailers, which is a necessary condition for each individual retailer to not having an incentive to outbid its opponent's offer (and thereby double its own customer base).

Lemma 2. *If both suppliers' participation constraints are satisfied, the profit of either supplier H or supplier L is maximized subject to* (6)*. If only the efficient supplier L's participation constraint is satisfied, L's profit is maximized subject to equation* (7)*.*

For some intuition for Lemma 2, consider Figure 3.2, where $w_r = a - 2bq_r$ depicts the zero-profit line of the retailers for the case where both suppliers accept an offer. From equation (4), we can state supplier s's marginal rate of substitution between q_r and w_r at $(\hat{q}_r, \hat{w}_r)$ as[17]

[17] As can be seen in Figure 3.2 (and will be of importance later on), the absolute value of $\text{MRS}_L(\hat{q}_r, \hat{w}_r)$ exceeds the absolute value of $\text{MRS}_H(\hat{q}_r, \hat{w}_r)$. This is because the lower marginal cost of supplier L leads to higher markups for L than for H ($\hat{w}_r - c_L > \hat{w}_r - c_H$). Accordingly,

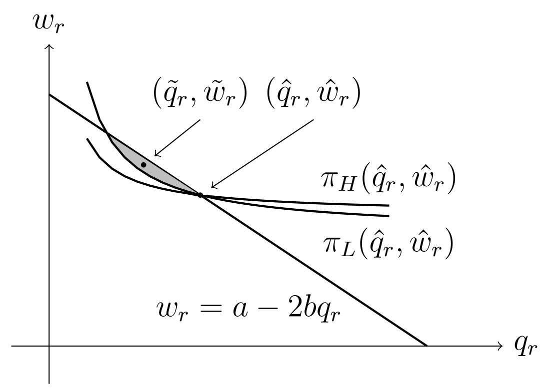


Figure 3.2: If no suppliers' profit is maximized conditional on $w_r = a - 2bq_r$, a deviating retailer can attract both suppliers and obtain a positive profit.

$$\mathrm{MRS}_s(\hat{q}_r, \hat{w}_r) := -\frac{\hat{w}_r - c_s}{\hat{q}_r} > -2b,$$

In the example of Figure 3.2, in contrast to Lemma 2, neither supplier L's nor supplier H's profit is maximized conditional on the retailers' zero-profit condition. Specifically, we have $\mathrm{MRS}_s(\hat{q}_r, \hat{w}_r) > -2b$ for $s \in \{H, L\}$. Due to the intersection of the suppliers' isoprofit curves with the retailers' zero-profit line, a deviating retailer r could instead offer $(\tilde{q}_r, \tilde{w}_r)$ within the shaded region in Figure 3.2. Since for both suppliers such an offer would lead to higher profits than $(\hat{q}_r, \hat{w}_r)$, the deviating retailer r would attract both H and L. Hence, r's zero-profit line would be unaffected, which implies that $(\tilde{q}_r, \tilde{w}_r)$ would lead to positive profits. In this case, as well as in all other cases where not at least either H's or L's profit is maximized conditional on the retailers' zero-profit condition, $(\hat{q}_r, \hat{w}_r)$ cannot be part of an equilibrium.

It directly follows from Lemma 2 that in the case of homogenous suppliers, both retailers offer (half of) the monopoly quantity.

Corollary 1. *For $\Delta = 0$, we have $q_r^* = (a - c)/4b$ and $w_r^* = (a + c)/2$.*

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to sustain the profit at a constant level, a reduction in q_r requires a higher compensation for L than for H.

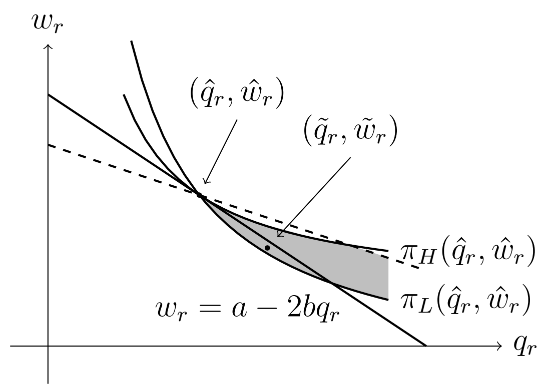

Figure 3.3: If supplier H's profit is maximized conditional on $w_r = a - 2bq_r$, a deviating retailer can attract supplier L and obtain a positive profit.

Simple as it is, Corollary 1 conveys a central result of this chapter: The retailers' exclusivity clauses imply that only the best offer has a chance of being accepted. Hence competition is vigorous. Not only do the retailers offer wholesale prices which equal the price they charge their final customers. In addition, total quantities aggregate to the amount a monopolistic vertically integrated firm would sell. Once all retailers offer their share of the monopoly quantity, the final price is independent of the suppliers' allocation. Thus if an individual retailer lowers its quantity, it would be disregarded by the suppliers. By offering more, it would make losses.

In a similar way as in Lemma 2, next we rule out equilibria where the profit of supplier H is maximized.

Lemma 3. *For $\Delta > 0$, an outcome where the profit of supplier H is maximized subject to* (6) *cannot be an equilibrium.*

In a putative equilibrium $\hat{\Phi}_r$ where the profit of the less efficient supplier H is maximized (conditional on $w_r = a - 2bq_r$), the (absolute value of the) marginal rate of substitution of supplier L exceeds the slope of the zero-profit line of the retailers (see Figure 3.3). Due to the intersection of L's isoprofit curve with the retailers' zero-profit line, a deviating retailer r could instead offer $\tilde{\Phi}_r$, which lies in the region of positive profits for the retailers, even with the original zero-profit line. Furthermore, by offering $\tilde{\Phi}_r$, supplier r's zero-profit line becomes $w_r = a - b\hat{q}_r - bq_r$. That is, it rotates around $(\hat{q}_r, \hat{w}_r)$, as indicated by the dashed line in Figure 3.3. If retailer r offered any contract within the shaded area in Figure 3.3, the inefficient supplier H would strictly prefer the opponent retailer $-r$ (which offers $\hat{\Phi}$), and the efficient supplier L would strictly prefer r. Hence, by exclusively attracting supplier L, r could increase its profit, which eliminates $\hat{\Phi}_r$.

From Lemmas 1 through 3, it follows that in any symmetric equilibrium, the profit of supplier L must be maximized subject to (6) or (7), that is, to a zero-profit condition of the retailers. As we show next, whenever this implies that both suppliers' participation constraints are satisfied, there are profitable deviations possible (unless if $\Delta = 0$, see Corollary 1), and there is no equilibrium. However, if c_L and c_H are sufficiently far apart from each other, there exists a unique equilibrium where only the efficient supplier L is attracted by the retailers.

Lemma 4. *For $\Delta > 0$, there exists no equilibrium where both suppliers' participation constraints are satisfied. If $\Delta > (a - c)/3$, there exists a unique equilibrium in which H's participation constraint is violated.*

To see this, first consider a candidate equilibrium where both suppliers participate. From Lemmas 2 and 3, we know that $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) = -2b$ in such an equilibrium. Since $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) < \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r)$, it is generally possible to increase q_r and decrease w_r in a way that the new contract $\tilde{\Phi}_r$ is only attractive for supplier L. As a consequence, the zero-profit line rotates around $(\hat{q}_r, \hat{w}_r)$ (see Figure 3.3) such that $\tilde{\Phi}_r$, which leads to negative profits according to the original zero-profit line, becomes profitable.[18]

Deviations of this type, however, are not feasible if an equilibrium contract Φ_r^* violates supplier H's participation constraint. In this case, it follows from

[18] Instead of attracting each supplier with probability 1/2, the deviating retailer attracts supplier L for sure.

