Networks with Spatially Distributed Externalities

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Sarina Katja Steinmann

from Lauperswil, Bern

Department of Economics

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Introduction

Environmental pollution is one of the biggest problems we face today. While some forms of environmental pollution mainly have negative impacts at a local level, others cross borders through pathways like water and air and affect entire regions or the whole planet making them a transnational or world-wide problem. In other words, environmental pollution may have a spatial component. For example, the release of green house gases into the atmosphere is the main driver of man-made global climate change, itself responsible for the rise of sea levels, heat waves, melting of glaciers, droughts, and floods worldwide. Furthermore, industrial and agricultural activities have led to soil contamination, destroying fertile soil and water ecosystems locally, but also depleting natural resources such as rainforests affecting the climate worldwide. Moreover, the discharge of commercial and industrial waste into rivers has drastically deteriorated their water quality, affecting the entire biosphere along their course. Further forms of pollution, such as artificial light and noise pollution, drown out natural landscapes locally, disturbing wildlife in their natural habitat.

In my thesis, I consider environmental pollution that is spatially distributed. Countries, regions or cities, represented by agents, may thus not only pollute their direct environment but their pollution may spread and accumulate across agents. Hence, whether and by how much the agents are affected by the pollution of the others depends on their location, their distance to the polluting sources and the accumulative nature of the pollution considered. The agents and the pollution emitted build a network, in which the agents represent the nodes and are arranged according to a geographical structure and in which the pollution flows represent the edges that connect the agents. The pollution emitted exerts a negative externality on the agents in the network. Since the others mostly bear the negative consequences of the pollution produced and these external costs are not taken into account by the polluters themselves, the polluters have little incentive to incur the costs to reduce pollution. Consequently, they choose to pollute more as if they were required to pay all associated costs. Thus, in general, unregulated markets in goods with externalities generate prices that do not reflect the full social cost of their transactions and therefore allocate the resources inefficiently. As a consequence, an intervention by an external authority or a voluntary agreement among the agents seems necessary to correct for the externalities and to improve on the inefficient status-quo.

There is a broad range of policy instruments available for the mitigation of pollution by an external authority, including the establishment of property rights, command-and-control regulation and market-based approaches. However, these approaches may be far from efficient or impractical to implement if, for example, heterogeneously distributed pollution or asymmetric information is present. In addition, pollution is rarely confined within territorial boundaries with the consequence that regulations cannot be instated in other jurisdictions and the polluters cannot be held liable for the pollution damage they cause in these jurisdictions. Thus, if no authority to enforce any regulation exists, voluntary cooperation among the polluters and their victims is the only way to improve on the inefficient outcome. In this thesis, I analyse if and to what extent the efficient levels of pollution can be implemented by voluntary agreements or regulatory instruments when pollution is spatially distributed and when many polluters share a network.

In the *first chapter*, co-authored with Ralph Winkler, we address the problem of efficient emission abatement in a multi-polluter network, where the polluters are sovereign states with no supranational authority to enforce any action. In this chapter, we consider a specific network in which the polluters are hierarchically ordered along a graph from upstream to downstream. In this setting, the emissions released upstream accumulate while moving downstream so that the pollution emitted upriver induces negative externalities on all downstream agents. An example for this structure is an international river being polluted by the riparians along its course. The polluters may abate some of their pollution by incurring abatement costs to reduce the damages caused to their downstream agents. As the agents do not take into account the externalities on the other agents when deciding on their actions, they decide to abate inefficiently little. Coase (1960) argued that the problem of having inefficiently low levels of pollution could be solved by establishing property rights. Given property rights, agents have an incentive to find a way to make mutually beneficial deals that lead them to take the externalities into account. However, concerning environmental problems, it is often unclear how these property rights should be defined. As an example, reconsider river pollution. There is an absence of clearly defined property rights over the river as all riparians sharing the river usually claim property rights over it or at least the part of the river flowing through their territory. As a consequence, none of the riparians is willing to reduce its pollution or pay compensations to the others suffering from it. Thus, one of the reasons for the inefficiently low amount of pollution abatement is the absence of well-defined property rights or/and the lack of an authority to assign such property rights. Hence, the only way to tackle pollution crossing borders is voluntary cooperation among the agents. In order for all externalities to be internalized, the grand coalition needs to form. Thus, all agents need to cooperate to attain the efficient solution.

An obvious difficulty of any cooperation is the tendency for agents to seek a free ride. This is because the formation of a coalition, in which agents act cooperatively and therefore reduce pollution, exerts a positive externality on potential non-members. Thus, the incentive to deviate from the grand coalition hinges on how much these deviating agents can achieve by themselves. Clearly, this value depends on how the remaining agents will behave once one or many agents have left the grand coalition. These remaining agents could either fully cooperate, form partial coalitions or, as is often assumed, they may behave fully non-cooperatively and break into singletons. Thus, for an agent not to deviate from the grand coalition, an agent should be at least as well off if he is part of the grand coalition as if he were on his own or cooperating with some others.

To achieve the grand coalition and the consequent reduction of pollution to efficient levels may involve some agents making losses relative to the status-quo. This is especially true for spatially distributed pollution as well as non-uniform damages and mitigation costs, where moving towards an efficient solution without having the victims pay compensations to the polluters for incurring the mitigation costs inevitably violates individual rationality. As a consequence, financial transfers between the agents have been advocated. But how should these transfers look like? While the abatement allocation determines the total welfare, these compensation transfers determine how this total welfare is shared among the agents. Clearly, there are infinitely many possibilities how this welfare can be split. One requirement for the transfer scheme to be accepted is to ensure that any polluter or coalition of polluters is compensated at least for what they would be able to achieve themselves if they were operating individually. Another requirement addresses the issue of fairness, in the sense that any transfer scheme should be perceived as fair. One possible way to define fairness is to demand that no agent or coalition of agents should be made better off than they were if the non-members fully mitigated the effects of the negative externality caused in their territories. The abatement allocation implemented together with the chosen transfer scheme is often called an agreement.

In the first chapter of this thesis, we therefore search for an efficient agreement that is sufficiently appealing to all agents involved for them to agree to it. Thus, it must fulfil the following conditions: First, all agents must participate and the efficient pollution levels need to be implemented. Second, it must satisfy the so-called participation constraints for all possible coalitions while assuming that non-coalition members behave non-cooperatively. This means that all agents or possible coalitions are made better off by the agreement than they were if they acted alone. Third, is must fulfil a fairness criterion in the sense that no agent and no coalition is made better off by the agreement than it were being alone in the network with no other polluters than themselves. We find that there exists only one agreement that fulfils all participation constraints and fairness constraints at the same time. This agreement, called the *Downstream Incremental Distribution*, assigns each agent his marginal contribution to the coalition composed of its predecessors. It favours downstream agents, as the cooperation gain goes to the agent most downstream while the agent most upstream is set indifferent to the status-quo.

Even if we do not have the problem of a missing authority to enforce any joint action, there are other reasons why an inefficient abatement allocation prevails, for example asymmetric information. In this case, one agent may have more information than another agent, for example more than the government. This divergence in knowledge may lead to a misallocation of resources because the better informed agent has a comparative advantage. Thus, an additional difficulty in forging an agreement among the agents or implementing a regulatory instrument is that each participant may retain private information on his mitigation costs or damages. In particular, polluters may have an incentive to exaggerate their privately known abatement costs in order to reduce the abatement level they have to supply with the consequence that most of the burden of abatement is left to the others. Due to this discrepancy in information, it may even be beneficial for an external authority to delegate its power to enforce actions to one of the agents in the network.

In the *second chapter*, I address this problem of asymmetric information in multi-polluter networks. For this, I extend the linear hierarchical setting with downstream oriented externalities introduced in the first chapter to include asymmetric information and a federalist governance structure. The lower tiers, representing the agents along the graph, are assumed to have private information about their abatement costs. An example for this structure is the Aare, a river which flows through several cantons of Switzerland. I propose that one of the lower tiers is nominated by the federal government to offer a set of mitigation contracts to the remaining agents. These contracts specify an abatement level and a compensation payment for the abatement effort. The selected principal can either propose the contracts in a centralized manner, where he simultaneously offers contracts to all agents or he contracts only with his direct up- and downstream neighbours and delegates to those two agents the authority to contract with their direct up- respectively downriver neighbours and so on till all agents received a contract. I establish that given full information, both models implement the first-best optimal allocation and the choice of the principal only determines which agent attains the full cooperation gain.

In the presence of asymmetric information, an agreement must not only satisfy the participation constraints of all agents, but also the so-called incentive compatibility constraints. When fulfilled, these incentive compatibility constraints guarantee that agents do not lie about their true costs. The abatement allocation attained may thus not be efficient as information rents in return for disclosing information have to be paid. Given asymmetric information, I demonstrate that even though the delegated principal agent model is prone to a control loss, the abatement allocation implemented in the centralized principal agent model can be replicated in the delegated principal agent model while matching the expected costs of the principal. However, to counteract and to avoid the tendency of the intermediate agents to bias the abatement allocation in their favour, the nominated principal must be able to monitor the abatement levels as well as the reports of the intermediate agents and it must be ensured that the set of contracts is executed after all intermediate agents have accepted their subcontracts.

As all potential principals implement a different abatement allocation, the choice of the prime principal matters for the total expected costs occurring in the river basin. I show that the tier located the most downriver, which is subject to the same informational constraints as the federal government, is never the best choice to nominate as a principal. In case of linear damages, I establish that choosing the agent at the source of the river will implement the abatement allocation leading to least expected total costs for the river basin. For other functional forms of damage and abatement costs, the nomination of the best principal depends not only on the position of the potential principal along the river but also on the damage cost parameters as well as the exogenously given pollution levels.

Even if we have full information on all relevant parameters and there exists an authority to enforce regulatory instruments, inefficient abatement allocations may still prevail in case of spatially distributed pollution. This is because the regulatory instruments are inefficient or impractical to implement. For example, command-and-control regulation, which includes policies to prescribe how much pollution a source is allowed to emit or what types of control equipment it must use to meet such requirements, have often been criticised. First of all, they offer no incentive to abate pollution beyond the standard set by the authority or to rethink their production methods. Second, they are inflexible. It is a "one-size-fits-all" approach that does not consider varying performance of polluters. Third, selecting the appropriate standards demands a high level of information, which may be missing or misrepresented. Thus, command-and-control regulation is unlikely to achieve a least-cost reduction in pollution and often leads to inefficient pollution levels. In contrast, market-based approaches seek to address the market failure caused by the externalities by incorporating the external cost created through taxes and charges or by creating property rights and facilitating the establishment of a proxy market for the use of environmental services. In other words, market-based approaches create markets where they did not previously exists to give polluters incentives to reduce pollution. These policies can either be quantity-based or price-based. Quantity-based policies include cap-and-trade systems where the total quantity of pollution is set at the optimal level of pollution, pollution allowances in this amount are distributed to the polluters and a market is established in which the allowances may be traded. Price-based policies are often a tax, where polluters have to pay a fixed price per unit of emissions. In theory, marked-based policies are often favoured over command-and-control regulation because they tend to be least costly, place a lower information burden on the authority and provide incentives for technological advances. However, with heterogeneously dispersed pollution, such as river pollution, the social welfare optimum can only be achieved by source-dependent taxes or prices of the tradable pollution rights in cap-and-trade systems, which makes their implementation questionable. Being restricted to one price or tax, the question arises, whether the enforcing authority could set a cap or a tax other than the efficient total pollution level to reduce overall costs.

In the *third chapter* of this thesis, I address this issue and propose a second best optimal solution to the problem of emission abatement in multi-polluter models. I extend the multi-polluter model introduced in the previous chapters to a general multi-polluter network with heterogeneously dispersed pollution, where there exists an external authority enforcing a cap-and-trade system to control pollution with one price for tradable pollution permits. While with regulating uniformly distributed pollution the first-best optimal solution can be implemented, controlling heterogeneously dispersed pollution in multi-polluter networks is more complex. This is because with heterogeneously dispersed pollution, the emissions released at the sources and the damage-inducing ambient pollution levels accumulated at the receptors may not coincide. Because emissions from different sources induce different damage costs for the receptors and as the marginal damage costs may vary across receptors, the first-best optimal abatement allocation can only be implemented by source-dependent taxes or prices of the tradable pollution rights in cap-and-trade systems. Hence, setting only one price for the tradable pollution permits will result in inefficiency.

To improve on this situation, I propose a second-best optimal solution. Instead of taking an exogenously given and predetermined pollution cap in a cap-and-trade system, the pollution cap is endogenized so that it is determined by the total cost-minimizing equilibrium of a cap-and-trade system. I show that with quadratic abatement costs and linear damage costs, the first-best optimal pollution cap implements the second-best cost-minimizing equilibrium of the cap-and-trade system for any network. However, the second-best optimal abatement allocation differs from the first-best optimal abatement allocation, implying higher second-best optimal total costs than first-best optimal. In particular, second-best optimal total abatement costs fall short of first-best total abatement costs, while second-best optimal total damage costs exceed first-best optimal damage costs. For other functional forms of damage and abatement cost functions, first-best pollution caps are not second-best optimal. The second-best optimal pollution caps may either exceed or deceed the first-best optimal pollution caps and have to be determined for each specific network. These findings hold for two different cap-and-trade systems, the emission permit market and the ambient pollution market.

To sum up, in this thesis, I studied if and to what extent the efficient solution can be implemented in multi-polluter networks with spatially distributed externalities. For this, I analysed three different circumstances: First, when there is transboundary pollution with no supranational authority to enforce any regulation, implying that a self-enforcing agreement among all agents is required to reach efficiency. We show that for a specific network there exists such a voluntary and efficient agreement that is both stable to deviations and perceived as fair. Second, when there is asymmetric information in regard to the mitigation costs, implying that acceptable agreements among all agents have to be found that lead to the revelation of the true costs with the smallest welfare loss possible. We show that for a specific network with a federalist setting the government should elect one of the lower tiers as a principal to offer contracts to the remaining agents. The elected principal is then indifferent between offering contracts simultaneously or delegating its power to his neighbours starting a sequential contracting process. And third, when there is a market-based approach instated in which only one price of a tradable pollution permit can be set, implying that a second-best optimal market-based approach has to be designed which implements the maximal welfare possible given the one price restriction. We propose a second-best optimal solution for this problem in which the pollution cap is endogenized in such a way that the social costs are minimized given the one-price-only restriction.

Evidently, regulating spatially distributed pollution poses a big challenge. This is because the standard regulating instruments such as cap-and-trade or command-and-control systems that usually implement an efficient allocation do not work in case of spatially distributed pollution or an authority to enforce them is missing. The problem of optimal pollution control is further aggravated by the presence of asymmetric information as many governments face limitations in monitoring the mitigation costs or the pollution flows. As illustrated, there is no one-size-fits-it-all solution to the problem of spatially distributed pollution, so that each situation needs to be analysed separately. Particularly, the design of second-best optimal mechanisms for pollution control in case of spatially distributed pollution together with multi-dimensional asymmetric information suggests an interesting avenue for future research. Also, the design of voluntary and fair agreements that are stable over time given uncertainties in the future pollution flows might yield interesting insights.

Chapter 1

Sharing A River With Downstream Externalities

Abstract

We consider the problem of efficient emission abatement in a multi-polluter setting, where agents are located along a river in which net emissions accumulate and induce negative externalities to downstream riparians. Assuming a cooperative transferable utility game, we seek welfare distributions that are in the non-cooperative core and satisfy a specific fairness constraint. Meaning, we search for welfare distributions that satisfy all agents' participation constraints, in that each coalition is at least as well off as it were if acting on its own and that is perceived to be fair, in that no coalition is better off than it were if all non-members of the coalition do not pollute the river at all. We show that the downstream incremental distribution, as introduced by Ambec and Sprumont (2002), is the only welfare distribution satisfying both constraints. In addition, we show that this result holds true for numerous extensions of our model.

1 Introduction

Industries and cities all around the world have historically been concentrated along rivers, since rivers provide means of transportation, food production, energy generation and drinking water. Because of this intensive utilization, many rivers and streams have been and still are being heavily polluted. Excessive pollution worsens water quality, which reduces economic profits and negatively impacts wildlife and human health. One specific characteristic of rivers is that pollutants discharged into the river are carried downriver. As a consequence, it is the downstream riparians rather than the polluter himself who bears the negative consequences of the emissions discharged into the river. Moreover, if upstream polluters and downstream riparians belong to different jurisdictions, polluters may have little incentive to abate their emissions, because they cannot be held liable for the pollution damage caused in other jurisdictions.

In this chapter, we consider the problem of efficient emission abatement among agents located along a river, where upstream emissions cause negative externalities to all downstream agents. This setting can be characterized as a cooperative transferable utility game with two sources of externalities. First, upstream emissions impose negative externalities on downstream agents. Second, cooperative behaviour among a subset of agents (a so-called coalition) imposes positive externalities upon agents located in between different connected subsets of this coalition. Due to this second kind of externalities, the core is, in general, empty. As a consequence, we restrict our attention to the non-cooperative core, i.e. the set of partitions which consists of one coalition and only singletons otherwise. The non-cooperative core imposes cost upper bounds for any coalition, which can be interpreted as a participation constraint that has to be satisfied by any cost distribution to be acceptable to all agents. In addition, we impose cost lower bounds, which are inspired by the aspiration welfare principle, i.e. no coalition of agents should have lower costs than it can secure for itself if all non-members of the coalition would not pollute the river at all. We show that the downstream incremental distribution, as introduced by Ambec and Sprumont (2002), is the only distribution simultaneously satisfying the non-cooperative core upper cost bounds and the aspiration lower cost bounds.

The existing literature on transboundary pollution in river basins mainly focuses on the case of two jurisdictions. Notable exceptions include Ni and Wang (2007) and Gengenbach et al. (2010). Ni and Wang (2007) derive cooperative sharing rules for the costs of cleaning a river from two principles of international water law: *Absolute Territorial Sovereignty* (ATS) claims that every jurisdiction has exclusive rights to use the water on its territory, while *Unlimited Territorial Integrity* UTI expands these exclusive use rights to all water originating within and upstream of a respective jurisdiction. They adapt these principles to the case of pollution responsibility and derive axioms characterizing the two resulting cost sharing principles. They also show that these cost-sharing principles correspond to the Shapley value solutions of the corresponding cost-sharing games. However, Ni and Wang (2007) assume exogenously given costs for cleaning the river. Thus, they are only concerned with the distribution of these costs. In contrast, pollution levels in our model are endogenously determined by the actions of the agents. Thus, we are concerned about finding cost sharing distributions that are acceptable to all agents and, at the same time, give incentives to choose efficient emission abatement levels in the first place. In line with the literature on international environmental agreements, Gengenbach et al. (2010) model river pollution as a two-stage-cartel-formation game. In the first stage, agents decide whether to join a coalition, while pollution abatement levels are chosen in the second stage. In the absence of a supranational authority, abatement levels are in general inefficiently low, as all agents have an incentive to free ride on the abatement efforts of their upstream neighbours. Analysing the formation of stable coalitions they find that the location of agents has no impact on coalition stability but rather impacts on environmental outcomes. In contrast to Gengenbach et al. (2010), we employ a cooperative game setting.

In fact, our research is most closely related to Ambec and Sprumont (2002) and Ambec and Ehlers (2008), who apply an axiomatic cooperative game theoretic approach to the efficient sharing of water along a river basin. In Ambec and Sprumont (2002), agents derive strictly increasing benefits from water consumption, while Ambec and Ehlers (2008) generalize the results to agents which may exhibit satiation in water consumption. Ambec and Ehlers (2008) show that the downstream incremental distribution is the only welfare distribution satisfying the non-cooperative core lower bounds and the aspiration welfare upper bounds. Several other papers propose alternative sharing rules to the downstream incremental distribution in settings similar to the one proposed by Ambec and Sprumont (2002). Interpreting the river sharing problem as a line-graph game, Van den Brink et al. (2007) derive four different efficient solutions including the downstream incremental distribution by imposing various properties with respect to deleting edges of the line-graph. However, they do not address fairness issues and consider non-satiable agents. Allowing for multiple springs and satiable agents with respect to water consumption, Van den Brink et al. (2012) propose a class of weighted hierarchical welfare distributions based on the Territorial Integration of all Basin States (TIBS) principle, which includes the downstream incremental distribution as a special case. Ansink and Weikard (2012) concentrate on reallocations of the resource itself instead of the reallocation of welfare by an appropriate transfer scheme. In case of water scarcity, the agents' overlapping claims to river water render it a contested resource similar to a bankruptcy problem. They propose a class of sequential sharing rules based on bankruptcy theory and compare them to other sharing rules, including the downstream incremental distribution. Demange (2004) considers hierarchies without externalities and shows that the *hierarchical outcome* satisfies the core bounds for all connected coalitions for all super-additive cooperative games. However, the hierarchical outcome may violate core bounds for non-connected coalitions. If the hierarchy is a river, then the hierarchical outcome corresponds to the counterpart of the downstream incremental distribution.

Our research can be interpreted as a generalization of the results of Ambec and Ehlers (2008) to commodities with public good properties. While water consumption is a purely private good, emission abatement exhibits public good characteristics, as it imposes negative externalities on all downstream agents. These additional externalities impose non-trivial complications for proving that the downstream incremental distribution satisfies the non-cooperative cost upper bounds and the aspiration cost lower bounds in the formulation of our river pollution model.

2 A River Sharing Model with Downstream Pollution Externalities

Consider a set of agents $N = \{1, ..., n\}$, which are located along a river. Without loss of generality, agents are numbered from upstream to downstream, i.e. $i < j$ indicates that agent j is located downriver of agent i. We follow Ambec and Sprumont (2002) and Ambec and Ehlers (2008) in defining the set of agents preceding agent i by $P_i = \{1, ..., i\}$, with the strict predecessors of agent i indicated as $P_i \backslash i = \{1, ..., i-1\}$. Analogously, the set of agents following agent i is defined by $F_i = \{i, ..., n\}$, where $F_i \backslash i = \{i+1, ..., n\}$ denotes the set of agents strictly located downriver of agent i.

Each agent i along the river produces gross emissions in exogenously given amount e_i. An agent i may choose to abate the amount x_i with $0 \leq x_i \leq e_i$, the costs of which are given by the strictly increasing, twice differentiable and strictly convex abatement cost function $c_i(x_i)$. Without loss of generality, we assume that abating nothing induces no abatement costs, i.e. $c_i(0) = 0$. Net emissions $e_i - x_i$ are passed into the river where they accumulate and are carried along its course. Assuming that net emissions of agent i are discharged into the river after agent i's but before agent $i+1$'s location, and that there is no pollution at the rivers' source, the ambient pollution level q_i at the location of agent i is given by the sum of net emissions of all strict predecessors of agent i:

$$q_i = \sum_{j \in P_i \backslash i} \gamma_{ji}(e_j - x_j) \,, \quad \forall\, i \in N$$

with $0 < \gamma_{ji} \leq 1$. γ_{ji} represents the assimilative capacity of the river, i.e. what fraction of the net emissions released by agent j actually reach agent i. As the vector of abatement efforts $x = (x_1, \ldots, x_n)$, together with the vector of exogenously given emissions $e = (e_1, \ldots, e_n)$, fully determine the vector of ambient pollution levels, we shall often write the ambient pollution levels as a function of the vector x:

$$q(x) = \big(q_1(x), \ldots, q_n(x)\big).$$

The ambient pollution level q_i causes damage costs to agent i, the amount of which is given by the increasing, twice differentiable and convex damage cost function $d_i(q_i)$. Thus, the net emissions $e_i - x_i$ released by agent i induce negative externalities for all downriver agents $j > i$, but not for agent i himself or all upstream agents $j < i$.

The total costs k_i agent i faces are the sum of abatement and damage costs:

$$k_i(x_i, q_i) = d_i(q_i) + c_i(x_i) \ .$$

A river sharing problem is characterized by (N, e, c, d), where $c = (c_1, \ldots, c_n)$ and $d = (d_1, \ldots, d_n)$ denote the vectors of abatement and damage cost functions. Given a river sharing problem, the distribution of total costs k_i among all agents i is determined by the emission abatement allocation x. Our assumptions about the accumulation of emissions along the river, as described in the previous paragraph, imply the following proposition.

Proposition 1 (No abatement is dominant strategy)**.** *Given a river sharing problem (N, e, c, d) and for given emission abatement levels of all agents $j \in N \backslash i$ it is a dominant strategy for agent i not to abate at all, i.e. $x_i = 0$.*

Proof. The damage costs of agent i only depend on q_i which are not influenced by x_i. As costs c_i are strictly increasing in the amount of emission abatement x_i, given q_i, total costs are minimized by setting $x_i = 0$. □

Proposition 1 states that agents who only consider their own total costs will never abate. In particular, this implies that if the river sharing problem (N, e, c, d) is considered to be a non-cooperative game among the agents $i \in N$, the unique Nash equilibrium is given by $\hat{x}_i = 0$ for all $i \in N$ (no matter whether agents are considered to decide sequentially or simultaneously). However, this outcome is, in general, inefficient. In particular, if we assume that money transfers between agents are possible and agents have unbounded resources for such transfers, the efficient emission abatement allocation $x^\star$ minimizes the sum of total costs k_i among all agents. The following proposition establishes that such an allocation exists and is also unique.

Proposition 2 (Existence and uniqueness of efficient allocation)**.** *Given a river sharing problem (N, e, c, d) there exists a unique vector $x^\star$ which is the solution to the following constrained minimization problem:*

$$\begin{aligned} \min_{\{x_i\}_{i=1}^n} \sum_{i=1}^{n} k_i\big(x_i, q_i(x)\big) \quad & \textit{subject to} \\ q_i(x) = \sum_{j \in P_i \backslash i} \gamma_{ji}(e_j - x_j) \ , \quad & \forall\, i \in N \ , \\ 0 \leq x_i \leq e_i \ , \quad & \forall\, i \in N. \end{aligned}$$

Proof. Existence and uniqueness follow directly from the strict convexity of the total costs functions $k_i(x_i, q_i)$. □

Let t_i denote the money payments. We impose $\sum_{i=1}^{n} t_i = 0$ and define agent i's after transfer costs

z_i as:

$$z_i = k_i(x_i, q_i) + t_i \ .$$

Obviously, any vector $z = (z_1, \ldots, z_n)$ with $\sum_{i=1}^n z_i = \sum_{i=1}^n k_i\big(x_i^\star, q_i(x^\star)\big)$ is an efficient cost distribution, as it implies a unique vector of transfer payments $t_i = z_i - k_i\big(x_i^\star, q_i(x^\star)\big)$ with $\sum_{i=1}^N t_i = 0$ (no waste of money) and achieves the cost minimum $\sum_{i=1}^N k_i\big(x_i^\star, q_i(x^\star)\big)$. In the following, we call any efficient cost distribution a *river sharing agreement*. The main problem will be which one to choose among this infinite set.

3 Coalitions and Cost Upper Bounds

A non-empty subset of agents $S \subset N$ is called a coalition if the agents of S choose their emission abatements such as to minimize the sum of total costs among all coalition members. Denoting by $minS$ and $maxS$ the most upstream, respectively the most downstream member of coalition S, the coalition S is *connected* or *consecutive* if all agents j with $minS < j < maxS$ are also members of the coalition S.

We define the secure costs $v(S)$ of a coalition S as the minimum value of the sum of the total costs k_i over all members of the coalition:

$$v(S) = \sum_{i \in S} k_i\big(x_i^v(S), q_i(x_i^v(S))\big) \ ,$$

where $x^v(S) = \big(x_1^v(S), \ldots, x_n^v(S)\big)$ denotes the solution to

$$\min_{\{x_i\}_{i\in S}} \sum_{i \in S} k_i\big(x_i(S), q_i(x(S))\big) \quad \text{subject to} \tag{3.1}$$

$$q_i(x) = \sum_{j \in P_i \setminus i} \gamma_{ji}(e_j - x_j) \ , \quad \forall\, i \in N \tag{3.2}$$

$$0 \le x_i \le e_i \ , \quad \forall\, i \in S \ , \tag{3.3}$$

$$x_j \text{ given} \ , \quad \forall\, j \notin S \ . \tag{3.4}$$

It is obvious from the above definition that both the allocation of abatement efforts $x^v(S)$ and the secure costs $v(S)$ of the coalition S depend, in general, on the behaviour of the agents not belonging to the coalition S. As an example, consider the coalition $S = \{k, \ldots, n\}$. In particular the pollution level q_k (but also the pollution levels q_i with $i > k$) depends on the amount of emission abatement undertaken by the agents i with $i < k$. According to Proposition 1, if these agents $i < k$ only minimize their own sum of abatement and damage costs, they would not abate at all, implying a pollution level of $q_k = \sum_{j \in P_k \setminus k} \gamma_{jk} e_j$. If however, the agents 1 to $k-1$ form a coalition T and minimize their joint total costs, they will, in general, choose $x_j > 0$ for at least some $j \in 1, \ldots, k-1$. This implies a pollution level of $q_k < \sum_{j \in P_k \setminus k} \gamma_{jk} e_j$ which reduces the minimal costs $v(S)$ coalition S can secure for itself. Thus, analogously to Ambec and Ehlers (2008), cooperation exerts a positive externality on the coalition S.

In the following, we restrict our attention to the *non-cooperative core*, i.e. we assume that all non-members of a coalition S behave non-cooperatively, which according to Proposition 1 implies that they do not abate at all. Then, condition (3.4) is replaced by $x_j = 0$ for all $j \notin S$, and the secure costs $v(S)$ of a coalition S are well defined and unique (as the resulting optimization problem is a sub-problem of the one analysed in Proposition 2). The reason is like in Ambec and Ehlers (2008): the structure of the river sharing problem (N, e, c, d), as described in detail in Section 2, is such that only the non-cooperative core is guaranteed to be non-empty.

Like Ambec and Ehlers (2008), we impose the secure costs as the participation constraint of any coalition S. A coalition S will only agree to a river sharing agreement if it is not worse off than without the agreement. Thus, a river sharing agreement should at most assign the secure costs $v(S)$ to any coalition S as otherwise the coalition would block the agreement knowing that it can achieve at least $v(S)$ on its own. Hence, $v(S)$ defines cost upper bounds for any coalition S a river sharing agreement must satisfy in order not to be blocked.

4 Cost Lower Bounds

Ambec and Ehlers (2008) also impose welfare upper bounds that are inspired by the *unlimited territorial integrity* (UTI) doctrine. In case of water consumption, UTI claims that all agents are entitled to consume the full stream of water originating upstream from their location and, thus, have a legitimate claim to the corresponding welfare level such a consumption generates. As such claims are, in general, incompatible if water is scarce, Ambec and Sprumont (2002) and Ambec and Ehlers (2008) interpret them as welfare upper bounds agents may legitimately aspire to.

The straightforward translation of these aspiration welfare upper bounds to the case of our river pollution model is to define the minimal costs a coalition S can ensure if all non-members of the coalition would abate all their emissions, and thus, not pollute the river at all. Formally, these cost lower bounds $a(S)$ are given by:

$$a(S) = \sum_{i \in S} k_i\big(x_i^a(S), q_i(x_i^a(S))\big) ,$$

where $x^a(S) = \big(x_1^a(S), \ldots, x_n^a(S)\big)$ denotes the solution to

$$\begin{aligned} \min_{\{x_i\}_{i\in S}} \sum_{i \in S} & k_i(x_i(S), q_i(x(S))) \quad \text{subject to} \\ & q_i(x) = \sum_{j \in P_i \setminus i} \gamma_{ji}(e_j - x_j) , \quad \forall\, i \in N , \\ & 0 \le x_i \le e_i , \quad \forall\, i \in S , \\ & x_j = e_j , \quad \forall\, j \notin S . \end{aligned}$$

The cost lower bounds $a(S)$ can be interpreted as a fairness condition: no coalition S should enjoy lower costs than the costs it were to secure itself if all non-members of the coalition did not pollute the river at all.

5 The Downstream Incremental Distribution

As in Ambec and Sprumont (2002) and Ambec and Ehlers (2008), there is a connection between the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$: For the coalition of all predecessors of agent i they coincide, i.e. $v(P_i) = a(P_i)$. Thus, for any coalition of predecessors P_i it is clear that the only river sharing agreement satisfying both the cost upper and cost lower bounds is the so called *downstream incremental distribution* (*DID*) defined by

$$z_i^\star = v(P_i) - v(P_i \backslash i) , \quad \forall\, i \in N .$$

The *DID* assigns every agent his marginal contribution to the coalition composed of his predecessors along the river. As a consequence, the *DID* is the only candidate for a river sharing agreement that at the same time satisfies the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S. The following theorem establishes that the *DID*, in fact, satisfies the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S.