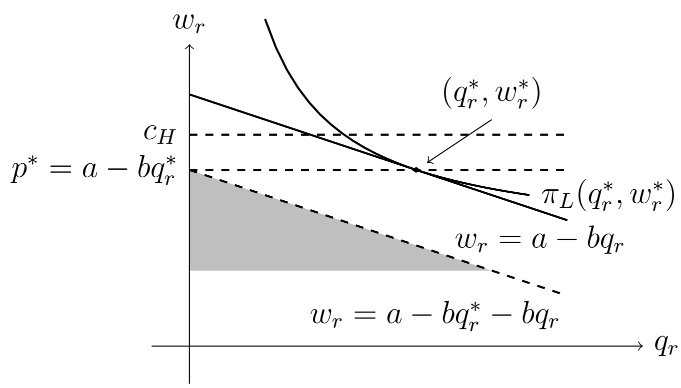

Figure 3.4: If supplier L's profit is maximized conditional on $w_r = a - bq_r$, and c_H is sufficiently high, no contract for which $w_H \geq c_H$ is profitable.

Lemmas 1 and 2 that any profitable deviation involves attracting the inefficient supplier H. Since the region of profitable contracts with two suppliers, $w_r < a - b2q_r$, is a subset of the region of profitable contracts with one supplier, $w_r < a - bq_r$, there is no profitable deviation which involves attracting both suppliers. Hence, the only possible deviation would be attracting supplier H without attracting supplier L. However, since L would stay with the other retailer, any quantity provided by supplier H would further deteriorate the customer price p. As can be seen in Figure 3.4, it follows from the retailers' zero-profit condition ($w_r^* = p^*$) and H's non-participation condition ($w_r^* < c_H$) that $p^* < c_H$. Hence, no contract $\tilde{\Phi}_r$ with $\tilde{q}_r > 0$ and $\tilde{w}_r \geq c_H$ would lead to positive profits of the retailer, indicated by the shaded area in Figure 3.4. Therefore, whenever maximizing supplier L's profit conditional on the retailers' zero-profit condition (7) leads to a wholesale price below c_H, there exists an equilibrium where the less efficient supplier H does not accept any offer. This is the case if and only if

$$w_r^* = \frac{a + c_L}{2} < c_H \;\;\Leftrightarrow\;\; \Delta > \frac{a - c}{3}.$$

What Happens if Exclusivity Clauses are Banned? Suppose now that exclusivity clauses are illegal. In this case, supplier $s \in \{H, L\}$ accepts any offer with $w_r \geq c_s$. Accordingly, each retailer sets $w_r^{NE} = c_L$. Furthermore, from quantity competition in the downstream market, both retailers purchase the Cournot quantity $q_r^{NE} = (a - c_L)/3b$, and obatain $\pi_r^{NE} = (a - c_L)^2/9b$.

Will Exclusivity Clauses Be Used? When exclusivity clauses are allowed, will they actually be used?

From the previous paragraph, we know that in a candidate equilibrium without exclusivity clauses, both suppliers obtain zero profit, that is, $\pi_s^{NE} = 0$ for $s \in \{H, L\}$.

If $\Delta > 0$, by offering $\tilde{\Phi}_r$ with $\tilde{w}_r = c_L + \varepsilon$ with $\varepsilon > 0$ sufficiently small, even in combination with an exclusivity clause, r could attract L without attracting H. As a monopolist in the downstream market, r could set $\tilde{q}_r = (a - \tilde{w}_r)/2b$. Since $\tilde{\pi}_r = (a - \tilde{w}_r)^2/4b$ exceeds π_r^{NE} for $\varepsilon < (a - c + \Delta)/3$, this eliminates the candidate equilibrium without exclusivity clauses.

If $\Delta = 0$, r would attract both suppliers by offering $\tilde{w}_r = c + \varepsilon$, and could therefore set $\tilde{q}_r = (a - c)/4b$. For $\varepsilon < (a - c)/3$, this constitutes a profitable deviation.

Regarding equilibria with exclusivity clauses, a unilateral deviation from $e_r^* = 1$ to $\tilde{e}_r = 0$ is effectless. As the other retailer maintains $e_{-r}^* = 1$, the suppliers still have to choose between the two retailers. Hence, the above results, together with $e_r^* = 1$ constitute an equilibrium indeed.

Proposition 1 summarizes the results of this section.

Proposition 1. *Consider the case where retailers can engage in exclusive provision. For $\Delta = 0$, there is a unique equilibrium, where each retailer offers Φ_r^* with $q_r^* = (a - c)/4b$, $w_r^* = (a + c)/2$, and $e_r^* = 1$. Both suppliers choose each retailer with probability $1/2$. The corresponding output price is $p^* = (a + c)/2$. Both retailers make zero profit. The profit of each supplier $s \in \{H, L\}$ is $\pi_s^* = (a - c)^2/8b$. For $0 < \Delta < (a - c)/3$, there is no equilibrium. For*

$\Delta \geq (a-c)/3$, *there is a unique equilibrium, where each retailer offers* Φ_r^* *with* $q_r^* = (a-c+\Delta)/4b$, $w_r^* = (a+c-\Delta)/2$, *and* $e_r^* = 1$. *Supplier* L *chooses each retailer with probability* $1/2$. *Supplier* H *does not accept any offer. The output price is* $p^* = (a+c-\Delta)/2$. *Both retailers and supplier* H *make zero profit. Supplier* L *obtains* $\pi_L^* = (a-c+\Delta)^2/4b$.

3.4 Exclusive Dealing

Now we look at the opposite setting where the suppliers $s \in \{H, L\}$ make binding offers, and the retailers select among these. Offers from the suppliers are of the type $\Phi_s := (q_s, w_s, e_s)$, where $q_s = q_{sr}$ for $r = \{1, 2\}$, and $w_s = w_{sr}$ for $r = \{1, 2\}$.[19] Analogous to the case above, e_s indicates whether s stipulates an exclusivity clause. If $e_s = 1$, each retailer which purchases q_s at w_s from s is not allowed to purchase the product from the other supplier. If $e_s = 0$ for $s = \{H, L\}$, the retailers are allowed to purchase from both suppliers.

In Figure 3.1, we illustrate exclusive dealing. When either none or both suppliers impose exclusivity clauses, the firms' possibilities to cooperate are equally restricted as with exclusive provision. Thus we refrain from replicating the respective diagrams from Figure 3.1. If only one supplier requires exclusivity, the choice of the retailers is restricted. Each of them can only purchase from one supplier. In the left panel, we depict a situation where retailer r_2, which purchases from supplier s_2, is contractually restricted to not purchase from supplier s_1. Retailer r_1 is not restricted by its own supplier, s_1. However, r_1's choice is equally limited, since s_2 refuses to deal with r_1 as long as r_1 maintains its relation with s_1. Otherwise put, in order to purchase from s_2, r_1 has to forego its relation with s_1 (see the right panel).

In any equilibrium (Φ_s^*, Φ_{-s}^*) where both retailers accept the offer of supplier s, accepting this offer must be a weakly dominant strategy. To avoid repetitions,

[19] By analogy with our previous notation (see footnote 15), we use (Φ_H^*, Φ_L^*) to denote both equilibria and equilibrium contracts. The same applies to putative equilibria $(\hat{\Phi}_H, \hat{\Phi}_L)$.

$e_1 = 0, \;\; e_2 = 1$ $\qquad\qquad$ $e_1 = 0, \;\; e_2 = 1$

Figure 3.1: Exclusive Dealing

we occasionally refer to Result 1, which states that dominance can be broken down to a single condition.