Theorem 1 (Only *DID* satisfies cost upper and lower bounds)**.** *The downstream incremental distribution (DID) $z^\star$ is the only river sharing agreement satisfying the non-cooperative core upper bounds $v(S)$ and the cost lower bounds $a(S)$ for any coalition S.*

Proof. The proof is split into three parts. In the first part, we show that the *DID* satisfies the non-cooperative core upper bounds for any coalition S. In part two, we prove that the *DID* also satisfies the cost lower bounds for any coalition S and, finally, in the third part, we show that any river sharing agreement that satisfies the cost upper and lower bounds for an arbitrary coalition S is identical to the *DID*.

We prove that the *DID* satisfies the non-cooperative core upper bounds for any coalition S by induction. The idea is that any coalition S can be created from the grand coalition N by consecutively deleting all non-members $m_j \in \{m_1, \ldots, m_z\}$ of S starting with the most downstream agent m_z. This procedure creates a sequence of intermediate coalitions $N = S_z, S_{z-1}, \ldots, S_1, S$. We show that the *DID* satisfies the core upper bounds for any intermediate coalition S_j, $j \in 1, \ldots, z$ and also for S.

For the first part of the proof we need the following proposition, the proof of which is given in the Appendix.

Proposition 3. *For any $T \subset N$ with $minT > j$ and any $j \in N$ the following inequality holds:*

$$v(P_{m_j} \cup T) - v(P_{m_j} \backslash m_j \cup T) \leq v(P_{m_j}) - v(P_{m_j} \backslash m_j) . \tag{5.1}$$

For the grand coalition $N = S_z$, the non-cooperative core upper bounds are satisfied. Now, suppose the *DID* satisfies the non-cooperative core upper bounds for some intermediate coalition S_j, i.e.

$$\sum_{i \in S_j} z_i^\star \leq v(S_j) . \tag{5.2}$$

We generate the intermediate coalition S_{j-1} by deleting the non-member m_j from the intermediate coalition S_j. By construction the intermediate coalition S_{j-1} consists of all strict predecessors of agent m_j and all agents $i > m_j$ who belong to the coalition S. Rearranging inequality (5.2) and applying the definition of the *DID* implies

$$\sum_{i \in S_{j-1}} z_i^\star \leq v(S_j) - z_{m_j}^\star = v(S_j) - v(P_{m_j}) + v(P_{m_j} \backslash m_j) \ .$$

We have to show that the *DID* satisfies the non-cooperative core upper bounds for the intermediate coalition S_{j-1}, i.e.

$$\sum_{i \in S_{j-1}} z_i^\star \leq v(S_j) - v(P_{m_j}) + v(P_{m_j} \backslash m_j) \leq v(S_{j-1}) \ .$$

Rearranging this inequality yields

$$v(S_j) - v(S_{j-1}) \leq v(P_{m_j}) - v(P_{m_j} \backslash m_j) \ . \tag{5.3}$$

If the coalition S does not have any members $i > m_j$, then the inequality is trivially satisfied as then $S_j = P_{m_j}$ and $S_{j-1} = P_{m_j} \backslash m_j$. Otherwise, define the set T consisting of all members i of the coalition S with $i > m_j$. Then, $S_j = P_{m_j} \cup T$ and $S_{j-1} = P_{m_j} \backslash m_j \cup T$ and by virtue of Proposition 3, inequality (5.3) holds.

For the second part of the proof, the following proposition is needed

Proposition 4. *For any $S \subset T \subset N$ and $i \notin S, T$ the following inequality holds:*

$$a(S \cup i) - a(S) \leq a(T \cup i) - a(T). \tag{5.4}$$

The proof of Proposition 4 is given in the Appendix.
To show that the *DID* satisfies the cost lower bounds for any coalition S, we employ $v(P_i) = a(P_i)$ to rewrite the definition of the *DID*:

$$z_i^\star = v(P_i) - v(P_i \backslash i) = a(P_i) - a(P_i \backslash i) \ .$$

Summing up over all agents $i \in S$ and employing Proposition 4 yields

$$\sum_{i \in S} z_i^\star = \sum_{i \in S} a(P_i) - a(P_i \backslash i) \geq \sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) \ .$$

The right hand side of the inequality simplifies to

$$\begin{aligned}\sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) &= a(P_{minS}) + a(\{P_{minS}, P_{minS} + 1\}) - a(\{P_{minS}\}) + ... \\ &+ a(\{P_{minS}, ..., P_{maxS}\}) - a(\{P_{minS}, ..., P_{maxS} - 1\}) \\ &= a(\{P_{minS}, ..., P_{maxS}\}) = a(S) \ .\end{aligned}$$

Thus, we obtain

$$\sum_{i \in S} z_i^\star = a(P_i) - a(P_i \backslash i) \geq \sum_{i \in S} a(P_i \cap S) - a(P_i \backslash i \cap S) = a(S) \ ,$$

which proves that the *DID* satisfies the cost lower bounds for any coalition S:

$$\sum_{i \in S} z_i^\star \geq a(S) \ .$$

Finally, we prove that the *DID* is the only river sharing agreement that simultaneously satisfies the cost upper and lower bounds for any coalition S. Therefore, we have to show that whenever a river sharing agreement z satisfies both the cost upper and lower bounds, then for each agent i it holds that $z_i = z_i^\star$. Again, the proof is by induction.

Similar to Ambec and Ehlers (2008), for agent 1, any river sharing agreement z fulfilling both constraints satisfies $v(\{1\}) \geq z_1 \geq a(\{1\})$. As $v(\{1\}) = a(\{1\})$ this implies $z_1 = z_1^\star$. Now, suppose that $z_i = z_i^\star$ holds for all agents i upstream of some agent j, i.e. $i \leq j < n$. Summing up over all $i \in P_j$, we obtain

$$\sum_{i \in P_j} z_i = \sum_{i \in P_j} z_i^\star = v(P_j) \ .$$

As $v(P_{j+1}) = a(P_{j+1})$ and because any river sharing agreement z satisfies both the cost upper and lower bounds, $\sum_{i \in P_{j+1}} z_i = v(P_{j+1}) = a(P_{j+1})$ has to hold. Hence,

$$z_{j+1} = \sum_{i \in P_{j+1}} z_i - \sum_{i \in P_j} z_i = v(P_{j+1}) - v(P_j) = z_{j+1}^\star .$$

Therefore, the cost distribution z is identical to the *DID*. □

Theorem 1 is the exact counterpart to Theorem 1 of Ambec and Ehlers (2008). However, it is neither obvious nor straightforward to prove that the *DID* is the only distribution satisfying the cost upper and lower bounds in case of our river pollution model. The main challenge in Ambec and Ehlers (2008) arose from the fact that cooperation among agents impose positive externalities on any coalition S. As a consequence, the welfare level a coalition could secure for itself crucially depends on the partition of all non-members. The same is true for our river pollution model. Cooperative behaviour among non-members of a coalition S induces, in general, positive abatement levels, which benefits the members of the coalition.

In contrast to Ambec and Sprumont (2002) and Ambec and Ehlers (2008), however, the decision variable in our model is emission abatement not water consumption. While water consumption only benefits the consumer and, thus, is a purely private commodity, emission abatement is not. In fact, in our model emission abatement does not benefit the abating agent but only all downstream agents, as it reduces the river's downstream pollution level. Thus, emission abatement imposes positive downstream externalities, i.e. pollution abatement is a commodity with public good properties. This is also reflected in the agents' welfare: agents' welfare in the water consumption models of Ambec and

Sprumont (2002) and Ambec and Ehlers (2008) is simply given by some benefit function $b_i(x_i)$ which depends on the water consumption x_i of agent i. In our model, the costs agent i faces consist of two parts: first, the abatement costs $c_i(x_i)$, which only depend on the emission abatement of agent i and, second, the damage cost function $d_i(q_i)$ depending on the pollution level q_i, which itself is a function of the emission abatement levels of all upstream agents.

6 Discussion and Extensions

The model detailed in Section 2 relies on a number of assumptions which can be relaxed without impairing the statement of Theorem 1. First, we assumed that there is no initial pollution at the source of the river and that the net emissions of agent i do not harm agent i himself but only all downstream agents. As a consequence, agent 1 does not face any pollution and the specification of agent 1's damage function d_1 is optional. The first assumption simplified the specification of the pollution level q_i, while the latter assumption implied that in the non-cooperative Nash equilibrium no agent would abate at all. However, the proof of Theorem 1 does not draw on these assumptions and would still be valid if the pollution level agent i faces would be defined as

$$q_i = q_0 + \sum_{j \in P_i} \gamma_{ji}(e_j - x_j) \ ,$$

where q_0 denotes an initial pollution level at the source of the river.

Second, we framed the model as a pollution abatement model. Obviously, emissions and the corresponding pollution levels are prime examples for downstream externalities, yet there are many other contexts to which our model is applicable. As an example, think of the case of flooding. Then, e_i corresponds to the water discharges from the territory of agent i into the river and x_i denotes the amount of water agent i withdraws from the stream (e.g. by the controlled flooding of designated flooding areas) and q_i is the amount of excess water at agent i's location. In this interpretation it would also be reasonable to assume that the water withdrawn x_i is not limited by the discharge e_i but could sum up to the total amount of excess water in the river basin, i.e.

$$0 \leq x_i \leq q_i \ .$$

These modifications also would not impact the validity of Theorem 1.

Third, particularly in case of flood protection, agents may have different means of protection. While the withdrawal of water induces costs to agent i and benefits all his downstream agents, there are other protection techniques which are purely private goods. As an example, consider that agent i could build a levee that protects his own territory from flooding, but does not induce any positive externalities to the downstream agents. Then, the damage to agent i does not only depend on the total amount of water q_i but also on the agent's investment into private damage protection m_i, i.e. $d_i = d_i(q_i, m_i)$. Assuming that an interior solution is optimal, i.e. $m_i^\star > 0$, the optimal level of private protection $m_i^\star(q_i)$ is given by the solution of the first order condition

$$\frac{\partial d_i(q_i, m_i)}{\partial m_i} = 0 \, .$$

Thus, we can re-write $d_i(q_i, m_i)$ as $d_i\big(q_i, m_i^\star(q_i)\big)$. Whenever these newly specified damage functions $d_i\big(q_i, m_i^\star(q_i)\big)$ are increasing, twice differentiable and convex in q_i, we are back at the model specification introduced in Section 2.

7 Conclusion

We showed that the main result of Ambec and Ehlers (2008) that the downstream incremental distribution is the only welfare distribution that satisfied the non-cooperative core bounds and the aspiration welfare bounds simultaneously, can be generalized to the case of commodities with public good characteristics. Like their water consumption model, our river pollution problem is a cooperative game with externalities, since cooperation among non-members imposes a positive externality to the members of any coalition S. However, our model comprises an additional source of externalities because the emissions discharged into the river induce negative externalities on all downstream agents. In addition, our results are robust with various extensions of our baseline model.

Appendix

Proof. of proposition 3
Set $S_j = P_{m_j} \cup T$ and $S_{j-1} = P_{m_j \setminus m_j} \cup T$. Let us parametrize the damage functions for agents $j > m_j$ with a parameter $\alpha \in [0, \infty)$. Due to this parametrization, the secure costs $v(S_{j-1}, \alpha)$ of the intermediate coalitions S_{j-1} now depend on the parameter α and amount to

$$v(S_{j-1}, \alpha) = \sum_{i \in P_{m_j} \setminus m_j} k_i(x^v(S_{j-1}, \alpha)) + \sum_{i \in F_{m_j} \setminus m_j \cap S} c_i(x^v(S_{j-1}, \alpha)) + \\ \alpha \cdot \sum_{i \in F_{m_j} \setminus m_j \cap S} d_i(q_i(x^v(S_{j-1}, \alpha))).$$

Furthermore, inequality (5.1) changes to

$$v(S_j, \alpha) - v(S_{j-1}, \alpha) \leq v(P_{m_j}) - v(P_{m_j} \setminus m_j) \quad \forall \alpha \in [0, \infty). \tag{7.1}$$

By showing that (7.1) holds for all $\alpha \in [0, \infty]$, then it holds, in particular, for $\alpha = 1$ and inequality (5.1) is satisfied.

Thus, in a next step, we show that inequality (7.1) holds for all $\alpha \in [0, \infty]$. For $\alpha = 0$, we have $v(S_j, 0) = v(P_{m_j})$ and $v(S_{j-1}, 0) = v(P_{m_j} \setminus m_j)$, therefore inequality (7.1) holds with equality. For all other α, we differentiate inequality (7.1) with respect to α, i.e.

$$\frac{\partial v(S_j, \alpha)}{\partial \alpha} - \frac{\partial v(S_{j-1}, \alpha)}{\partial \alpha} \leq 0. \tag{7.2}$$

Hence, we partially differentiate $v(S_j, \alpha)$ with respect to α and apply the envelope theorem, i.e.

$$\begin{aligned} \frac{\partial v(S_j, \alpha)}{\partial \alpha} &= \frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial \alpha} + \underbrace{\frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial x(S_j, \alpha)}}_{0} \frac{\partial x(S_j, \alpha)}{\partial \alpha} \\ &= \frac{\partial v(x^v(S_j, \alpha), \alpha)}{\partial \alpha} \\ &= \sum_{i \in F_{m_j} \setminus m_j \cap S_j} d_i(q_i(x^v(S_j, \alpha))) \end{aligned} \tag{7.3}$$

and analogously,

$$\begin{aligned} \frac{\partial v(S_{j-1}, \alpha)}{\partial \alpha} &= \frac{\partial v(x^v(S_{j-1}, \alpha), \alpha)}{\partial \alpha} \\ &= \sum_{i \in F_{m_j} \setminus m_j \cap S_{j-1}} d_i(q_i(x^v(S_{j-1}, \alpha))). \end{aligned}$$

Hence, inequality (7.2) can be rewritten to

$$\sum_{i \in F_{m_j} \backslash m_j \cap S_j} d_i(q_i(x^v(S_j), \alpha)) \leq \sum_{i \in F_{m_j} \backslash m_j \cap S_{j-1}} d_i(q_i(x^v(S_{j-1}, \alpha))).$$

Clearly, this inequality is satisfied whenever

$$\sum_{l \in P_k} x_l^v(S_{j-1}, \alpha) \leq \sum_{l \in P_k} x_l^v(S_j, \alpha), \forall k \in S_{j-1}, S_j,$$

which is stated in the following lemma.

Lemma 1. For any agent $k \in S, T$ the following inequality is satisfied

$$\sum_{l \in P_k} x_l^v(S_{j-1}, \alpha) \leq \sum_{l \in P_k} x_l^v(S_j, \alpha), \forall k \in S_{j-1}, S_j. \tag{7.4}$$

Proof. of Lemma 1
Consider two coalitions S and $T = S \cup m$ and an agent $m \notin S$. Given this notation, inequality (7.4) changes to

$$\sum_{j \in P_k \cap S} x_j^v(S, \alpha) \leq \sum_{j \in P_k \cap T} x_j^v(T, \alpha), \forall k \in S, T.$$

Let us prove Lemma 1 by contradiction, i.e. assume that

$$\sum_{j \in P_k \cap T} x_l^v(T, \alpha) < \sum_{j \in P_k \cap S} x_l^v(S, \alpha). \tag{7.5}$$

According to the parametrized minimization problem, the following first order conditions have to be satisfied

$$\begin{aligned} c_i'(x_i) &\leq \sum_{j \in F_i \backslash i \cap T \cap P_m} d_j'(q_j(x_j)) + \alpha \sum_{j \in F_m \backslash m \cap T} d_j'(q_j(x_j)) \\ &\leq \sum_{j \in F_i \backslash i \cap T \cap P_m} d_j' \left(\sum_{k \in P_j \backslash j} \gamma_{kj} e_k - \sum_{k \in P_j \backslash j \cap T} \gamma_{kj} x_k \right) \\ &\quad + \alpha \cdot \sum_{j \in F_m \backslash m \cap T} d_j' \left(\sum_{k \in P_j \backslash j} \gamma_{kj} e_k - \sum_{k \in P_j \backslash j \cap T} \gamma_{kj} x_k \right), \forall i \in T, \end{aligned} \tag{7.6}$$

and

$$\begin{aligned} c_i'(x_i) &\leq \sum_{j \in F_i \backslash i \cap S \cap P_m \backslash m} d_j'(q_j(x_j)) + \alpha \sum_{j \in F_m \backslash m \cap S} d_j'(q_j(x_j)) \\ &\leq \sum_{j \in F_i \backslash i \cap S \cap P_m \backslash m} d_j' \left(\sum_{k \in P_j \backslash j} \gamma_{kj} e_k - \sum_{k \in P_j \backslash j \cap S} \gamma_{kj} x_k \right) \\ &\quad + \alpha \sum_{j \in F_m \backslash m \cap S} d_j' \left(\sum_{k \in P_j \backslash j} \gamma_{kj} e_k - \sum_{k \in P_j \backslash j \cap S} \gamma_{kj} x_k \right), \forall i \in S \end{aligned} \tag{7.7}$$

Due to assumption (7.5), the right hand side of (7.6) for $i \in T$ is higher than the right hand side of (7.7) for $i \in S$. This implies $c_i'(x_i(T)) \geq c_i'(x_i(S))$ for all agents $i \in S, T$ and thus, due to the characteristics of the cost function $c_i(.)$, $x_i^v(T) \geq x_i^v(S), \forall i$. This, however, implies

$$\sum_{j \in P_k \cap T} x_j(T) > \sum_{j \in P_k \cap S} x_j(S).$$

Therefore, by contradiction, inequality $\sum_{j \in P_k \cap S} x_j(S) > \sum_{j \in P_k \cap S} x_j(T)$ cannot hold.