Result 1. *Choosing supplier s is a (weakly) dominant strategy (over choosing supplier $-s$) if and only if*

$$(a - 2bq_s - w_s)q_s > (\geq) \;\; (a - bq_s - bq_{-s} - w_{-s})q_{-s}. \tag{8}$$

Accordingly, whenever s caters both retailers, each retailer would also accept s's offer if the other retailer would not. Therefore, in the subgames where the retailers choose their suppliers, every equilibrium where s prevails is an equilibrium in dominant strategies.

As in the previous section, we derive (Φ_H^*, Φ_L^*) by means of a number of lemmas. In Lemmas 5 through 8, we suppress that we refer to equilibria where the suppliers make offers. At the end of this section, we show that in most circumstances equilibria necessarily feature exclusivity clauses, and in the remaining cases where equilibria without exclusivity clauses exist, this is because there is a de-facto monopoly on behalf of the more efficient supplier L. For the moment, we simply assume that $e_s^* = 1$ for $s \in \{H, L\}$.

Lemma 5. *The less efficient supplier H is not chosen by any retailer.*

The intuition for Lemma 5 is simple: If supplier H's contract was to be accepted, it must be strictly better than L's offer. Thus L could undercut H by mimicking H and marginally lower the offered wholesale price. This is possible unless $\hat{w}_H = c_H$ and $c_H = c_L$. Nevertheless, also this remaining case cannot be an equilibrium. Since H's offer would be strictly better that L's offer, H could marginally decrease its offered wholesale price. Its offer would still be strictly better, and H could obtain a positive profit. This eliminates all putative equilibria of this type.

Lemma 6. *Supplier H sets $w_H^* \geq c_H$, supplier L sets $w_L^* \leq c_H$.*

The first statement of Lemma 6 directly follows from Assumption 4: Each putative equilibrium where supplier H sets its wholesale price below marginal cost violates subgame perfection. The second statement stems from the fact that H could obtain a positive profit by slightly undercutting L's offer whenever $\hat{w}_L > c_H$. This is in contradiction with Lemma 5.

Lemma 7. *If $\Delta \geq 3(a-c)/5$, there exists a continuum of equilibria with $q_L^* = (a-c_L)/4b$ and $w_L^* = (a+c_L)/2$. If $0 \leq \Delta < 3(a-c)/5$, every equilibrium features $w_L^* = w_H^* = c_H$.*

From Result 1 we know that a necessary condition for supplier H to profitably enter is to attract one retailer, given the other retailer sticks with L. Although the retailers make zero profit at L, to offer a mutually profitable contract $\tilde{\Phi}_H$, the inefficient supplier H must ensure that the resulting total quantity $\tilde{q}_H + q_L^*$ is low enough to cause the customer price taking on a value above $\tilde{w}_H$. Since the low marginal cost c_L of supplier L further implies a high (monopoly) quantity q_L^*, there is no low enough $\tilde{q}_H > 0$ which attracts a retailer, given $\tilde{w}_H > c_H$.[20]
If Δ is small, supplier L cannot maintain its monopoly contract. In this case, if we had $\hat{w}_L < \hat{w}_H$, and the retailers strictly preferred L's offer, L could do better by slightly increasing w_L such that choosing L would still be a dominant strategy for the retailers. Similarly, if the retailers are indifferent between the

[20] If H offered a sufficiently small $\tilde{q}_H$ in combination with $c_H < \tilde{w}_H < a$, both retailers (and thus the deviating supplier H) would profit by mutually accepting H's offer. However, the retailers suffer from a "prisoners' dilemma".

two suppliers' offers (and accept L's offer, see Assumption 3), we show in Appendix 3.A.9 that L's best reaction to any of H's offers Φ_H involves $w_L^* \geq w_H$. Together with Lemma 6, this implies $w_H^* = w_L^* = c_H$.

Lemma 8. *If $0 \leq \Delta < 3(a-c)/5$, in any equilibrium each supplier $s \in \{H, L\}$ offers Φ_s^* with $q_s^* = (a - c_{-s})/3b$. Together with Lemma 7, this implies that there is no equilibrium whenever $0 < \Delta < 3(a-c)/5$. If $\Delta = 0$, there exists a unique equilibrium.*

Lemma 8 states that, if $\Delta = 0$, both suppliers offer the Cournot duopoly quantity at a wholesale price equal to the suppliers' marginal cost. Accordingly, both retailers are provided with the quantity which they would have chosen themselves. If, say, supplier $-s$ would offer a different quantity, the contract of supplier s which includes the Cournot quantity would be strictly better. This would give s an incentive to increase its wholesale price w_s.

If $\Delta > 0$, Lemma 8 requires that each supplier offers the Cournot quantity with marginal costs of the *other* supplier. If this could be done by simultaneously offering a wholesale price which equals the other supplier's marginal cost, there would not be a way to profitably deviate. This, however, is not possible for the inefficient supplier H which would incur losses by doing so. That is, L is more competitive, and it follows from standard results of Cournot theory that L's quantity lies above H's quantity. In this case, L wants to decrease its wholesale price, which is in contrast to the result of Lemma 7. Accordingly, there is no equilibrium for $0 < \Delta < 3(a-c)/5$.

Before we summarize the results of Lemmas 5 through 8 in Proposition 2, we discuss the possibility of equilibria without exclusivity clauses.

What Happens if Exclusivity Clauses are Banned? As in the previous section, we compare the above equilibria with results which emerge under a ban on exclusivity clauses.

In a "no-exclusivity" equilibrium Φ_s^{NE} with $e_s = 0$ for $s = \{H, L\}$, first suppose that both suppliers offer positive quantities which are accepted by the retailers.

If this is true, retailer r has no incentive to deviate by not purchasing from supplier s. That is,

$$\begin{aligned} & (a-2bq_s-2bq_{-s}-w_s)q_s \\ + \ & (a-2bq_s-2bq_{-s}-w_{-s})q_{-s} \\ \geq \ & (a-bq_s-2bq_{-s}-w_{-s})q_{-s}. \end{aligned} \tag{9}$$

Furthermore, (9) has to hold as an equality, as otherwise supplier s could increase its wholesale price w_s without deterring any retailer. Upon simplification, this implies $w_s = a - b(2q_s + 3q_{-s})$, and in an equilibrium each supplier $s \in \{H, L\}$ solves

$$\begin{aligned} (q_s^{NE}, w_s^{NE}) = & \arg\max_{q_s, w_s} (w_s - c_s)2q_s \\ & \text{s.t. } \ w_s = a - b(2q_s + 3q_{-s}), \end{aligned} \tag{10}$$

which yields the reaction function of supplier s,

$$q_s^{NE}(q_{-s}^{NE}) = \frac{a-c_s}{4b} - \frac{3}{4}q_{-s}^{NE}. \tag{11}$$

Together with the constraint in (10), equation (11) implies

$$q_s^{NE} = (a + 3c_{-s} - 4c_s)/7b, \tag{12}$$

$$w_s^{NE} = (2a + 6c_{-s} - c_s)/7, \tag{13}$$

and

$$\pi_s^{NE} = (2a + 6c_{-s} - 8c_s)^2/49b.$$

If $\Delta > (a-c)/7$, we have $q_H^{NE} = 0$, and L acts as a monopolist in the upstream market. Therefore, L sells $q_L^{NE} = (a - c + \Delta)/4b$ at $w_L^{NE} = (a + c - \Delta)/2$. In this case, H cannot make an offer which the retailers accept *in addition* to L's offer. H can, however, attempt to make an offer which the retailers choose *instead* of L's offer. Since each retailer r obtains $\pi_r^{NE} = 0$ at L, H could attract r and obtain a positive profit by offering $\tilde{\Phi}_H$ with $\tilde{w}_H = c_H + \varepsilon$, $\varepsilon > 0$, and $(a - bq_L^{NE} - b\tilde{q}_H - \tilde{w}_H)\tilde{q}_H > 0$. As it is possible to find such an offer whenever $0 < \Delta < 3(a-c)/5$, there is no equilibrium within this range.