Hence, given Lemma 1, inequality (7.1) is satisfied for all $\alpha \in [0, \infty]$, thus also for $\alpha = 1$ implying that inequality (5.1) holds for the coalition S_{j-1}. By induction, inequality (5.1) holds for all intermediate coalitions S_j, $j = 1, ..., z$ with $S_1 = S$, therefore the *DID* is stable for all non-consecutive coalitions S. $\square$

Proof. of proposition 4
For the proof of Proposition 4 the following lemma is required.

Lemma 2. For any two coalitions S, T, the following relationships among the abatement levels of an agent $j \in T, S$ hold

$$x_j^a(T \cup i) \geq x_j^a(S \cup i) \geq x_j^a(S) \text{ and } x_j^a(T \cup i) \geq x_j^a(T).$$

Proof. of Lemma 2
It suffices to show that these inequalities hold for two coalitions S, T, with $T = S \cup t$, $t \in N \backslash S$. Let us first establish that $x_j^a(G \cup i) \geq x_j^a(G)$ for all coalitions $G = T, S$. The first order conditions for an agent $j \in G$ respectively $j \in G \cup i$ read

$$c_j'(x_j) \leq \sum_{k \in F_j \backslash j \cap G \cup i} d_k'(\sum_{m \in P_k \backslash k \cap G \cup i} \gamma_{mk}(e_m - x_m)) \tag{7.8}$$

respectively

$$c_j'(x_j) \leq \sum_{k \in F_j \backslash j \cap G} d_k'(\sum_{m \in P_k \backslash k \cap G} \gamma_{mk}(e_m - x_m)). \tag{7.9}$$

The right hand side of the first order condition in (7.8) is either larger than the right hand side of (7.9), if $j \leq i$, or equal to it, if $j > i$. Thus, $x_j^a(G \cup i) \geq x_j^a(G), \forall j \in G, G \cup i$ and $G = T, S$. Due to $T \cup i = S \cup i \cup t$, it follows that $x_j^a(T \cup i) \geq x_j^a(S \cup i)$. Thus, lemma 2 holds.

Recall inequality (5.4) in Proposition 4. We restate the two differences in the inequality in the following way

$$\begin{aligned} a(T \cup i) - a(T) &= k_i(x_i^a(T \cup i)) + \sum_{j \in T} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \\ &= k_i(x_i^a(T \cup i)) + \sum_{j \in T \backslash S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) + \\ &\quad \sum_{j \in S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \end{aligned}$$

and

$$a(S \cup i) - a(S) = k_i(x_i^a(S \cup i)) + \sum_{j \in S} k_j(x_j^a(S \cup i)) - k_j(x_j^a(S)).$$

Thus, by rearranging and using the above expressions, inequality (5.4) can expressed as

$$
\begin{aligned}
&k_i(x_i^a(T\cup i)) - k_i(x_i^a(S\cup i)) + \sum_{j\in T\setminus S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \\
&\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) \geq 0.
\end{aligned} \tag{7.10}
$$

To prove that inequality (7.10) is satisfied, we divide the terms into three groups I, II, III as represented in the following

$$
\begin{aligned}
&\underbrace{\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) +}_{I} \\
&\underbrace{\sum_{j\in T\setminus S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T))}_{II} + \underbrace{k_i(x_i^a(T\cup i)) - k_i(x_i^a(S\cup i))}_{III} \geq 0.
\end{aligned} \tag{7.11}
$$

In the three lemmas presented below, we will show that for all subgroups I, II, III we have $I, II, III \geq 0$. As a result, we conclude that inequality (7.10) holds.

Lemma 3. Given the terms in subgroup I of (7.11), it holds that $I \geq 0$, i.e.

$$
\sum_{j\in S} k_j(x_j^a(T\cup i)) - k_j(x_j^a(T)) + \sum_{j\in S} k_j(x_j^a(S)) - k_j(x_j^a(S\cup i)) \geq 0. \tag{7.12}
$$

Proof. of Lemma 3
Let us rewrite inequality (7.12) by splitting it into cost and damage functions, i.e.

$$
\begin{aligned}
&\sum_{j\in S} c_j(x_j^a(T\cup i)) - c_j(x_j^a(T)) + c_j(x_j^a(S)) - c_j(x_j^a(S\cup i)) \\
&\quad + \sum_{j\in S} d_j(q_j(x_j^a(T\cup i))) - d_j(q_j(x_j^a(T))) + d_j(q_j(x_j^a(S))) - d_j(q_j(x_j^a(S\cup i))) \geq 0.
\end{aligned}
$$

We prove the inequality above graphically. Due to the convexity of both the damage and cost functions of each agent $j \in S$ and due to the relationships $x_j^a(T\cup i) \geq x_j^a(S\cup i) \geq x_j^a(S)$ and $x_j^a(T\cup i) \geq x_j^a(T)$ established in Lemma 2, for each agent j it holds that

$$
\begin{aligned}
c_j(x_j^a(T\cup i)) - c_j(x_j^a(S\cup i)) &= m, m \geq 0 \\
c_j(x_j^a(S)) - c_j(x_j^a(T)) &= n, n \leq 0,
\end{aligned}
$$

with $|m| \geq |n|$ as depicted[1] in Figure 1. Similarly,

$$
\begin{aligned}
d_j(q_j(x_j^a(T\cup i))) - d_j(q_j(x_j^a(S\cup i))) &= m, m \leq 0 \\
d_j(q_j(x_j^a(S))) - d_j(q_j(x_j^a(T))) &= n, n \geq 0, \forall j
\end{aligned}
$$

with $|n| \geq |m|$ as depicted in Figure 2. Thus, we conclude that inequality (7.12) holds.

[1]As no general relationship can be established for $x_j^a(T)$ and $x^a(S\cup i)$, both cases are depicted in the Figures 1 and 2.

Figure 1: Cost functions

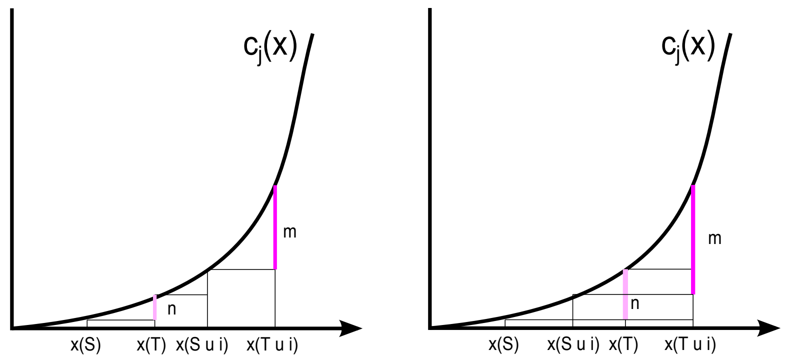
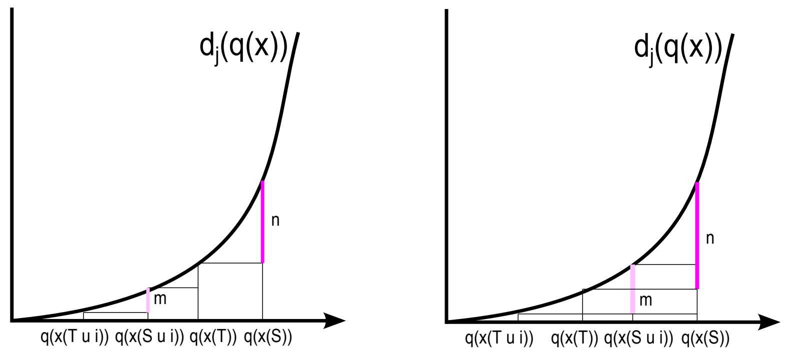


Figure 2: Damage cost functions



Lemma 4. Given the terms in subgroup II of (7.11), it holds that $II \geq 0$, i.e.

$$\sum_{j \in T \setminus S} k_j(x_j^a(T \cup i)) - k_j(x_j^a(T)) \geq 0. \tag{7.13}$$

Proof. of Lemma 4
The following first order conditions have to be satisfied

$$c_j'(x_j(T)) \leq \sum_{k \in F_j \setminus j \cap T} d_k' \left(\sum_{t \in P_k \setminus k \cap T} \gamma_{tk}(e_t - x_t(T)) \right), \forall j \in T, \tag{7.14}$$

and

$$c_j'(x_j(T \cup i)) \leq \sum_{k \in F_j \setminus j \cap T \cup i} d_k' \left(\sum_{t \in P_k \setminus k \cap T} \gamma_{tk} e_t + \gamma_{ik} e_i - \gamma_{tk} x_t(T) - \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} \Delta x_t \right), \forall j \in T \cup i, \tag{7.15}$$

with $\sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} \Delta x_t = \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} x_j^a(T \cup i) - \sum_{t \in P_k \setminus k \cap T} \gamma_{tk} x_t^a(T)$.
As $x_j^a(T \cup i) \geq x_j^a(T)$ derived in Lemma 2, the right hand side of inequality (7.15) has to be larger than the right hand side of (7.14). In order for this to be satisfied, the following needs to hold

$$\gamma_{ik} e_i \geq \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} \Delta x_t = \sum_{t \in P_k \setminus k \cap T \cup i} \gamma_{tk} x_j^a(T \cup i) - \sum_{t \in P_k \setminus k \cap T} \gamma_{tk} x_t^a(T).$$

Consequently,

$$d_j\left(\sum_{k\in P_j\backslash j\cap(T\cup i)}\gamma_{kj}(e_j-x_j^a(T\cup i))\right)\geq d_j\left(\sum_{k\in P_j\backslash j\cap T}\gamma_{kj}(e_j-x_j^a(T))\right),\forall j\in T. \tag{7.16}$$

In addition, due to $x_j^a(T\cup i)\geq x_j^a(T)$ and $c_i(x_i)$ increasing and convex, we have $c_j(x_j^a(T\cup i))\geq c_j(x_j^a(T))$. Combining this with (7.16) implies $k_j(x_j^a(T\cup i))\geq k_j(x_j^a(T))$. Summing up over all $j\in T\backslash S$ yields the desired inequality (7.13).

Lemma 5. Given the terms in subgroup III of (7.11), it holds that $III\geq 0$, i.e.

$$k_i(x_i^a(T\cup i))-k_i(x_i^a(S\cup i))\geq 0.$$

Proof. of Lemma 5
Recall that for each $j\in S\cup i$, the following first order condition must hold

$$c_j'(x_j(S\cup i))\leq\sum_{k\in F_j\backslash j\cap S\cup i}d_k'\left(\sum_{t\in P_k\backslash k\cap S\cup i}\gamma_{tk}(e_t-x_t(S\cup i))\right). \tag{7.17}$$

Similarly, for each $j\in T\cup i$ it must hold that

$$\begin{aligned}c_j'(x_j(T\cup i))\leq\sum_{k\in F_j\backslash j\cap T\cup i}d_k'(&\sum_{t\in P_k\backslash k\cap S\cup i}\gamma_{tk}e_t+\sum_{t\in P_k\backslash k\cap T\backslash S}\gamma_{tk}e_t\\&-\sum_{t\in P_k\backslash k\cap S\cup i}\gamma_{tk}x_t(S\cup i)-\sum_{t\in P_k\backslash k\cap T\cup i}\gamma_{tk}\Delta x_t).\end{aligned} \tag{7.18}$$

Given that $x_j^a(T\cup i)\geq x_j^a(S\cup i)$ derived in Lemma 2, the left hand side of inequality (7.18) is larger than the left hand side of inequality (7.17). Consequently, it has to hold that

$$\sum_{t\in P_k\backslash k\cap T\backslash S}\gamma_{tk}e_t\geq\sum_{t\in P_k\backslash k\cap T\cup i}\gamma_{tk}\Delta x_t=\sum_{t\in P_k\backslash k\cap T\cup i}\gamma_{tk}x_t^a(T\cup i)-\sum_{t\in P_k\backslash k\cap S\cup i}\gamma_{tk}x_t^a(S\cup i).$$

The agents $j\in T\cup i$ do not abate more than the additional pollution flow passing through their region compared to what they would optimally abate if they belonged to the smaller coalition $S\cup i$. As a result,

$$d_i\left(\sum_{j\in P_i\backslash i\cap T\cup i}\gamma_{ji}(e_j-x_j^a(T\cup i))\right)\geq d_i\left(\sum_{j\in P_i\backslash i\cap S\cup i}\gamma_{ji}(e_j-x_j^a(S\cup i))\right). \tag{7.19}$$

In addition, as $x_j^a(T\cup i)\geq x_j^a(S\cup i)\ \forall j$ derived in Lemma 2 and $c_i(x_i)$ is increasing and convex, we have

$$c_i(x_i^a(T\cup i))\geq c_i(x_i^a(S\cup i)). \tag{7.20}$$

Combining (7.19) with (7.20) implies

$$k_i(x_i(T \cup i)) \geq k_i(x_i(S \cup i)).$$

Proposition 4 then follows from combining Lemmas 3, 4 and 5. $\square$

Chapter 2

Sharing a River with Asymmetric Information

Abstract

In this chapter, we analyse the contractual mitigation of a global public bad along a river in the presence of a federalist governance structure, where the lower tiers have private information about their abatement costs. We propose that the federal government nominates one of the lower tiers to be the principal, who is authorized to offer mitigation contracts to the other tiers sharing the river. The elected principal can do so either in a centralized manner, i.e. he offers contracts simultaneously to all other tiers, or in a delegated manner, i.e. he starts an upstream and downstream sequential contracting process by contracting with his up- and downriver neighbouring tiers, to which he then gives the authority to subcontract with their respective neighbouring tiers till all tiers received a contract. We show that under certain conditions, a nominated principal can achieve the same abatement allocation with the delegated as with the centralized contracting method while matching his expected costs. As all potential principals implement a different abatement allocation, the choice of the prime principal matters for the total expected costs occurring in the river basin. We show that the tier located most downriver, which is subject to the same informational constraints as the federal government, is never the best choice to be nominated as the principal.

1 Introduction

One negative aspect of sharing a river with others is water pollution. Water pollution not only negatively impacts wildlife and human health but may also deteriorate economic profits. A special characteristic of rivers is their unidirectional flow and, as a consequence, they carry the pollution downriver where it accumulates. In addition, the polluters may not bear the negative consequences of their emissions themselves and may thus have little incentive to reduce their emissions. Moreover, rivers may flow through several countries, cantons and numerous municipalities so that upstream polluters and downstream riparians may belong to different jurisdictions. As a consequence, polluters may have little motivation to abate their emissions, because they cannot be held liable for the pollution damage caused in other jurisdictions.