If $\Delta \geq 3(a-c)/5$, the inefficient supplier H has no means to challenge L's monopoly behavior. Accordingly, the outcome under a ban on exclusivity clauses mirrors the outcome when exclusivity clauses are allowed (see Lemma 7). In fact, due to the absence of H's competitiveness, the market structure is the same as with only one supplier. Of course, under such circumstances it is irrelevant to distinguish between varying exclusivity regimes.

Will Exclusivity Clauses Be Used? First consider the case where, under a ban of exclusivity clauses, both suppliers sell positive quantities; that is, $\Delta \leq (a-c)/7$. In this case, once exclusivity clauses are allowed, $(\Phi_H^{NE}, \Phi_L^{NE})$ with $e_s^{NE} = 0$ for $s = \{H, L\}$ no longer constitutes an equilibrium. More precisely, supplier H could profitably deviate by offering an alternative contract $\tilde{\Phi}_H$ with $\tilde{q}_H = 2(a-c)/7b$, $\tilde{w}_H = (2a+5c)/7 - \varepsilon$, and $\tilde{e}_H = 1$. That is, in addition to imposing an exclusivity clause, H could offer the total quantity of both suppliers, given in (12), at a wholesale price which is slightly below the average wholesale price, given in (13). To see this, first note from Result 1, it is a dominant strategy for retailer r to choose H's offer whenever

$$
\begin{aligned}
&(a - 2b\tilde{q}_H - \tilde{w}_H)\tilde{q}_H > (a - b\tilde{q}_H - bq_L^{NE} - w_L^{NE})q_L^{NE} \\
\Leftrightarrow\ &\varepsilon[(a-c)/7] > -\Delta^2.
\end{aligned}
$$

which holds for all $\Delta \geq 0$ and $\varepsilon > 0$. Consequently, H's deviation profit would exceed H's equilibrium profit in the case of a ban on exclusivity clauses whenever

$$(\tilde{w}_H - c_H)2\tilde{q}_H > \pi_H^{NE} = 4(a - c - 7\Delta)/49b$$
$$\Leftrightarrow \varepsilon[28(a-c)^2] < (a-c)^2 - 49\Delta^2 + 7(a-c)\Delta. \tag{14}$$

Since for $\Delta < (a-c)/7$ it holds that $49\Delta^2 < (a-c)^2$, the right-hand side of (14) is positive for all $\Delta \in [0, (a-c)/7]$. Hence, there exists $\varepsilon > 0$ such that both retailers would prefer H's alternative contract $\tilde{\Phi}_H$, and H would increase its profit.

Next, consider the case where only L sells a positive quantity in the absence of exclusivity clauses. As we have shown in the previous paragraph, in this case it is not possible for supplier H to persuade a retailer to choose H instead of L, neither with nor without exclusivity clauses. Furthermore, L obtains its unconstrained profit maximum. Therefore, if $\Delta \geq 3(a-c)/5$, equilibria require no restrictions on exclusivity clauses of either supplier.

Finally, regarding equilibria with exclusivity clauses, unilateral deviations from $e_s^* = 1$ to $\tilde{e}_s = 0$ are effectless, and the offers Φ_s^* of Lemma 8, together with $e_s^* = 1$ for $s = \{H, L\}$ constitute an equilibrium indeed. However, there are two additional equilibria where only one supplier imposes an exclusivity clause. If $\Delta = 0$, and the opponent supplier sets $e_{-s}^* = 0$, supplier s which sets $e_s^* = 1$ could remove bilateral exclusivity by individually switching to $\tilde{e}_s = 0$. Nevertheless, in this case, inequality (9), together with $q_{-s}^* = (a-c)/3b$, requires $\tilde{w}_s \leq c - 2b\tilde{q}_s$, which prevents s from obtaining a positive profit.

We summarize the results of this section in Proposition 2.

Proposition 2. *Consider the case where suppliers can engage in exclusive dealing. For $\Delta = 0$, each supplier $s \in \{H, L\}$ offers Φ_s^* with $q_s^* = (a-c)/3b$, $w_s^* = c$, and $e_r^* \in \{0, 1\}$. At least one supplier sets $e_r^* = 1$. Both retailers accept the offer of supplier L. The output price is $p^* = (a+2c)/3$. Both suppliers make zero profit. The profit of each retailer is $\pi_r^* = (a-c)^2/9b$. For $0 < \Delta < 3(a-c)/5$, there is no equilibrium. For $\Delta \geq 3(a-c)/5$, there is a con-*

tinuum of equilibria where supplier L offers $\Phi_L^* = \Phi_L^{NE}$ with $q_L^* = (a-c+\Delta)/4b$, $w_L^* = (a+c-\Delta)/2$, and $e_L^* \in \{0,1\}$. Supplier H's offer is such that both retailers accept Φ_L^*. Both retailers and supplier H make zero profit. The profit of supplier L is $\pi_L^* = (a-c+\Delta)^2/4b$.

3.5 Discussion

In order to discuss welfare properties of both unconstrained outcomes and equilibria in the case of a ban on exclusivity clauses, it is convenient to compare the customer prices arising from final demand. Since the retailers' and suppliers' participation constraints ensure that no inefficiently high quantity is provided, equilibrium customer prices can be used to compare utilitarian welfare levels.

In Table 3.1, we provide an overview of the customer prices as they depend on the suppliers' difference in efficiency, on the policy towards exclusivity clauses, and on whether the retailers or the suppliers offer contracts.

If the retailers make offers, customer prices are higher once exclusivity clauses are allowed. This holds because, without exclusivity clauses, the retailers do not have to compete for suppliers, as the latter accept any offer which covers their production cost. Accordingly, the retailers' appetite for higher quantities is only restrained by the negatively-sloped downstream demand curve. This leads them to engage in Cournot competition, which implies $p^* = (a + 2(c-\Delta))/3$. In contrast, in the presence of exclusive provision, suppliers only accept the most favorable contract, which induces the retailers to strongly compete. As a result, the retailers are obliged to promise up to the maximum they can reap in the downstream market. Consequently, they have to offer the monopoly surplus, which they obtain from the customers thereupon. Even if the retailers only compete for the efficient supplier L, the resulting customer price, $p^* = (a+c-\Delta)/2$, still exceeds the customer price without exclusivity clauses, $p^{NE} = (a + 2(c-\Delta))/3$.

$\Delta \in$	0	$\left(0, \frac{a-c}{7}\right]$	$\left(\frac{a-c}{7}, \frac{a-c}{3}\right)$	$\left[\frac{a-c}{3}, \frac{3(a-c)}{5}\right)$	$\left[\frac{3(a-c)}{5}, a-c\right)$
			Exclusive Provision		
Exclusivity allowed	$\frac{a+c}{2}$	—	—	$\frac{a+c-\Delta}{2}$	$\frac{a+c-\Delta}{2}$
Exclusivity banned	$\frac{a+2c}{3}$	$\frac{a+2(c-\Delta)}{3}$	$\frac{a+2(c-\Delta)}{3}$	$\frac{a+2(c-\Delta)}{3}$	$\frac{a+2(c-\Delta)}{3}$
			Exclusive Dealing		
Exclusivity allowed	$\frac{a+2c}{3}$	—	—	—	$\frac{a+c-\Delta}{2}$
Exclusivity banned	$\frac{3a+4c}{7}$	$\frac{3a+4c}{7}$	—	—	$\frac{a+c-\Delta}{2}$

Table 3.1: Values of the customer price for various regimes and values of Δ.

This result is reversed once we examine the opposite setting where the suppliers make offers. In this case, customer prices are lower with exclusivity clauses than without. Similarly as described above, exclusivity clauses force the suppliers to intensely compete for the retailers. Thus they have to offer the best possible contract from an individual retailer's perspective; that is, they sell Cournot quantities at wholesale prices which equal their marginal cost of production. A ban on exclusivity clauses, in principle, alleviates competition, as in this case suppliers do not have to outperform their competitors in order to be taken into consideration. However, when faced with large quantities, retailers may profit from denying a contract, as this increases the customer price they obtain in the downstream market. Therefore, from equation (9), each supplier s is constrained by

$$w_s = a - b(2q_s + 3q_{-s}). \tag{15}$$

By solely considering (15) (and supposing that $w_s = c_s$), we would find that a ban on exclusive dealing increases the altogether provided quantity. However, if wholesale prices were set at marginal cost, the suppliers would obtain zero profit. This can be avoided by increasing w_s, conditional on ensuring that (15) still holds. Thus q_s has to be reduced, which leads to a higher customer price than in the case with exclusivity clauses.[21]

When allowed, irrespective of which side of the market makes offers, exclusivity clauses are used (as long as there is no de-facto monopoly of a substantially more efficient supplier L). Taking this into account, Table 3.1 shows that offers of the retailers lead to higher customer prices than offers of the suppliers. In both instances the firms necessarily maximize the profit of the other side of the market. If the suppliers propose contracts, however, maximizing an individual retailer's surplus implies offering a higher quantity, since part of the negative customer-price effect is passed on to the competing retailer. Such externalities are absent if the retailers make offers.[22] Therefore, retailers are better capable of maximizing the joint surplus of the vertical supply chain, which leads to higher customer prices.

Which Side of the Market Makes Offers? The question which consequently arises is whether we rather expect exclusive provision (Section 3.3) or exclusive dealing (Section 3.4) to prevail. Instead of building up a theoreti-

[21] If $\Delta \geq 3(a-c)/5$, there is no difference between contracts with and without exclusivity clauses. As discussed in Section 3.4, such a setting corresponds to a quasi-monopoly of the more efficient supplier, where exclusivity regimes do not matter anymore.

[22] Externalities are also absent in case of exclusive dealing with a monopolistic retailer, as in Bernheim and Whinston (1998). They find that "the form of representation (i.e., exclusivity or common representation) is chosen to maximize the joint surplus of the [vertical supply chain]". This result does not apply here, as the suppliers are forced to use exclusivity clauses, although the thus generated externalities result in a jointly less profitable allocation.

cal superstructure, we refer to the Coase's (1960) theorem, which essentially postulates that voluntary contracts between firms necessarily maximize these firms' joint benefit. Accordingly, contracts will be specified in such a way that there is no alternative contract with a higher joint surplus. As profit increases can be split between the contracting parties, profit-maximizing contracts will be agreed upon. Therefore, by reasoning along the lines of the previous sections, exclusive dealing cannot be sustained once we allow for counteroffers on behalf of the retailers. The latter are more capable to extract the customers' willingness to pay. Therefore, in principle, equilibria where the suppliers make offers (as in Section 3.4) can be eliminated. Retailers find a counteroffers which lead to a Pareto-superior allocations. Whether the above discussed instruments (quantites, wholesale prices, exclusivity clauses) are sufficient to reach the equilibrium of Section 3.3, or whether side-payments are required, clearly depends on the structure of the "counteroffer game". For the present chapter, we leave this issue aside.

3.6 Conclusion

In contrast to previous work, in our setup exclusivity clauses are neither used to foreclose, nor do they align incentives. Instead they are necessary features of competitive contracts. Nevertheless, their welfare effect may be detrimental, because competitive forces are not directed towards the well-being of customers. Rather, to stay in business, contract-issuing firms have to maximize the profit of their counterparts in the upstream market.

Of course, our results hinge on the specification of the framework. In particular, firms are able to commit on an array of variables such as prices, quantities, and exclusivity clauses. Hence possible extensions of the model would allow firms to offer "menu contracts" or include bargaining on the terms of trade.

Regarding policy implications, our analysis suggests that prohibiting exclusivity clauses potentially improves welfare. An overall ban, however, seems to be premature. Despite our conjecture that "exclusive provision" arises endogenously, in practice we often observe "exclusive dealing". For this case, we have shown that a prohibition of exclusivity clauses might reduce efficiency. Moreover, firms could be induced to substitute exclusivity clauses by other vertical restraints such as retail price maintenance, quantity discounts, exclusive territories, tying and the like.

Against this ambiguous background, it seems consistent that both the United States[23] and the European Union[24] treat exclusive contracts under a rule-of-reason approach, in which economic efficiencies are balanced against anticompetitive effects. Nevertheless, as exemplified, policy makers should not be mislead by the competition-promoting effect of exclusivity clauses: In fact, it is precisely this effect which may force contract-issuing firms to exploit their final customers.

[23]See Continental T.V., Inc. v. GTE Sylvania, Inc., 433 U.S. 36 (1977).
[24]See European Commission (2010).

3.A Appendices to Chapter 3

3.A.1 Proof of Lemma 1

Consider the case where both suppliers' participation constraints are satisfied. Then, with $w_r^* = a - b2q_r^*$, it follows from equation (5) that both retailers obtain zero profit.
To the contrary, suppose there is an equilibrium $\hat{\Phi}_r$ with $\hat{w}_r > a - b2\hat{q}_r$. In this case, both retailers make losses. By offering $\tilde{\Phi}_r$ with $\tilde{q}_r(\tilde{w}_r - c_s) < \hat{q}_r(\hat{w}_r - c_s)$ for $s \in \{H, L\}$, a deviating retailer r would not be chosen by any supplier, which improves r's profit to 0.
Suppose next that $\hat{w}_r < a - b2\hat{q}_r$. In this case,

$$\hat{\pi}_r = (a - 2b\hat{q}_r - \hat{w}_r)\hat{q}_r > 0. \tag{A.1}$$

Due to the continuity of (A.1), for all $\varepsilon > 0$ there exist an alternative contract $\tilde{\Phi}_r$ with $\tilde{q}_r > \hat{q}_r$ and $\tilde{w}_r > \hat{w}_r$ such that

$$(a - 2b\tilde{q}_r - \tilde{w}_r)\tilde{q}_r > \hat{\pi}_r - \varepsilon.$$

But if $\tilde{q}_r > \hat{q}_r$ and $\tilde{w}_r > \hat{w}_r$, we have $\tilde{q}_r(\tilde{w}_r - c_s) > \hat{q}_r(\hat{w}_r - c_s)$ and both suppliers approach the deviating firm r. In this case, r's profit is

$$\tilde{\pi}_r = (a - 2b\tilde{q}_r - \tilde{w}_r)2\tilde{q}_r > 2(\hat{\pi}_r - \varepsilon). \tag{A.2}$$

By comparing (A.1) with (A.2), we observe that $\tilde{\pi}_r > \hat{\pi}_r$ for all $\varepsilon < \hat{\pi}_r/2$.
The proof where only the efficient supplier L's participation constraint is satisfied is analogous.