In this chapter, we analyse contractual mitigation of a global public bad along a river in the presence of a federalist governance structure, where the lower tiers have private information about their abatement costs. In countries with a strong federalist structure like Switzerland, the US or Canada levels directly below the Federal government (States, Provinces or Cantons) as well as the lowest tiers of government (communities) have considerable authority in terms of the allocation and distribution of a public bad. Thus, mitigation contracts that have to be accepted by all tiers in the river basin may be proposed by two sources; either by the federal government, which acts as a social planner or by one of the lower tiers. Even though the lower tiers follow their self-interest, it might be beneficial for the federal government to let one of the lower tiers propose mitigation contracts because the lower tiers are expected to know the preferences of their constituents better than the higher level and thus are subject to fewer informational constraints. Yet, as all lower tiers would try to seize the opportunity to propose contracts to the other tiers and each of them would implement a different allocation of the public bad, the federal government should nominate the tier that instates the allocation leading to the lowest expected total costs for the federalist state. However, as is well known, the contractual mitigation of a global public bad in adverse selection models with voluntary participation encounters several difficulties. First, all tiers have to voluntarily agree to a set of contracts offered by the principal. Second, tiers may have an incentive to exaggerate their privately known abatement costs in order to reduce the abatement level they have to supply with the consequence that most of the burden of abatement is left to the other riparians. Thus, the proposed set of contracts has to ensure that all tiers are not worse off than in the status quo without any contracts and that it is in their best interest to reveal their true abatement costs.

Only a few papers consider asymmetric information in the costs and benefits of reducing pollution and even fewer address environmental agreements taking a mechanism design perspective. One of the first to consider environmental agreements in a mechanism design setting was Dasgupta et al. (1980). In their paper, they focus on optimal pollution control with imperfect information about the abatement costs of the agents. The polluting agents communicate with the regulator but not with one another and there is only one victim of pollution, the society. In particular, they look at incentive compatible direct revelation schemes and propose a simple adaptation of the Groves scheme, while neglecting the issue of voluntary participation. Caparros et al. (2004) consider a bargaining model in which northern countries suffer from emissions of southern countries and negotiate the transfer necessary to reduce these emissions. Thus, they consider one-sided asymmetric information and bargaining between one polluter and one victim. Baliga and Maskin (2003) are the first to consider environmental agreements between countries that are at the same time victims and polluters as a mechanism design problem under asymmetric information. However, they do not consider participation constraints and thus find that first best agreements are possible. Helm and Wirl (2011) consider a two-country model, where bargaining power is asymmetrically distributed and an uninformed country designs a mechanism controlling collective emissions. They show that the uninformed party must jointly use subsidies and its own emissions to incentivize the informed party and ensure its participation. Martimort and Sand-Zantman (2013) analyse international environmental agreements and highlight the trade-off between the free riding problem due to asymmetric information and voluntary participation. They concentrate on second best mechanisms and show that the optimal mechanism admits a simple approximation by menus. We contribute to this strand of literature by adding directed externalities between the affected agents, by studying environmental agreements in a federalist setting and by analysing the choice of the principal offering contracts.

The design of incentive schemes for implementing optimal plans in organizations where information differs across agents has been much studied in recent years. In adverse selection environments, Myerson (1982) has shown that, in general, any non-cooperative equilibrium outcome of an arbitrary decentralized organization can be mimicked by a centralized one, where agents communicate their private information directly to the principal without any interactions among them. Our research is closest related to Melumad et al. (1995), who show in a three agent model, that both organizational structures are equivalent when, in the decentralized structure, the principal can make the payment to the intermediate agent contingent on the contract set up between this agent and his partner. When this is not so, the centralized structure is strictly superior. We generalize their model to n agents, extend it by adding directed externalities among the intermediate agents and set it in a federalist structure with the option of having different principals.

In particular, we consider two forms of contracting, a centralized and a delegated principal agent model. In the centralized contracting model, the lower tier chosen by the federal government to be the principal contracts directly and simultaneously with all other tiers. For the delegated contracting model, upstream and downstream delegated contracting seem appropriate due to the nature of the river. Hence, the nominated principal contracts with two tiers, his upriver and downriver direct neighbours, to which he then gives the authority to sub-contract with their up respectively downriver neighbouring tier and so on till all riparians have accepted their contracts. The elected principals are allowed to choose among the two contracting methods. Delegated contracting may have the advantage that it reduces the communication requirements between the principal and the other tiers. A possible disadvantage of delegation is that it may exacerbate incentive problems. Intermediate tiers who have been given authority over certain decision may pursue their own self-interest rather than that of the principal.

We show that given full information about the abatement costs, the first best optimal solution, i.e. the abatement allocation minimizing the sum of all abatement and damage costs in the river, can be implemented in both the centralized and the delegated principal agent models independent on who proposes the set of contracts. The total cooperation gain goes to the chosen principal, whereas all other tiers are equally well off as in the status quo without any mitigation. Thus, the choice of the principal is only a matter of who attains the cooperation gain and does not affect the abatement allocation. In the case of privately known abatement costs, the first best optimal abatement allocation cannot be attained in either contracting model. The abatement levels are distorted due to the information rents that have to be paid with the result that they fall short of the first best optimal abatement levels. In line with the literature, we show that, under certain conditions, a principal can achieve the same abatement allocation in the delegated contracting model as in the centralized contracting model while matching his expected costs. For this result to hold, we assume that the contracts, generally consisting of an assigned abatement level and compensation payment from the benefactors of the abatement done to the tiers doing the abatement, are realized after all riparians have accepted them and that all intermediate principals are able to monitor the abatement levels and reported types. In addition, we show that the selection of the prime principal plays an important role for the total costs occurring in the river in the presence of asymmetric information. We establish that the riparian most downriver, who faces the same informational constraints as the federal government, offers contracts that lead to the highest expected costs in the river basin. Thus, the federal government should delegate its power to any other riparian as total expected costs for the river basin amount to less with any of them acting as the prime principal. Intuitively, the reason for this is that by electing any other tier to be the principal, one uncertainty drops as this principal knows his own type and thus one less information rent has to be paid. If damage costs are linear, the tier most upriver is the best choice.

However, if damage costs take any other functional form, the choice of the best principal depends on the exogenously given pollution levels, the position of the principal along the river and the damage parameters as illustrated by an example.

The remainder of this chapter is organized as follows. Section 2 introduces the model. Section 3 characterizes the centralized and delegated principal agent model and presents the optimal contracts in case of full and in case of asymmetric information in regard to the abatement costs. In Section 4, we investigate which riparian should be chosen by the federal government as a principal.

2 The River Sharing Model with Asymmetric Information

We analyse contractual mitigation of a global public bad in a federalist structure where the abatement costs are private information. Specifically, consider a country with a federalist government G with n lower tiers sharing a river. The federal government aims to minimize the expected total costs incurred due to the public bad in the river basin. Let the lower tiers be represented by agents $i = 1, ..., n$. The agents are numbered from upstream to downstream, with $i < j$ indicating that agent i is upriver of agent j. Agents may pollute the river by discharging pollutants in exogenously given amount $e_i > 0$. Agents have the possibility to abate pollution in the amount of $x_i \leq e_i$ by facing abatement costs $C_i(x_i, \theta_i) = \theta_i c_i(x_i)$. Assume, $c_i(.)$ is increasing and strictly convex in the abatement efforts x_i. The abatement costs depend on an agent-specific abatement cost parameter $\theta_i \in [\underline{\theta}, \overline{\theta}]$, which is observable only by agent i and not by the other agents $j \neq i$. Thus, the parameter θ_i represents the agent's private information, i.e. his type. The cost parameters θ_i are drawn independently from a commonly known prior distribution $F(\theta_i)$ with a positive density function $f(\theta_i)$. Denote by $x = (x_1, ..., x_{n-1})$ the vector of abatement levels and by $\theta = (\theta_1, ..., \theta_n)$ the vector of abatement cost parameters. Furthermore, let $\theta_{-i} = \theta \backslash \theta_i$. The net emissions released by the agents accumulate while moving downriver, so that the ambient pollution level at agent i's location amounts to

$$q_i(x) = \sum_{j=1}^{i-1} e_j - x_j.$$

The ambient pollution level $q_i(x)$ causes damage costs $D_i(q_i(x))$ that are increasing and convex in $q_i(x)$ and are known to all agents.

Summing up, we face a pollution abatement problem $\mathcal{S}$ in a federalist setting with asymmetric information characterized by (N, e, c, d, θ), where $N = n$ is the number of agents sharing the river, $e = (e_1, ..., e_n)$ is the vector of gross emissions and $C = (C_1, ..., C_n)$ and $D = (D_1, ..., D_n)$ denote the vectors of abatement and damage cost functions.

Furthermore, note that

$$\frac{\partial^2 C_i(x_i, \theta_i)}{\partial x_i \partial \theta_i} = c_i'(x_i) \geq 0, \text{ and } \frac{\partial^3 C_i(x_i, \theta_i)}{\partial x_i \partial^2 \theta_i} = 0.$$

The term on the left hand side represents the single crossing property saying that an agent i's marginal costs from increasing x_i is increasing in his type θ_i. This implies that higher types will be asked to abate less, since the marginal costs of abating x_i increases everywhere as θ_i increases. Let us make the following standard assumption in adverse selection models.

Assumption 1.

$$\frac{F(\theta_i)}{f(\theta_i)} \text{ is increasing in } \theta_i.$$

Note that we assume pollution to be a directed externality in that agents only suffer from the net emissions released by agents located upriver. As a consequence, if the pollution abatement problem is considered to be a non-cooperative game among the agents, then agents have no incentive to abate, as this would only increase individual costs. Thus, in order to do abatement, agents have to be compensated for the costs they bear by transfer payments made by the other agents who benefit from this abatement. Hence, an incentive mechanism or a set of contracts has to be designed that gives the agents incentives to voluntarily abate pollution. Formally, a set of contracts consists of abatement levels $x(.) = \{x_1(.), ..., x_{n-1}(.)\}$ and compensation transfers $t(.) = \{t_1(.), ..., t_n(.)\}$ so that given such a set of contracts, an agent i's costs amount to

$$k_i(.) = C_i(x_i, \theta_i) + D_i(q_i(x)) - t_i(x),$$

where $t_i(x)$ can either be positive or negative. This set of contracts may be proposed by two sources, either the federal government, which acts as a social planner or one of the agents is elected by the federal government to be the principal who subcontracts with all the other agents. In the first concept, the social planner faces the problem of aggregating the (announced) costs of the agents into a collective decision and has to design an appropriate compensation scheme. In the later concept, the principal minimizes his expected costs given the announced costs of the other agents and adopts an appropriate compensation scheme. According to the constrained efficiency theorem (Mas-Colell et al. (1995)), the federal government is not able to improve upon a decentralized outcome (even if that outcome is inefficient), if it is limited by the same informational constraints. Thus, the federal government cannot achieve a better outcome than agent n by designing a set of contracts for all n agents, as agent n and the social planner face the same informational constraints. However, it might be beneficial for the federal government to elect one of the other agents to design contracts because these agents have more information with regard to their abatement costs than the federal government and thus are limited by fewer informational constraints. The nomination of a specific principal itself is relevant due to two reasons. First, all agents have an incentive to act as a principal and second, each potential principal will implement a different abatement allocation. Furthermore, the elected principal can offer these contracts in two different ways, either in a centralized or a delegated manner.

We first discuss centralized contracting models, where the elected principal offers contracts to all other agents simultaneously. Second, we consider delegated contracting models, in which one agent is chosen to be the prime principal, who offers contracts to his direct downstream and upstream neighbour and gives them the authority to sequentially subcontract with their direct neighbours. In either model, the principal faces two problems: first, all agents need to have an incentive to accept a proposed contract, i.e. they must be at least as well off as in the status quo and second, abatement costs are privately known, i.e. agents may have an incentive to overstate their true abatement costs. Thus, a principal has to design a set of contracts for which all agents may find it in their best interest to reveal their abatement costs truthfully and in which all agents are made better off as in the non-cooperative outcome. In other words, a set of contracts must fulfil all incentive compatibility constraints as well as all individual rationality constraints.

According to the revelation principle (Gibbard (1973)), we can restrict our attention to incentive compatible direct revelation contracts which satisfy the individual rationality of all agents. In such contracts, agents are asked to announce their type, denoted by $\hat{\theta}_i$, and in return receive a transfer $t_i(\hat{\theta}_i)$ and an abatement level $x_i(\hat{\theta}_i)$. Furthermore, we assume that the principal observes all reported types as well as the abatement levels chosen. In addition, contracts are realised, i.e. abatement levels and payments are executed, after all agents have accepted their contracts.

Let us have a closer look at the *individual rationality constraints*. We assume that if one agent does not accept the contract, no other contracts will be established among the remaining agents. Thus, a contract fulfils the individual rationality constraints, if no agent can do better on his own given the assumption that the other agents behave non-cooperatively and do not mitigate. Formally, a contract satisfies the individual rationality constraints as long as

$$k_i(.) \leq D_i\left(\sum_{j=1}^{i-1} e_j\right) := \bar{k}_i.$$

Next, let us consider the *incentive compatibility constraints*. Let θ_i denote the true type of agent i and $\hat{\theta}_i$ the reported type. Given any contract, let $k_i(\hat{\theta}_i, \theta_i)$ be the total costs of agent i if he is of type θ_i and reports $\hat{\theta}_i$ while the other agents report truthfully, i.e.

$$k_i(\hat{\theta}_i, \theta_i) = C_i(x_i(\hat{\theta}_i), \theta_i) + D_i(q_i(x(\theta_{-i}))) - t_i(\hat{\theta}_i).$$

Then, incentive compatibility requires

$$k_i(\hat{\theta}_i, \theta_i) \geq k_i(\theta_i, \theta_i), \forall \theta_i,$$

where $k_i(\theta_i, \theta_i)$ is the cost level of agent i of type θ_i if he announces his true type. Fortunately, Mirrlees (1971) introduced a way to reduce the number of incentive constraints by replacing them with the corresponding first order conditions. The trick is as follows: If we think of an agent i's problem as choosing an announcement $\hat{\theta}_i$, then his minimization problem can be written as

$$min_{\hat{\theta}} k_i(\hat{\theta}_i, \theta_i).$$

Thus, the incentive constraint can be stated as follows

$$\theta_i \in argmin_{\hat{\theta}_i} k_i(\hat{\theta}_i, \theta_i).$$

Therefore, for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ at which the objective function is differentiable, which it is by assumption, the following first order condition must hold

$$0 = \frac{\partial}{\partial \hat{\theta}} k_i(\hat{\theta}_i, \theta_i)|_{\hat{\theta}_i = \theta_i}.$$

That is, truth-telling implies that the first order condition of $k_i(\hat{\theta}_i, \theta_i)$ is satisfied when $\hat{\theta}_i = \theta_i$.

Let $\hat{\theta}_i(\theta_i) \in argmin\ C_i(x_i(\hat{\theta}_i), \theta_i) - t_i(\hat{\theta}_i)$ and let $k_i(\theta_i, \hat{\theta}_i) = C_i(x_i(\hat{\theta}_i(\theta_i)), \theta_i) - t(\hat{\theta}_i(\theta_i))$ be the equilibrium cost level of agent i. Note that this cost level depends on θ_i in two ways: on the agent's true type and on his announcement. Thus, differentiating $k_i(\theta_i, \hat{\theta}_i)$ with respect to θ_i, yields

$$\frac{dk_i(\hat{\theta}_i, \theta_i)}{d\theta_i} = \frac{\partial C_i(x_i(\hat{\theta}_i(\theta_i)), \theta_i)}{\partial \theta_i} + \frac{\partial C_i(x_i(\hat{\theta}_i(\theta_i)), \theta_i)}{\partial x_i(\hat{\theta}_i(\theta_i))} \frac{\partial x_i(\hat{\theta}_i(\theta_i))}{\partial \hat{\theta}_i(\theta_i)} \frac{\partial \hat{\theta}_i(\theta_i)}{\partial \theta_i} - \frac{\partial t_i(\hat{\theta}_i(\theta_i))}{\partial \hat{\theta}_i(\theta_i)} \frac{\partial \hat{\theta}_i(\theta_i)}{\partial \theta_i}$$

Applying the envelope theorem yields

$$\frac{dk_i(\hat{\theta}_i, \theta_i)}{d\theta_i} = \frac{\partial C_i(x_i(\hat{\theta}_i(\theta_i)), \theta_i)}{\partial \theta_i}.$$

Thus, in case of incentive compatibility, i.e. where $\hat{\theta}_i(\theta_i) = \theta_i$, we receive

$$\frac{dk_i(\hat{\theta}_i, \theta_i)}{d\theta_i} = \frac{\partial C_i(x_i(\theta_i), \theta_i)}{\partial \theta_i}. \tag{2.1}$$

This condition ensures local incentive compatibility. Meaning that an agent i does not gain by misrepresenting θ_i around the neighbourhood of θ_i. By itself, it does not ensure that agent i does not want to misrepresent θ_i by a large amount. Hence, reporting θ_i might be a local minimum, but not a global one. However, global incentive compatibility is guaranteed if in addition to condition (2.1), monotonicity of $x(.)$ holds.

Proposition 1. *(Myerson's Characterization Theorem) A contract $\{x(.), t(.)\}$ is globally incentive compatible iff*

1. $x_i(\theta_i, \theta_{-i})$ *is decreasing in* θ_i *(monotonicity)*
2. $k_i(\theta_i, \theta_i) = k_i(\overline{\theta_i}, \overline{\theta_i}) - \int_{\theta_i}^{\overline{\theta_i}} \frac{\partial C_i(x_i(s), s)}{\partial \theta_i} ds$

The proof of proposition 1 may be found in the appendix. As a consequence, when proposition 1 holds, it is ensured that truth-telling is a global minimum.

3 Optimal Contracts in Principal Agent Models in a River

Assume the federal government nominates one of the agents to design a set of contracts for all other agents. The agent chosen has two possibilities to offer contracts. Either he simultaneously and directly contracts with the remaining agents in a centralized manner or he starts a delegated contracting

process in which he subcontracts with his two direct neighbours who are given the authority to subcontract with their direct neighbour and so on till all agents received and accepted a contract.
One advantage of the delegated structure is that the intermediate agents can design their subcontracts while having full knowledge about their own costs. On the other hand, a potential drawback of delegated contracting is that the prime principal may experience a control loss, owing to the monopsony power granted to the other agents. Melumad et al. (1995) consider an upwards-oriented sequential contracting model with three agents. They show that the principal, in their case agent 3, can alleviate the control loss completely by constructing a sustainable transfer for the outsourcing to agent 2 so that the allocation is the same as in the centralized model. To calibrate the transfer correctly, however, the principal has to know agent 2's true costs θ_2 as this determines the magnitude of the monopsony distortion. To elicit this information, the principal offers agent 2 a contract with a continuum of contingencies, corresponding to different possible true values of θ_2. Similarly, we find that the prime principal can achieve the same abatement allocation in the delegated as in the centralized model while he is indifferent ex-ante between the centralized and delegated model.

3.1 The Centralized Principal Agent Model

Let us first have a look at the centralized principal agent model. Let P_i denote the structure in which agent i, $i = 1, ..., n$, is the principal elected by the federal government. Assume that agent i has to compensate all agents $j < i$ for their abatement efforts, and is paid by all agents $j > i$ for his abatement effort.

Full Information

Suppose that agent i has full information about the abatement costs of the other agents. Agent i's minimization problem can be written as follows

$$min_{x_1,...,x_{n-1},t_1(.),...,t_n(.)} C_i(x_i, \theta_i) + D_i(q_i(x)) + \sum_{j=1}^{i-1} t_j(x) - \sum_{j=i+1}^{n} t_j(x),$$

subject to the binding individual rationality constraints of the agents $j \neq i$, i.e.

$$C_j(x_j, \theta_j) + D_j(q_j(x)) - t_j(x) = \bar{k}_j, \text{ for } j < i$$
$$C_j(x_j, \theta_j) + D_j(q_j(x)) + t_j(x) = \bar{k}_j, \text{ for } j > i,$$

with $\bar{k}_j$ being the cost level an agent j incurs in the status quo. The individual rationality constraints must bind, because if they would not, the principal could lower his costs by decreasing $t_j(.), \forall j < i$, or by increasing $t_j(.), \forall j > i$, while still satisfying the individual rationality constraints. From the binding individual rationality constraints, the following transfer payments can be attained

$$t_j(x) = C_j(x_j, \theta_j) + D_j(q_j(x)) - \bar{k}_j, \text{ for } j < i$$
$$t_j(x) = \bar{k}_j - C_j(x_j, \theta_j) - D_j(q_j(x)), \text{ for } j > i.$$

Substituting these $t_j(.)$ into the minimization problem of agent i yields

$$min_{x_1,...,x_{n-1}} \sum_{j=1}^{n-1} C_j(x_j, \theta_j) + \sum_{i=2}^{n} D_j(q_j(x)) - \sum_{j=1, j\neq i}^{n} \bar{k}_j.$$

The solution to this minimization problem is the first best abatement allocation $x^{FB} = (x_1^{FB}, ..., x_{n-1}^{FB})$, with $x_n^{FB} = 0$.

Asymmetric Information

Let us now assume that the agents have no knowledge about the abatement costs of the other agents. Finding the optimal direct-revelation set of contracts for the principal in case of asymmetric information demands minimizing the principal's expected total costs over the set of mechanisms that induce truthful revelation of the agents' types and full participation. Thus, the principal's problem of designing the optimal contract can be stated equivalently as minimizing the principal's expected total costs subject to the local incentive compatibility constraints, the individual rationality constraints and the monotonicity constraints. Hence, the principal's minimization problem can be expressed as

$$min_{x_1,...,x_n, t_1(.),...,t_{i-1}, t_{i+1},...,t_n(.)} E_{\theta_{-i}}[C_i(x_i, \theta_i) + D_i(q_i(x)) + \sum_{j=1}^{i-1} t_j(x) - \sum_{j=i+1}^{n} t_j(x)] \tag{3.1}$$

subject to the individual rationality constraints, i.e.

$$\begin{aligned} &C_j(x_j, \theta_j) + D_j(q_j(x)) - t_j(x) \leq \bar{k}_j, \text{ for } j < i \\ &C_j(x_j, \theta_j) + D_j(q_j(x)) + t_j(x) \leq \bar{k}_j, \text{ for } j > i, \\ &D_n(q_n(x)) + t_n(x) = \bar{k}_n, \end{aligned}$$

the incentive compatibility constraints, i.e.

$$\begin{aligned} &C_j(x_j(\theta_j), \theta_j) + D_j(q_j(x(\theta_j))) - t_j(\theta_j) \leq C_j(x_j(\hat{\theta}_j), \theta_j) + D_j(q_j(x(\hat{\theta}_j))) - t_j(\hat{\theta}_j), \text{ for } j < i \\ &C_j(x_j(\theta_j), \theta_j) + D_j(q_j(x(\theta_j))) + t_j(\theta_j) \leq C_j(x_j(\hat{\theta}_j), \theta_j) + D_j(q_j(x(\hat{\theta}_j))) + t_j(\hat{\theta}_j), \text{ for } j > i, j \neq n \end{aligned}$$

and the feasibility constraints

$$\sum_{k=1}^{j} x_k \leq \sum_{k=1}^{j} e_k, \forall k.$$

The optimal set of contracts is summarized in the following proposition.

Proposition 2. *The optimal abatement levels $x_1^{P_i}, ..., x_{n-1}^{P_i}$ in the centralized model with agent i as the principal satisfy*

$$x_1^{P_i}, ..., x_{n-1}^{P_i} \in argmin_{x_1,...,x_{n-1}} C_i(x_i, \theta_i) + D_i(q_i(x)) + \sum_{j=1}^{i-1} t_j(x) - \sum_{j=i+1}^{n} t_j(x),$$

whereby

$$t_j(x,\theta_j) = C_j(x_j,\theta_j) + \frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j}\frac{F(\theta_j)}{f(\theta_j)} + D_j(q_j(x)) - \bar{k}_j, \forall j < i$$
$$t_j(x,\theta_j) = -C_j(x_j,\theta_j) - \frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j}\frac{F(\theta_j)}{f(\theta_j)} - D_j(q_j(x)) + \bar{k}_j, \forall j > i$$
$$t_n(x) = D_n(\sum_{j=1}^{n-1} e_j) - \bar{k}_n.$$

We refer to these transfers as the *standard principal agent transfers*. They reflect the relevant costs for the principal that have to be paid to the agents. In expectation, the cost level each agent $i \neq n$ incurs by accepting his contract falls short of the non-cooperative status quo by an amount equal to the agent's informational rent $r := \frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j}\frac{F(\theta_j)}{f(\theta_j)}$. The proof of proposition 2 is in the appendix.

3.2 The Delegated Principal Agent Model

Let us now introduce the delegated principal agent model. Due to the nature of the river, we consider upstream-oriented and downstream-oriented sequential contracting. Assume agent i has been selected as the prime principal. Suppose, he first subcontracts with his direct upriver neighbour agent $i-1$, who in turn subcontracts with his direct upriver neighbour agent $i-2$ and so on, till agent 1 is reached. After all upriver agents $j < i$ have accepted their contract, agent i offers agent $i+1$ a contract, who in turn is given the authority to subcontract with agent $i+2$ and so on till all agents $j > i$ have accepted the contract proposed by their upriver neighbour[2]. Similarly to the centralized model, we assume that an agent j has to compensate his direct upriver neighbour $j+1$ for his abatement effort whereas he is paid by his direct downriver neighbour $i+1$ for his abatement effort.

Full Information

Assume first, that types are common knowledge. By backwards induction, agent i, knows that all other agents $j \neq i$ will set their direct up or downriver contracting partner indifferent between accepting their contracts of the non-cooperative outcome. In addition, to maximize his cooperation gain, agent i is confronted with the following minimization problem

$$min_{x_1,\ldots,x_n} \sum_{j=1}^{n} C_j(x_j) + D_j(q_j(x)) - \sum_{j=1,j\neq i}^{n} \bar{k}_j.$$

The abatement allocation minimizing agent i's costs corresponds to the first best optimal abatement allocation x^{FB}. However, in contrast to the centralized model, agent i cannot choose all abatement levels simultaneously to fully incorporate the externalities. Thus, agent i has to construct the two contracts to his direct upriver neighbour $i-1$ and direct downriver neighbour $i+1$ in such a way that the first best allocation is sequentially implemented by all agents. We claim that agent i is able to skim off the maximal cooperation gain by inducing the following contracting process: He first offers

[2]Note that delegating downriver first and upriver afterwards will lead to the same results.

agent $i-1$ a transfer t_{i-1} contingent on the abatement levels of the upriver agents and lets agent $i-1$ decide on the abatement level x_{n-1}. After accepting the transfer payment, agent $i-1$ will offer a transfer to agent $i-2$ and so on. We claim that if agent i offers agent $i-1$ a transfer of

$$\begin{aligned} t_{i-1}(x) = &\sum_{i=1}^{n-1} C_i(x_i^{FB}, \theta_i) + \sum_{i=1}^{n} D_i(q_i(x^{FB})) - \sum_{j=i}^{n} D_j(q_j(x_1, ..., x_i, x_{i+1}^{FB}, ..., x_{n-1}^{FB})) \\ &- \sum_{j=i}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=1}^{i-1} \overline{k}_j, \forall i \end{aligned}$$

all upriver agents $j < i$ will sequentially choose first best optimal abatement levels. After all upriver agents $j < i$ have accepted their contracts, agent i offers a contract consisting of x_i^{FB} and t_i to agent $i+1$. We claim that if agent i offers agent $i+1$ a transfer of

$$t_i(x) = \sum_{j=i+1}^{n} \overline{k}_j - \sum_{j=i+1}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=i+1}^{n} D_j(q_j(x^{FB})),$$

all downriver agents $j > i$ will subsequently choose the first best optimal abatement levels x_j^{FB}. In the end, all agents $j \neq i$ will end up with their reservation costs $\overline{k}_j$ whereas the full cooperation gain goes to agent i.

Proposition 3. *Given full information, agent i offers agent* $i-1$ *a contract consisting of the transfer*

$$\begin{aligned} t_{i-1}(x) = &\sum_{i=1}^{n-1} C_i(x_i^{FB}, \theta_i) + \sum_{i=1}^{n} D_i(q_i(x^{FB})) - \sum_{j=i}^{n} D_j(q_j(x_1, ..., x_i, x_{i+1}^{FB}, ..., x_{n-1}^{FB})) \\ &- \sum_{j=i}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=1}^{i-1} \overline{k}_j, \forall i \end{aligned} \tag{3.2}$$

and offers agent $i+1$ *a contract consisting of an abatement level of* x_i^{FB} *and a transfer of*

$$t_i(x) = \sum_{j=i+1}^{n} \overline{k}_j - \sum_{j=i+1}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=i+1}^{n} D_j(q_j(x^{FB})). \tag{3.3}$$

These transfers together with the first best optimal abatement allocation x^{FB} *constitute a Nash-equilibrium of the delegated contracting model.*

The proof of proposition 3 may be found in the appendix.

Clearly, the nominated principal can achieve the same abatement allocation in the centralized and delegated principal agent model. The transfers to be paid, however, differ in the two models. Nonetheless, the principal is equally well off in both models in that he attains the maximal cooperation gain possible.

Proposition 4. *Agent i, the principal, is indifferent between the delegated and centralized model.*

This proposition follows by construction.

Asymmetric Information
Suppose now, that the agents cannot observe the types of the other agents. Suppose the contracting goes as follows: the principal, agent i, first offers agent $i-1$ a contract consisting of a transfer $t_{i-1}(x_1, ..., x_{i-1}, \hat{\theta}_{i-1})$ contingent on the abatement levels of all upriver agents $j < i$ which agent $i-1$ accepts by reporting $\hat{\theta}_{i-1}$. Agent $i-1$ then chooses x_{i-1} and offers a contract to agent $i-2$, who in turn reports $\hat{\theta}_{i-2}$ and selects x_{i-2} optimally. This contracting process is continued till agent 1 reports his type $\hat{\theta}_1$ to agent 2 in exchange for a transfer $t_1(x_1, \hat{\theta}_1)$ and finally chooses to abate x_1. Then, agent i proposes a contract consisting of an abatement level $x_i(\hat{\theta}_{i+1})$ and transfer $t_i(\hat{\theta}_{i+1})$ to agent $i+1$, who accepts by reporting $\hat{\theta}_{i+1}$, followed by agent $i+1$ proposing an abatement level $x_{i+1}(\hat{\theta}_{i+2})$ and a transfer $t_{i+1}(\hat{\theta}_{i+2})$ to agent $i+2$, who in turn reports $\hat{\theta}_{i+2}$. This contracting process is continued till agent n accepts the contract.