3.A.2 Proof of Lemma 2

To the contrary, suppose that in a symmetric equilibrium $\hat{\Phi}_r$ the retailers obtain zero profits, but neither supplier's profits are maximized subject to the retailers' zero-profit condition. In this case, if both suppliers' participation constraints are satisfied, we have

$$\mathrm{MRS}_s(\hat{q}_r, \hat{w}_r) := -\frac{\hat{w}_r - c_s}{\hat{q}} \neq -2b,$$

for $s \in \{H, L\}$, and, if only the efficient supplier L's participation constraint is satisfied, we have

$$\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) = -\frac{\hat{w}_r - c_H}{\hat{q}} \neq -b.$$

If no participation constraint is satisfied, trivially, a deviating retailer could offer a sufficiently attractive contract, which would increase its profit from zero to a positive value.
Next consider putative symmetric equilibria $\hat{\Phi}_r$ where both suppliers' participation constraints are satisfied. Since, for all $(\hat{q}_r, \hat{w}_r)$, $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) < \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r)$, we can distinguish between the following three (exhaustive) cases for which $\mathrm{MRS}_s(\hat{q}_r, \hat{w}_r) \neq -2b$ for $s \in \{H, L\}$. For each case, by deriving profitable deviation strategies, we show that it cannot constitute an equilibrium.

Case 1: $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) \leq \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) < -2b$.

First note that in such a case, both suppliers' participation constraints are *strictly* satisfied. This is because, from $\hat{w}_r \leq c_H$ follows $\mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) \geq 0$, which is a contradiction to $\mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) < -2b$.

Because both suppliers' participation constraints are strictly satisfied, a deviating retailer r could attract both suppliers by marginally increasing q_r and adjusting (lowering) $w_r(q_r)$ in a manner that

$$\frac{\mathrm{d}w_r}{\mathrm{d}q_r} > \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r). \tag{A.3}$$

In this case, r's profit function would be

$$\pi_r = (a - 2bq_r - w_r(q_r))2q_r, \tag{A.4}$$

which equals $\hat{\pi}_r = 0$ at $(q_r, w_r) = (\hat{q}_r, \hat{w}_r)$. The first-order effect on π_r from increasing q_r would be

$$\left[\frac{\mathrm{d}\pi_r}{\mathrm{d}q_r}\right]_{(q_r,w_r)=(\hat{q}_r,\hat{w}_r)} = \left(-2b - \frac{\mathrm{d}w_r}{\mathrm{d}q_r}\right) 2\hat{q}_r. \tag{A.5}$$

Since, by assumption, there exists $\varepsilon > 0$ such that $\mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) < -2b - \varepsilon$, r could choose $\mathrm{d}w_r/\mathrm{d}q_r = -2b - \varepsilon$, which satisfies (A.3) and renders (A.5) positive.

Case 2: $-2b < \mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) \leq \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r)$.

Then, a deviating retailer r could attract both suppliers by marginally decreasing q_r and adjusting (increasing) $w_r(q_r)$ in a manner that

$$\frac{\mathrm{d}w_r}{\mathrm{d}(-q_r)} > -\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) \;\Leftrightarrow\; \frac{\mathrm{d}w_r}{\mathrm{d}q_r} < \mathrm{MRS}_L(\hat{q}_r, \hat{w}_r). \tag{A.6}$$

In this case, r's profit function would be (A.4), and the first-order effect on π_r from decreasing q_r would be

$$\left[\frac{\mathrm{d}\pi_r}{\mathrm{d}(-q_r)}\right]_{(q_r,w_r)=(\hat{q}_r,\hat{w}_r)} = \left(2b + \frac{\mathrm{d}w_r}{\mathrm{d}q_r}\right) 2\hat{q}_r. \tag{A.7}$$

Since, by assumption, there exists $\varepsilon > 0$ such that $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) > -2b + \varepsilon$, r could choose $\mathrm{d}w_r/\mathrm{d}q_r = -2b + \varepsilon$, which satisfies (A.6) and renders (A.7) positive.

Case 3: $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) < -2b < \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r)$.

In this case, we necessarily have $\Delta > 0$, and supplier L's participation constraint is strictly satisfied. Therefore, a deviating retailer r could attract supplier L without attracting supplier H by marginally increasing q_r and adjusting (lowering) $w_r(q_r)$ in a manner that

$$\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) < \frac{\mathrm{d}w_r}{\mathrm{d}q_r} < \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r). \tag{A.8}$$

In this case, r's profit function would be

$$\pi_r = (a - b\hat{q}_r - bq_r - w_r(q_r))q_r, \tag{A.9}$$

and the first-order effect on π_r from increasing q_r would be

$$\left[\frac{\mathrm{d}\pi_r}{\mathrm{d}q_r}\right]_{(q_r,w_r)=(\hat{q}_r,\hat{w}_r)} = \left(-b - \frac{\mathrm{d}w_r}{\mathrm{d}q_r}\right)\hat{q}_r. \tag{A.10}$$

By assumption, $\mathrm{d}w_r/\mathrm{d}q_r = -2b$ satisfies (A.8) and renders (A.10) positive.

Finally, we consider putative equilibria where only the efficient supplier L's participation constraint is satisfied, that is, $\hat{w}_r < c_H$. In this case, whenever $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) \neq -b$, there exists a combination $(\tilde{q}_r, \tilde{w}_r)$ below the zero-profit line $\hat{w}_r = a - b\hat{q}_r$, which is preferred by L as compared to $(\hat{q}_r, \hat{w}_r)$. Since $\hat{w}_r < c_H$, the less-efficient supplier H would not be attracted by marginal changes in (q_r, w_r), which leaves the zero-profit line unaffected. Thus, r can profitably deviate in a similar manner as described above.

3.A.3 Proof of Corollary 1

If $\Delta = 0$, Lemma 2 requires (w_r^*, q_r^*) to maximize $q_r(w_r - c)$ conditional on (6). Accordingly, we have

$$(q_r^*, w_r^*) = \arg\max_{q_r,w_r}(w_r - c)q_r \text{ s.t. } w_r = a - b2q_r = ((a-c)/4b, (a+c)/2).$$

3.A.4 Proof of Lemma 3

Suppose there exists an equilibrium $\hat{\Phi}_r$ with $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) < -2b = \mathrm{MRS}_H(\hat{q}_r, \hat{w}_r)$. A deviating retailer r could attract supplier L without attracting supplier H by marginally increasing q_r and adjusting (lowering) $w_r(q_r)$ in a manner that

$$\mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) > \frac{\mathrm{d}w_r}{\mathrm{d}q_r} > \mathrm{MRS}_L(\hat{q}_r, \hat{w}_r). \tag{A.11}$$

In this case, r's profit function would be given by (A.9), and the first-order effect on π_r from increasing q_r would be given by (A.10). Since, by assumption, there exists $\varepsilon > 0$ such that $\mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) + \varepsilon < -2b$, r could choose $\mathrm{d}w_r/\mathrm{d}q_r = -2b - \varepsilon$, which satisfies (A.11) and renders (A.10) positive.