The resulting sequential optimization problem can be represented as follows. Let us first consider the upstream-oriented contracting part of the game. In the last step of the upstream contracting, agent 1 receives $t_1(x_1, \hat{\theta}_1)$ and chooses x_1. In the second last step, agent 2 subcontracts with agent 1, given the transfer $t_2(x_1, x_2, \hat{\theta}_1, \hat{\theta}_2)$ and his report $\hat{\theta}_2$. Agent 2 then faces the following minimization problem

$$min_{x_1,x_2,t_1} E_{\theta_1}[C_2(x_2, \theta_2) + D_2(q_2(x)) + t_1(x_1, \theta_1) - t_2(x_1, x_2, \hat{\theta}_1, \hat{\theta}_2)] := min_{x_1,x_2} M_2(x, \hat{\theta}_2, \theta_2)$$

subject to the incentive compatibility constraint of agent 1

$$\theta_1 \in argmin_{\hat{\theta}_1} C_1(x_1(\hat{\theta}_1), \theta_1) - t_1(x_1, \hat{\theta}_1)$$

and the individual rationality constraint of agent 1

$$C_1(x_1(\theta_1), \theta_1) - t_1(x_1, \theta_1) \leq \bar{k}_1.$$

In the previous step, agent 3 subcontracts with agent 2, given the transfer $t_3(x_1, x_2, x_3, \hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)$ and solves

$$\begin{aligned} &min_{x_1,x_2,x_3,t_2} E_{\theta_1,\theta_2}[C_3(x_3, \theta_3) + D_3(q_3(x)) + t_2(x_1, x_2, \hat{\theta}_1, \hat{\theta}_2) - t_3(x_1, x_2, x_3, \hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)] \\ &:= min_{x_1,x_2,x_3} M_3(x, \hat{\theta}_3, \theta_3), \end{aligned}$$

subject to the incentive compatibility constraint of agent 2

$$\theta_2 \in argmin_{\hat{\theta}_2} M_2(x, \hat{\theta}_2, \theta_2),$$

the individual rationality constraint of agent 2

$$J_2(x, \hat{\theta}_2, \theta_2) \leq \bar{k}_2$$

and subject to

$$x_1, x_2 \in argmin_{x_1,x_2} M_2(x, \hat{\theta}_2, \theta_2).$$

This process is continued till agent i is reached. Next, consider the downstream-oriented contracting part of the game. Let $\tilde{x}_j = \sum_{k=1}^{j-1} x_k$, i.e. the joint abatement effort chosen by the agents $k < j$. Consider the second last step of the contracting, in which agent $n-2$ subcontracts with agent $n-1$, given all upriver agents j's reports $\hat{\theta}_j$, the transfer $t_{n-2}(\tilde{x}_{n-2}, \hat{\theta}_{n-1})$ and the abatement levels selected upriver. Agent $n-1$ will offer agent n a contract composed of an abatement level x_{n-1} satisfying

$$\begin{aligned} &min_{x_{n-1}} C_{n-1}(x_{n-1}, \theta_{n-1}) + D_{n-1}(q_{n-1}(\tilde{x}_{n-1})) + t_{n-2}(\tilde{x}_{n-2}, \hat{\theta}_{n-1}) - \bar{k}_n + D_n(q_n(\tilde{x}_{n-1}, x_{n-1})) \\ &:= min_{x_{n-1}} M_{n-1}(x, \hat{\theta}_{n-1}, \theta_{n-1}) \end{aligned}$$

and a transfer setting him indifferent between accepting and not. Moving one step upriver, agent $n-2$ subcontracts with agent $n-1$, given the transfer $t_{n-3}(\tilde{x}_{n-3}, \hat{\theta}_{n-2})$ as well as all upriver agents i's reports $\hat{\theta}_i, i = 1, ..., n-2$ and the abatement levels selected upriver. Agent $n-2$ thus faces the following minimization problem

$$\begin{aligned} &min_{x_{n-1}, x_{n-2}, t_{n-2}} E_{\theta_{n-1}}[C_{n-2}(x_{n-2}, \theta_{n-2}) + D_{n-2}(q_{n-2}(\tilde{x}_{n-2})) + t_{n-3}(\tilde{x}_{n-3}, \hat{\theta}_{n-2}) - t_{n-2}(x)] \\ &:= min_{x_{n-2}, x_{n-1}} M_{n-2}(x, \hat{\theta}_{n-2}, \theta_{n-2}), \end{aligned}$$

so that agent $n-1$ reports truthfully, i.e.

$$\begin{aligned} \theta_{n-1} \in argmin_{\hat{\theta}_{n-1}} & C_{n-1}(x_{n-1}(\hat{\theta}_{n-1}), \theta_{n-1}) + D_{n-1}(q_{n-1}(\tilde{x}_{n-1}(\hat{\theta}_{n-1}))) \\ & + t_{n-2}(\tilde{x}_{n-2}(\hat{\theta}_{n-1})) - t_{n-1}(\tilde{x}_{n-1}(\hat{\theta}_{n-1})), \end{aligned}$$

the individual rationality constraint of agent $n-1$ is fulfilled, i.e.

$$C_{n-1}(x_{n-1}, \theta_{n-1}) + D_{n-1}(q_n(\tilde{x})) + t_{n-2}(\tilde{x}_{n-2}) - t_{n-1}(\tilde{x}_{n-1}) \leq \bar{k}_{n-1}$$

and

$$x_{n-1} \in argmin M_{n-1}(x, \hat{\theta}_{n-1}, \theta_{n-1}).$$

Continue this process till agent i is reached. This implies the following minimization problem for agent i

$$\begin{aligned} &min_{x_1,...,x_{n-1},t_{i-1},t_i} E_{\theta_{-i}}[C_i(x_i, \theta_i) + D_i(q_i(x)) + t_{i-1}(x, \hat{\theta}_1, ..., \hat{\theta}_{i-1}) - t_i(x, \hat{\theta}_{i+1}, ..., \hat{\theta}_{n-1})] \\ &:= min_{x_1,...,x_{n-1}} M_i(x) \end{aligned}$$

subject to the incentive compatibility constraint of both agents $i-1, i+1$, i.e.

$$\theta_j \in argmin_{\hat{\theta}_j} M_j(x, \hat{\theta}_j, \theta_j), j = i+1, i-1,$$

the individual rationality constraint of both agents $i-1$,$i+1$, i.e.

$$M_j(x,\hat{\theta}_j,\theta_j) \leq \bar{k}_j, j = i+1, i-1$$

and subject to

$$x_1,...,x_{i-1} \in argmin_{x_1,...,x_{i-1}} M_{i-1}(x,\hat{\theta}_{i-1},\theta_{i-1}), \tag{3.4}$$

$$x_{i+1},...,x_{n-1} \in argmin_{x_{i+1},...,x_{n-1}} M_{i+1}(x,\hat{\theta}_{i+1},\theta_{i+1}). \tag{3.5}$$

The last two constraints (3.4),(3.5) reflect that the desired abatement levels must coincide with those that agent $i+1$ respectively agent $i-1$ will choose when they subcontract with their corresponding contracting partners $i+2$ respectively $i-2$.

For the class of contracts we consider, the revelation principle implies that a centralized arrangement weakly dominates all other arrangements. The relevant question is thus whether the principal can design a set of contracts that match his expected costs in the centralized model. We will show that by choosing appropriate transfers, agent i can achieve the same abatement allocation as in the centralized principal agent model and this is the best he can do.

Proposition 5. *Agent i, the principal, can achieve the same abatement allocation x^{P_i} in the delegated model as in the centralized model.*

The proof of proposition 5 is in the appendix and is based on the following idea. To achieve the same outcome in the delegated model as in the centralized principal agent model, agent i has to counteract the tendency of the other agents $j \neq i$ to bias the abatement levels in their favour. Thus, the prime principal i has to shift the preferences of both agent $i-1$ and $i+1$ in such a way that they fully internalize the prime principals objective. We claim that he achieves this by adopting the following incentive scheme: He first offers agent agent $i-1$ a transfer of

$$\begin{aligned} t_{i-1}(x) =& \omega(\hat{\theta}_{i-1}) - \sum_{j=i}^{n} D_j(q_j(x_1,...,x_i,x_{i+1}^{P_i},...,x_{n-1}^{P_i})) - \sum_{j=i}^{n-1} C_j(x_j^{P_i},\hat{\theta}_j) \\ &- \sum_{j=i}^{n-1} \frac{\partial C_j(x_j^{P_i},\hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} - \frac{\partial C_{i-1}(x_{i-1},\hat{\theta}_{i-1})}{\partial \hat{\theta}_{i-1}} \frac{F(\hat{\theta}_{i-1})}{f(\hat{\theta}_{i-1})} \end{aligned}$$

with

$$\begin{aligned} \omega(\hat{\theta}_{i-1}) = & \sum_{k=1}^{n-1} C_k(x_k^{P_i},\theta_k) + \sum_{k=2}^{n} D_k(q_k(x^{P_i})) + \sum_{k=i-1}^{n-1} \frac{\partial C_k(x_k^{P_i},\hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \sum_{l=1}^{i-1} \bar{k}_l \\ &+ \sum_{k=1}^{i-1} \int_{\hat{\theta}_k}^{\bar{\theta}_k} \frac{\partial C_k(x_k(\theta_{-k},s,s,),s)}{\partial \hat{\theta}_k} ds. \end{aligned}$$

After all upriver agents have accepted their respective offer, agent i offers agent $i+1$ an abatement level of $x_i^{P_i}$ and a transfer of

$$t_i(x) = \sum_{j=1}^{i-1} \frac{\partial C_j(x_j^{P_i}, \hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} + \frac{\partial C_{i+1}(x_{i+1}, \hat{\theta}_{i+1})}{\partial \hat{\theta}_{i+1}} \frac{F(\hat{\theta}_{i+1})}{f(\hat{\theta}_{i+1})} - \varphi(\hat{\theta}_{i+1}).$$

with

$$\varphi(\hat{\theta}_{i+1}) = \sum_{k=i+1}^{n-1} C_k(x_k^{P_i}, \theta_k) + \sum_{k=i+1}^{n} D_k(q_k(x^{P_i})) + \sum_{k=1,k\neq i}^{i+1} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)}$$
$$+ \sum_{k=i+1}^{n-1} \int_{\hat{\theta}_k}^{\overline{\theta}_k} \frac{\partial C_k(x_k(\theta_m, s, s), s)}{\partial \hat{\theta}_k} ds - \sum_{l=i+1}^{n} \overline{k}_l.$$

The functions $\varphi(.), \omega(.)$ in the transfers above are chosen so as to ensure full participation and truthful type revelation of all agents $j \neq i$. Since under any incentive-compatible scheme, the expected payments to either agent $i-2$ and $i+2$ must equal their standard principal agent transfers, the transfers $t_{i-1}(x), t_i(x)$ given above ensure, that both agents $i+1, i-1$ internalize the principal's objective. In the proof of proposition 5, we show that with the functions $\omega(.), \varphi(.)$ given above each agent j will indeed report truthfully at each stage and his expected profit will be equal to his reservation costs at $\theta_j = \overline{\theta}_j$, in other words his individual rationality constraint will be binding for the highest type. The claim then follows from the revenue equivalence theorem.

As established, a nominated principal can achieve the same abatement allocation in the delegated as in the centralized model. However, the transfer scheme offered differs in the two models and thus, total costs incurred by the nominated principal may vary in the two models. We show that the expected total costs of the nominated principal fall together in both models and thus, ex-ante, the principal is indifferent between choosing the delegated or centralized contracting method.

Proposition 6. *Agent i is indifferent ex-ante between the delegated and centralized model.*

The proof of proposition 6 may be found in the appendix.

4 Choice of the Principal

As a last point, we address the question which agent the federal government should nominate as a principal. In case of full information, all potential principals implement the first best optimal allocation. Thus, the source at which the contracts originate just determines who gets the cooperation gain. Instead of nominating one of the agents to propose contracts, the federal government could for example propose a set of contracts according to the *downstream incremental cost distribution* as proposed by Winkler and Steinmann (2015). This cost distribution is in the core and additionally satisfied a specific fairness criterion. In case of asymmetric information, the federal government may benefit from letting one of the agents propose a set of contracts. Even though the elected principal has a different objective than the federal government, nominating one of the agents to be the principal might still be beneficial for the government because this principal has full knowledge about his own abatement costs. In other words, the nominated principal has one less informational constraint to

consider while offering contracts. Furthermore, as established in proposition 5, a nominated principal implements the same abatement allocation whether he chooses a centralized or a delegated principal agent model. As evident from the minimization problem of the principal (3.1), however, the abatement allocation implemented differs across the prime principals. Thus, clearly, the government should nominate the agent who implements the abatement allocation leading to least costs for the river basin. But who is the best principal?
First and not surprisingly, all potential principals will lead to higher total costs for the river basin than first best optimal as all abatement allocations chosen have in common that there is less individual and thus less total abatement done than first best optimal.

Proposition 7. *The abatement levels under asymmetric information are weakly smaller than under full information*

$$x_j^{FB} \geq x_j^{P_i}, \forall i, j.$$

The proof of proposition 7 is in the appendix. There is efficiency at the top, i.e. the highest type agents abate pollution in the first best optimal amount. This is because for $\theta_i = \overline{\theta}_i$, each principal's problem in the asymmetric information case is equivalent to full information case. For all other types, the principals will distort the abatement levels to reduce the information rents.
Second, we find that agent n should never be the one proposing contracts for the agents along the river as his best set of contracts leads to higher total expected costs in the river basin as with any agent $i \neq n$ acting as a principal. Intuitively, by electing any agent $i \neq n$ as a principal, one uncertainty drops as agent i knows his own type and only $n-2$ information rents have to be paid, whereas with agent n proposing contracts, there are still $n-1$ unknown types and information rents. To show this formally, let us establish the condition under which an agent i is a better prime principal than an agent k.

Proposition 8. *Let $E[K^{P_i}]$ be total expected costs in the river basin with agent i being the principal, $i = 1, ..., n$. Then, for any two principals i,k*

$$E[K^{P_i}] \leq E[K^{P_k}], \forall i, k, i \neq k$$

if

$$E_\theta[\frac{\partial C_k(x_k^{P_i}, \theta_k)}{\partial \theta_k} - \frac{\partial C_i(x_i^{P_k}, \theta_i)}{\partial \theta_i}] \leq E_\theta[\frac{\partial^2 C_i(x_i^{P_k}, \theta_i)}{\partial \theta_i \partial x_i}(x_i^{P_i} - x_i^{P_k})].$$

The proof of proposition 8 may be found in the appendix. From proposition 8, we deduct, that choosing any agent $i, i \neq n$ as principal leads to lower total expected costs than selecting agent n and there is less total abatement in the river basin with agent n being the principal.

Corollary 1. Let $E[K^{P_i}]$ be total expected costs in the river basin with agent i being the principal, $i = 1, ..., n$. It holds that

$$E[K^{P_i}] \leq E[K^{P_n}], \forall i, i \neq n.$$

and

$$E_\theta[\sum_{j=1}^{n-1} x_j^{P_i}] \geq E_\theta[\sum_{j=1}^{n-1} x_j^{P_n}], \forall i, i \neq n$$

The proof of corollary 1 may be found in the appendix. According to the constrained efficiency theorem, the federal government is not able to achieve a better outcome for the river basin as agent n because it is subject to the same informational constraints.

According to proposition 8, the position of a potential principal along the river as well as the exogenously given pollution levels and the damage cost parameters play an important role in the abatement allocation chosen and thus in determining who of the principals is the best one. We establish, that with linear damages, it is best to assign the power to the agent at the source of the river.

Proposition 9. *Assume, the damage functions are of the form $D_i(q_i) = \alpha_i + \beta_i q_i$. Then, the federal government should nominate agent 1 to be the principal.*

The proof of proposition 9 may be found in the appendix. With linear damages, the abatement levels chosen are a function of the constant marginal damage costs. Because of this and due to his position, agent 1's expected abatement level is the highest, no information rent has to be paid for it and it has the most influence as it benefits all and not just a subgroup of agents. Thus, intuitively, total expected costs are smallest with agent 1 as the principal.

For any other functional form of the damage costs, there is no clear cut answer to who should be nominated as principal. This is illustrated by an example of a river shared by three agents with quadratic damage and abatement costs.

Example 1. Consider a river shared by three agents. Assume, the abatement costs are of the form $C_i(x_i) = \frac{1}{2}\theta_i x_i^2, i = 1, 2$ and the damage costs of the form $D_i(q_i(x)) = 1/2\beta_i q_i(x)^2, i = 2, 3$. As established in proposition 1, the agent most downriver, agent 3, should not be chosen to be the principal. Let us thus consider agents 1,2 as principals in dependence of the pollution levels e_1, e_2. Furthermore, set $e_1 = \alpha e_2$. As shown in the appendix, there exist values of α for which either agent 1 or agent 2 is the better principal.

Lemma 1. There exist $\alpha \in (0, \alpha^{id})$ with $\alpha^{id} < 1$ for which

$$E[K^{P_1}(\alpha)] > E[K^{P_2}(\alpha)],$$

and for all $\alpha > \alpha^{id}$,

$$E[K^{P_1}(\alpha)] < E[K^{P_2}(\alpha)].$$

5 Conclusion

In this chapter, we analyse contractual mitigation of a global public bad along a river in the presence of a federalist governance structure, where the lower tiers have private information about their abatement costs. In our model, one of the lower tiers is nominated by the federal government to offer a set of mitigation contracts to the remaining agents. The selected principal either proposes contracts in a centralized manner, where he simultaneously offers contracts to all agents, or he contracts only with his direct up- and downstream neighbour and delegates to those two agents the authority to contract with their direct up- respectively downriver neighbours, and so on, until all agents received a contract. We establish that given full information, both models implement the first best optimal allocation and the choice of the principal only determines which agent attains the full cooperation gain. In case of asymmetric information, we demonstrate that even though the delegated principal agent model is prone to a control loss, the abatement allocation implemented in the centralized principal agent model can be replicated in the delegated principal agent model while matching the expected costs of the principal. However, to counteract and to avoid the tendency of the intermediate agents to bias the abatement allocation in their favour, the nominated principal must be able to monitor the abatement levels as well as the reports of the intermediate agents and it must be ensured that the set of contracts is executed after all intermediate agents have accepted their subcontracts. As all potential principals implement a different abatement allocation, the choice of the prime principal matters for the total expected costs occurring in the river basin. We show, that the tier located most downriver, which is subject to the same informational constraints as the federal government, is never the best choice to nominate as a principal. In case of linear damages, we establish that choosing the agent at the source of the river will implement the abatement allocation leading to least expected total costs for the river basin. For other functional forms of damage and abatement costs, the nomination of the best principal depends not only on the position of the potential principal along the river but also on the damage cost parameters as well as the exogenously given pollution levels.