3.A.5 Proof of Lemma 4

First suppose that both suppliers' participation constraints are satisfied in a symmetric equilibrium $\hat{\Phi}$. From Lemmas 1 through 3 we know that in such an equilibrium

$$(q_r^*, w_r^*) = \arg\max_{q_r,w_r}(w_r - c_L)q_r \text{ s.t. } w_r = a - b2q_r = ((a-c_L)/4b, (a+c_L)/2),$$

$\hat{\pi}_r = 0$, $\hat{\pi}_L = (a - c_L)^2/8b$, and $\hat{\pi}_H = (a - c_L)/4b \times (a + c_L - 2c_H)/2$. By increasing q_r and adjusting (lowering) w_r such that the utility of supplier L is marginally increased, retailer r's offer would only be attractive for L, and H would stay with the competing retailer $-r$, which offers $(\hat{q}_r, \hat{w}_r)$. In this case, r's profit function would be

$$\pi_r = (a - b\hat{q}_r - bq_r - w_r(q_r))q_r,$$

where $w_r(q_r)$ is such that a marginal deviation of $(q_r, w_r(q_r))$ from $(\hat{q}_r, \hat{w}_r)$ only attracts supplier L. The first-order effect on π_r from increasing q_r then would be

$$\left[\frac{\mathrm{d}\pi_r}{\mathrm{d}q_r}\right]_{(q_r,w_r)=(\hat{q}_r,\hat{w}_r)} = \left(-b - \frac{\mathrm{d}w_r}{\mathrm{d}q_r}\right)\hat{q}_r. \tag{A.12}$$

Since $\mathrm{MRS}_H(\hat{q}_r, \hat{w}_r) > \mathrm{MRS}_L(\hat{q}_r, \hat{w}_r) = -2b$, by choosing $\mathrm{d}w_r/\mathrm{d}q_r = -2b + \varepsilon$, with ε sufficiently small, retailer r would attract supplier L, whereas supplier H would stay with retailer $-r$, which offers $(\hat{q}_r, \hat{w}_r)$. As a result, (A.12) would be positive, which eliminates the putative equilibrium.

Next, consider the case where H's participation constraint is violated in an equilibrium Φ_r^*. From Lemma 2, we know that in this case L's profit must be maximized conditional on the retailers' zero-profit condition (7). Therefore, from a similar computation as in Corollary 1, Φ_r^* involves $q_r^* = (a - c + \Delta)/2b$, $w_r^* = (a + c - \Delta)/2$, $\pi_r^* = 0$, $\pi_L^* = (a - c + \Delta)^2/4b$, and $\pi_H^* = 0$. As L's profit is maximized conditional on (7), every potentially profitable deviation $\tilde{\Phi}_r$ from Φ_r^* involves attracting H.

It follows from r's profit function (5) that any deviation $\tilde{\Phi}_r$ which attracts *both* suppliers is only profitable if $\tilde{w}_r < a - 2b\tilde{q}_r$. Since in the proposed equilibrium $\pi_L^* = q_r^*(w_r^* - c_L)$ is maximized conditional on $w_r^* \leq a - bq_r^*$, no profitable $\tilde{\Phi}_r$ which attracts both suppliers exists. On the other hand, if a deviation consists of attracting H without attracting L, the profit of the deviating retailer equals

$$\tilde{\pi}_r = (a - bq_r^* - b\tilde{q}_r - \tilde{w}_r)\tilde{q}_r.$$

As H's participation constraint ($w_r^* \geq c_H$) is violated if and only if $\Delta > (a - c)/3$, it follows that $q_r^* > 2(a - c)/3b$. Attracting supplier H further requires $\tilde{w}_r \geq c_H$, thus we have

$$\tilde{\pi}_r < \left(a - b\left[\frac{2(a-c)}{3b}\right] - b\tilde{q}_r - c_H\right)\tilde{q}_r.$$

By writing $c_H = c + \Delta$, $\Delta > (a - c)/3$ implies

$$\tilde{\pi}_r < -b\tilde{q}_r^2 < 0.$$

3.A.6 Proof of Result 1

We prove Result 1 for the case of strict dominance. For weak dominance, the proof is identical, except that weak inequalities are replaced by strict ones.

In addition to condition (8), strict dominance requires

$$(a - bq_s - bq_{-s} - w_s)q_s > \ (a - 2bq_{-s} - w_{-s})q_{-s}. \tag{A.13}$$

Since (8) can be rewritten as $(a - bq_s - bq_{-s} - w_s)q_s > (a - 2bq_s - bq_{-s} - w_{-s})q_{-s} + bq_s^2$, (8) implies (A.13) whenever

$$\begin{aligned} & (a - 2bq_s - bq_{-s} - w_{-s})q_{-s} + bq_s^2 > (a - 2bq_{-s} - w_{-s})q_{-s} \\ \Leftrightarrow\ & (q_s - q_{-s})^2 > 0. \end{aligned}$$

This is true whenever $q_s \neq q_{-s}$. If $q_s = q_{-s}$, inequality (8) requires $w_s < w_{-s}$, which also implies inequality (A.13).

3.A.7 Proof of Lemma 5

Any equilibrium $(\hat{\Phi}_H, \hat{\Phi}_L)$ where H is chosen by the retailers requires $\hat{w}_H \geq c_H$, because H could avert negative profits by offering a sufficiently unattractive contract $\tilde{\Phi}_H$.
If $\hat{w}_H > c_H$ and the retailers choose H, L could attract both retailers by offering $\tilde{\Phi}_L$ with $\tilde{q}_L = \hat{q}_H$ and $\tilde{w}_L = \hat{w}_H - \varepsilon$ with $\varepsilon > 0$ sufficiently small. In this case, choosing L would be a strictly dominant strategy for both retailers, and L's profit would increase from 0 to $(\tilde{w}_L - c_L)\tilde{q}_L > (c_H - c_L)\hat{q}_H \geq 0$.
To rule out the case that the retailers choose H when $\hat{w}_H = c_H$, recall that Assumption 3 requires

$$\hat{\pi}_r = (a - 2b\hat{q}_H - \hat{w}_H)\hat{q}_H > \hat{\pi}_r(L), \tag{A.14}$$

where $\hat{\pi}_r(L)$ denotes retailer r's profit which could be obtained by accepting supplier L's offer instead. But then, H could also offer $\tilde{\Phi}_H$ with $\tilde{q}_H = \hat{q}_H$ and $\tilde{w}_H = \hat{w}_H + \varepsilon$. For sufficiently small values of $\varepsilon > 0$, inequality (A.14) would remain valid, and both retailers would still opt for supplier H. Furthermore, H's profit would increase, which eliminates the remaining candidate equilibrium where H is chosen by the retailers.

3.A.8 Proof of Lemma 6

Assumption 4 rules out equilibria with $\hat{w}_H < c_H$, hence $w_H^* \geq c_H$.
Next suppose that $\hat{w}_L > c_H$. From Lemma 5, we know that both retailers choose supplier L, and thus the profit of supplier H is $\hat{\pi}_H = 0$. By offering $\tilde{\Phi}_H$ with $\tilde{q}_H = \hat{q}_L$ and $\tilde{w}_H = \hat{w}_L - \varepsilon$ with $\varepsilon > 0$ sufficiently small, choosing H would be a strictly dominant strategy for both retailers. In this case, H's profit would be $\tilde{\pi}_H = (\tilde{w}_H - c_H)2\tilde{q}_H = (\hat{w}_L - \varepsilon - c_H)2\hat{q}_L > 0$.

3.A.9 Proof of Lemma 7

As it follows from Lemma 5 that supplier H is not chosen in any equilibrium, we have

$$(a - 2b\hat{q}_L - \hat{w}_L)\hat{q}_L \geq \max\left\{(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H)\hat{q}_H, 0\right\}. \tag{A.15}$$

From Result 1, (A.15) implies that it is a strictly dominant strategy for the retailers to choose supplier L whenever (A.15) holds strictly. Then, however, L could offer $\tilde{\Phi}_L$ with $\tilde{q}_L = \hat{q}_L$ and $\tilde{w}_L = \hat{w}_L + \varepsilon$ with $\varepsilon > 0$ sufficiently small. In this case, inequality (A.15) would remain valid; that is, choosing L would still be a dominant strategy for both retailers. Furthermore, L's profit would be higher by offering $\tilde{\Phi}_L$ than by offering $\hat{\Phi}_L$, which eliminates all (candidate) equilibria with $(a - 2b\hat{q}_L - \hat{w}_L)q_L > \max\{(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H)q_H, 0\}$.

Otherwise, if (A.15) holds with equality, we distinguish between the following cases.

Case 1: $(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H)\hat{q}_H \leq 0$.
In this case, L solves

$$(\hat{q}_L, \hat{w}_L) = \arg\max_{q_L, w_L} (w_L - c_L)2q_L \text{ s.t. } w_L = a - 2bq_L,$$

which yields $\hat{q}_L = (a - c_L)/4b$ and $\hat{w}_L = (a + c_L)/2$. In this case, each retailer r obtains $\hat{\pi}_r = 0$ at L.
This, however, cannot be an equilibrium if $\Delta < 3(a - c)/5$, because then H could attract r and obtain a positive profit by offering $\tilde{\Phi}_H$ with $\tilde{w}_H = c_H + \varepsilon$ and $(a - b\hat{q}_L - b\tilde{q}_H - \tilde{w}_H)\tilde{q}_H > 0$. Therefore, the suggested equilibrium only exists for $\Delta \geq 3(a - c)/5$.

Case 2: $(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H)\hat{q}_H > 0$.
To show that any equilibrium with $(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H)\hat{q}_H > 0$ requires $w_H^* = w_L^* = c_H$, it follows from Lemma 6 that it is sufficient to show that $\hat{w}_H > \hat{w}_L$ cannot be part of an equilibrium in this case.
From L's objective function and from (A.15), in any such equilibrium, L solves

$$\begin{aligned}(\hat{q}_L, \hat{w}_L) = &\arg\max_{q_L, w_L} (w_L - c_L)2q_L \\ &\text{s.t. } w_L = a - 2bq_L - \frac{q_H}{q_L}(a - bq_L - bq_H - w_H),\end{aligned} \tag{A.16}$$

which yields

$$\hat{q}_L(\hat{q}_H) = \frac{a - c_L}{4b} + \frac{\hat{q}_H}{4}. \tag{A.17}$$

From the constraint in (A.16), it follows that

$$\hat{w}_L \geq a - 2b\hat{q}_L - \max_{\hat{q}_L, \hat{q}_H} \underbrace{\left\{ \frac{\hat{q}_H}{\hat{q}_L}(a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H) \right\}}_{=:\gamma_1}. \tag{A.18}$$

Furthermore, from the equality version of (A.15), and from the fact that in any equilibrium the retailers obtain non-negative profits, we have

$$\gamma_2 := a - b\hat{q}_L - b\hat{q}_H - \hat{w}_H \geq 0.$$

Therefore,

$$\frac{\partial \gamma_1}{\partial \hat{q}_L} = -\frac{\hat{q}_H}{\hat{q}_L^2}\gamma_2 + \frac{\hat{q}_H}{\hat{q}_L}(-b) < 0. \tag{A.19}$$

Now suppose that $\hat{q}_H \leq (a - c_L)/3b$. In this case, (A.17) implies

$$\hat{q}_H \leq \hat{q}_L \leq (a - c_L)/3b. \tag{A.20}$$

Therefore, from (A.19), subject to (A.20), γ_1 is maximized at $\hat{q}_L = \hat{q}_H$. Together with (A.18), this yields

$$\hat{w}_L \geq a - 2b\hat{q}_L - (a - 2b\hat{q}_L - \hat{w}_H) = \hat{w}_H, \tag{A.21}$$

which is the required contradiction to $\hat{w}_H > \hat{w}_L$ for the case that $\hat{q}_H \leq (a - c_L)/3b$.
Next, suppose that $\hat{q}_H \geq (a - c_L)/3b$. In this case, (A.17) implies

$$\hat{q}_H \geq \hat{q}_L \geq (a - c_L)/3b. \tag{A.22}$$

Therefore, from (A.19), subject to (A.22), γ_1 is maximized at $\hat{q}_L = (a - c_L)/3b$. From (A.17), this further requires $\hat{q}_H = (a - c_L)/3b$, and thus $\hat{q}_H = \hat{q}_L$. Therefore again, (A.18) implies (A.21), which constitutes the required contradiction to $\hat{w}_H > \hat{w}_L$ for the second case that $\hat{q}_H \geq (a - c_L)/3b$.

3.A.10 Proof of Lemma 8.

First suppose that supplier L instead offers $\hat{q}_L = (a - c_H)/3b + \delta$ with $\delta \neq 0$. In this case, H could offer $\tilde{\Phi}_H$ with $\tilde{q}_H = (a - c_H)/3b$ and $\tilde{w}_H = c_H + \varepsilon$. From Result 1, choosing H is a dominant strategy for both retailers if and only if

$$\begin{aligned} & \tilde{\pi}_r := (a - 2b\tilde{q}_H - \tilde{w}_H)\tilde{q}_H > \hat{\pi}_r := (a - b\tilde{q}_H - b\hat{q}_L - c_H)\hat{q}_L \\ \Leftrightarrow \;& (b\delta)^2 > \varepsilon(a - c)/3. \end{aligned}$$

Hence, by setting $\varepsilon > 0$ sufficiently small, H would attract both retailers, and H would obtain a positive profit. This eliminates all equilibria with $\hat{q}_L \neq (a - c_H)/3b$.
Next suppose that supplier H offers $\hat{q}_H = (a - c_L)/3b + \delta$ with $\delta \neq 0$. In this case, we know from the strict inequality versions of (A.20), (A.21), and (A.22) (see the proof of Lemma 7) that $\hat{w}_L > \hat{w}_H$. This, however, contradicts Lemma 7, and eliminates all equilibria with $\hat{q}_H \neq (a - c_L)/3b$.
Using $q_s^* = (a - c_{-s})/3b$, equation (A.16) in Appendix 3.A.9 implies $w_L^* \leq w_H^*$, where the inequality is strict if and only if $\Delta > 0$. But then it follows from Lemma 7 that there is no equilibrium for $\Delta > 0$.
If $c_H = c_H = c$, to show that Φ_s^* with $q_s^* = (a - c)/3b$ and $w_s^* = c$ constitutes an equilibrium, first consider potential deviations of supplier L. From the equality versions of (A.20), (A.21), and (A.22), we know that L's best answer to Φ_H^* indeed is Φ_L^* with $q_L^* = q_H^*$ and $w_L^* = w_H^*$. Regarding supplier H, first note that offering a contract with $\tilde{w}_H < c$ can never lead to a positive profit for H. Second, if H cannot attract any retailers by offering a contract with $\tilde{w}_H = c$, this is also not possible with any contract with $\tilde{w}_H > c$. If H offers $\tilde{\Phi}_H$ with $\tilde{w}_H = c$ and $\tilde{q}_H = (a - c)/3b + \delta$ with $\delta \neq 0$, both retailers accept the offer of L, since it follows from Result 1 that

$$\hat{\pi}_r := (a - 2b\hat{q}_L - \hat{w}_L)\hat{q}_L > \tilde{\pi}_r := (a - b\hat{q}_L - b\tilde{q}_H - \tilde{w}_H)\tilde{q}_H \Leftrightarrow \;\; \delta^2 > 0$$

implies that accepting L's offer is a dominant strategy for both retailers.

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Bern, 26. Februar 2014

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