This chapter can be extended in a number of directions, for example, to flood protection. In this case, the exogenously given pollution levels correspond to the water discharges from one tier to another, the abatement levels match the amount of water a tier withdraws from the stream by controlled flooding of designated flooding areas and the pollution flow is the amount of excess water in a tier's area. These modifications would not impact on the validity of any propositions made in this chapter. A more challenging extension of our work would be to allow for the formation of sub-coalitions, meaning that agents may propose a set of contracts to a sub-group of agents. By incorporating this cooperative behaviour among the agents, more complex reservation costs respectively participation constraints would have to be considered. This would also be the approach to follow if sovereign countries were considered, where there is no federal government giving the power to offer contracts to one particular agent with the threat that the status quo were to prevail if one agent refuses the contract.

Our work suggests several other avenues for future research. Our model relies on perfect monitoring of both the communication among the agents as well as the abatement levels chosen by each agent. A realistic limitation in monitoring could be that an intermediate agent is confined to observing only the joint abatement effort of his upriver agents rather than each individual abatement level or/and may not be able to monitor the communication in regard to the type reports between the other agents. Thus, analysing the consequences of such reductions in the amount of monitoring between the agents might yield interesting insights. Additionally, making the exogenously given pollution levels private information, the adverse selection problem would be enriched by a moral hazard component. In this case, an intermediate agent can only observe the pollution flow at his location but cannot deduct the abatement effort done by the upriver agent. Finally, considering a setting where the asymmetric information involves two dimensions, the abatement costs and the damage costs, would allow to study the problems of multidimensional incentive design with type dependent reservation costs. This may induce countervailing incentives in the sense that an agent may want to overstate its abatement and damage costs to obtain a higher compensation and at the same time to understate his type to pretend a better outside option.

Appendix

Proof. of proposition 1

Assume, a contract is incentive compatible. Let us show (i) and (ii).

Suppose, $\hat{\theta}_i > \theta_i$ and θ_{-i} constant. From incentive compatibility, we have

$$k_i(\hat{\theta}_i, \theta_i) \geq k_i(\theta_i, \theta_i).$$

By definition

$$\theta_i c_i(x(\hat{\theta}_i)) - t_i(\hat{\theta}_i) \geq \theta_i c_i(x(\theta_i)) - t_i(\theta_i).$$

Rearranging implies

$$k_i(\hat{\theta}_i, \hat{\theta}_i) - k_i(\theta_i, \theta_i) \quad \geq \quad (\hat{\theta}_i - \theta_i) c_i(x_i(\hat{\theta}_i, \theta_{-i})). \tag{5.1}$$

The same must hold for $\hat{\theta}_i$, i.e. from

$$k_i(\theta_i, \hat{\theta}_i) \geq k_i(\hat{\theta}_i, \hat{\theta}_i),$$

we attain

$$\hat{\theta}_i c_i(x(\theta_i)) - t_i(\theta_i) \geq \hat{\theta}_i c_i(x(\hat{\theta}_i)) - t_i(\hat{\theta}_i)$$

and thus

$$k_i(\theta_i, \theta_i) - k_i(\hat{\theta}_i, \hat{\theta}_i) \geq (\theta_i - \hat{\theta}_i) c_i(x_i(\theta_i)). \tag{5.2}$$

Combining inequalities (5.1) and (5.2) yields

$$(\hat{\theta}_i - \theta_i) c_i(x_i(\hat{\theta}_i)) \leq k_i(\hat{\theta}_i, \hat{\theta}_i) - k_i(\theta_i, \theta_i) \leq (\hat{\theta}_i - \theta_i) c_i(x_i(\theta_i)).$$

The above inequality can be rewritten to

$$\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(\hat{\theta}_i), s)}{\partial \theta_i} ds \leq k_i(\hat{\theta}_i, \hat{\theta}_i) - k_i(\theta_i, \theta_i) \leq \int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(\theta_i), s)}{\partial \theta_i} ds. \tag{5.3}$$

Ignoring the middle term, this implies that

$$\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(\theta_i), s)}{\partial \theta_i} ds - \int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(\hat{\theta}_i), s)}{\partial \theta_i} ds \geq 0,$$

which is identical to

$$\int_{\theta_i}^{\hat{\theta}_i} \int_{x_i(\hat{\theta}_i)}^{x_i(\theta_i)} \frac{\partial^2 C_i(z, s)}{\partial x_i \partial \theta_i} dz ds \geq 0.$$

By assumption $\frac{\partial^2 C_i(x_i,\theta_i)}{\partial x_i \partial \theta_i} \geq 0$, making the integrand positive. For the integral itself to be positive, we need

$$(\hat{\theta}_i - \theta_i)(x_i(\theta_i) - x_i(\hat{\theta}_i)) \geq 0.$$

By assumption $\hat{\theta}_i > \theta_i$, thus the above inequality is satisfied whenever $x_i(\theta_i) \geq x_i(\hat{\theta}_i)$. Thus, for incentive compatibility to hold, it is required that $x_i(\theta_i)$ is non-increasing in θ_i. Note that this implies that $x(.)$ is continuous almost everywhere. Moreover, by fixing one end point in (5.3) and letting the other converge towards it, we see that $k(\theta_i, \theta_i)$ must be continuous and thus almost everywhere differentiable. The derivative of $k(\theta_i, \theta_i)$ amounts to

$$k_i'(\theta_i, \theta_i) = \frac{\partial C_i(x_i, \theta_i)}{\partial \theta_i}.$$

Consequently, we attain (ii). The proof is similar for $\hat{\theta}_i < \theta_i$.

Next, we assume (i),(ii) and show incentive compatibility. Suppose $\hat{\theta}_i > \theta_i$. For incentive compatibility to hold, we need

$$k_i(\hat{\theta}_i, \hat{\theta}_i) \leq \hat{\theta}_i c_i(x_i(\theta_i)) - t_i(\theta_i) = k_i(\theta_i, \theta_i) + (\hat{\theta}_i - \theta_i) c_i(x_i(\theta_i)).$$

Thus, we need to check whether

$$k_i(\hat{\theta}_i, \hat{\theta}_i) - k_i(\theta_i, \theta_i) = \int_{\theta_i}^{\hat{\theta}_i} \frac{C_i(x_i(s), s)}{\partial \theta_i} ds \leq (\hat{\theta}_i - \theta_i) c_i(x_i(\theta_i)).$$

The right hand side of the above inequality can be rewritten to

$$(\hat{\theta}_i - \theta_i) c_i(x_i(\theta_i)) = \int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(\theta_i), s)}{\partial \theta_i} ds.$$

Hence, the inequality we have to check is equivalent to

$$\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i(s), s)}{\partial \theta_i} - \frac{\partial C_i(x_i(\theta_i), s)}{\partial \theta_i} ds \leq 0.$$

Given our functional forms, this is equivalent to

$$\hat{\theta}_i c_i(x_i(\hat{\theta}_i)) - \hat{\theta}_i c_i(x_i(\theta_i)) \leq 0.$$

This holds true as we assumed $\hat{\theta}_i > \theta_i$ and $x_i(\theta_i)$ decreasing in θ_i.

Next, let us prove that we have global incentive compatibility. As $C_i(x_i, \theta_i)$ satisfies the single crossing property and is continuously differentiable, $C_i(x_i, \theta_i)$ satisfies increasing differences. Increasing differences imply, that if $\hat{x} > x$ and $\hat{\theta} > \theta$,

$$\begin{aligned} \hat{\theta}_i c_i(\hat{x}_i) - t(\hat{\theta}_i) - (\hat{\theta}_i c_i(x_i) - t(\hat{\theta}_i)) &\geq \theta_i c_i(\hat{x}_i) - t(\theta_i) - (\theta_i c_i(x_i) - t(\theta_i)) \\ c_i(\hat{x}_i) &\geq c_i(x_i). \end{aligned}$$

Say that we have local incentive compatibility and monotonicity. Let $\hat{\theta}_i > \theta_i$, then by $\frac{\partial C_i(x_i,\theta_i)}{\partial x_i} \geq 0$ and $\frac{\partial C_i(x_i,\theta_i)}{\partial \theta_i} \geq 0$, we have

$$\frac{\partial C_i(x_i(\hat{\theta}_i),\theta_i)}{\partial x_i(\hat{\theta}_i)} \leq \frac{\partial C_i(x_i(\hat{\theta}_i),\hat{\theta}_i)}{\partial x_i(\hat{\theta}_i)}.$$

Then from

$$\frac{\partial}{\partial \hat{\theta}_i} k_i(\hat{\theta}_i,\theta_i) = \frac{\partial C_i(x_i(\hat{\theta}_i),\theta_i)}{\partial x_i(\hat{\theta}_i)} \cdot \frac{\partial x_i(\hat{\theta}_i)}{\partial \hat{\theta}_i} - \frac{\partial t_i(\hat{\theta}_i)}{\partial \hat{\theta}_i},$$

we get

$$\frac{\partial}{\partial \hat{\theta}_i} k_i(\hat{\theta}_i,\theta_i) \geq \frac{\partial C_i(x_i(\hat{\theta}_i),\hat{\theta}_i)}{\partial x_i(\hat{\theta}_i)} \cdot \frac{\partial x_i(\hat{\theta}_i)}{\partial \hat{\theta}_i} - \frac{\partial t_i(\hat{\theta}_i)}{\partial \hat{\theta}_i} = \frac{\partial}{\partial \hat{\theta}_i} k_i(\hat{\theta}_i,\hat{\theta}_i) = 0$$

Hence, $k_i(\hat{\theta}_i,\theta_i)$ is increasing in $\hat{\theta}_i$ as we assumed $\hat{\theta}_i > \theta_i$. Equivalently, if we assume $\hat{\theta}_i < \theta_i$, we receive $\frac{\partial}{\partial \hat{\theta}_i} k_i(\hat{\theta}_i,\theta_i) \leq 0$. Indicating that θ_i is a global minimum. □

Proof. of proposition 2

According to proposition 1, a contract is globally incentive compatible iff for all agents j,

$$\frac{dx_j^{P_i}}{d\theta_j} \leq 0, \forall j, j \neq i, n$$

and

$$k_j(\theta_j,\theta_j) = k_j(\overline{\theta}_j,\overline{\theta}_j) - \int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s,\theta_{-j}),s)}{\partial \theta_j} ds. \tag{5.4}$$

Note, that if equation (5.4) holds, then all individual rationality constraints will be satisfied iff $k_j(\overline{\theta_j},\overline{\theta_j}) \leq \overline{k}_j$. As a result, one can replace all individual rationality constraints with the one of the highest type, which has to be binding, i.e.

$$k_j(\overline{\theta}_j,\overline{\theta}_j) = \overline{k}_j.$$

From equation (5.4), we attain

$$\begin{aligned} t_j(x) &= \int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),s)}{\partial \theta_j} ds - \overline{k}_j + C_j(x_j,\theta_j) + D_j(q_j(x)), \forall j < i, \\ t_j(x) &= \overline{k}_j - \int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),s)}{\partial \theta_j} ds - C_j(x_j,\theta_j) - D_j(q_j(x)), \forall j > i \\ t_n(x) &= \overline{k}_n - D_n(q_n(x)). \end{aligned}$$

Hence, substituting these transfers into the principals objective function implies

$$min_{x_1,...,x_n} C_i(x_i,\theta_i) + D_i(q_i(x)) + \sum_{j=1,j\neq i}^{i-1} \int_{\underline{\theta}_j}^{\overline{\theta}_j} [\int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),s)}{\partial \theta_j} ds - \overline{k}_j + C_j(x_j,\theta_j) + D_j(q_j(x))] f(\theta_j) d\theta_j$$
$$- \overline{k}_n + D_n(q_n(x))$$

subject to

$$\frac{dx_j^{P_i}}{d\theta_j} \leq 0, \forall j \neq i.$$

Let us first ignore the monotonicity constraint and solve the resulting relaxed problem. Integrating by parts yields

$$\begin{aligned} \int_{\underline{\theta}_j}^{\overline{\theta}_j} \int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),s)}{\partial \theta_j} ds f(\theta_j) d\theta_j &= [\int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),s)}{\partial \theta_j} ds F(\theta_j)]_{\underline{\theta}_j}^{\overline{\theta}_j} + \int_{\underline{\theta}_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(s),\theta_j)}{\partial \theta_j} F(\theta_j) d\theta_j \\ &= \int_{\underline{\theta}_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(\theta_j),\theta_j)}{\partial \theta_j} F(\theta_j) d\theta_j. \end{aligned}$$

Thus, the principals minimization problem can be restated to

$$\begin{aligned} & min_{x_1,...,x_n} C_i(x_i,\theta_i) + D_i(q_i(x)) - \sum_{j=1,j\neq i}^{n} \overline{k}_j + D_n(q_n(x)) \\ & + \sum_{j=1,j\neq i}^{n-1} \int_{\underline{\theta}_j}^{\overline{\theta}_j} [\frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)} + C_j(x_j,\theta_j) + D_j(q_j(x))] f(\theta_j) d\theta_j. \end{aligned} \tag{5.5}$$

It remains to be checked that the solution $x_1^{P_i}, ..., x_{n-1}^{P_i}$ to the above minimization problem is indeed globally incentive compatible, i.e. whether $x_j^{P_i}$ is decreasing in $\theta_j, \forall j$. By using the implicit function theorem, we attain

$$\frac{dx_j^{P_i}}{d\theta_j} = -\frac{\frac{\partial FOC_j}{\partial \theta_j}}{\frac{\partial FOC_j}{\partial x_j}} = -\frac{\frac{\partial^2 C_j(x_j,\theta_j)}{\partial x_j \partial \theta_j} + \frac{\partial^3 C_j(x_j,\theta_j)}{\partial x_j \partial^2 \theta_j}\frac{F(\theta_j)}{f(\theta_j)} + \frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j}\frac{\partial F(\theta_j)}{f(\theta_j)} \partial \theta_j}{SOC_j} \leq 0,$$

with FOC_j being the first and SOC_j being the second order condition of the above minimization problem (5.5), whereby $SOC_j \geq 0$ as x_j is a minimum. Hence, the set of contracts is globally incentive compatible. □

Proof. of proposition 3

Let us first have a look at the upriver part of the contracting process, starting with agent i proposing a contract to agent $i-1$. With the transfer t_{i-1} given in (3.2) from agent i to agent $i-1$, agent $i-1$ is confronted with the following minimization problem

$$
\begin{aligned}
min_{x_1,...,x_{i-1},t_{i-2}(.)} C_{i-1}(x_{i-1},\theta_{i-1}) + D_{i-1}(q_{i-1}(x)) + \sum_{j=i}^{n-1} C_j(x_j^{FB}) \\
+ \sum_{j=i}^{n} D_j(q_j(x_1,...,x_{i-1},x_i^{FB},...,x_{n-1}^{FB})) - \sum_{j=1}^{n-1} C_j(x_j^{FB},\theta_j) \\
- \sum_{j=2}^{n} D_j(q_j(x^{FB})) + \sum_{j=1}^{i-1} \bar{k}_j + t_{i-2}(x),
\end{aligned}
$$

so that the individual rationality constraint of agent $i-2$ is binding, i.e.

$$
C_{i-2}(x_{i-2},\theta_{i-2}) + D_{i-2}(q_{i-2}(x)) - t_{i-2}(x) + t_{i-3}(x) = \bar{k}_{i-2},
$$

implying

$$
t_{i-2}(x) = \sum_{j=1}^{i-2} C_j(x_j,\theta_j) + \sum_{j=1}^{i-2} D_j(q_j(x)) - \sum_{j=1}^{i-2} \bar{k}_j.
$$

The best agent $i-1$ can do is to choose $x_{i-1} = x_{i-1}^{FB}$. To ensure that agent $i-2$ incorporates the externalities fully and to maximize his cooperation gain, agent $i-1$ proposes to agent $i-2$ the following transfer

$$
\begin{aligned}
t_{i-2}(x) = \sum_{i=1}^{n-1} C_i(x_i^{FB},\theta_i) + \sum_{i=2}^{n} D_i(q_i(x^{FB})) - \sum_{i=1}^{i-2} \bar{k}_i \\
- \sum_{j=i-1}^{n} D_j(q_j(x_1,...,x_{i-2},x_{i-1}^{FB})) - C_j(x_j^{FB},\theta_j).
\end{aligned}
$$

Then again, given that agent $i-2$ sets his upriver agent $i-3$ indifferent between accepting the contract or not, agent $i-2$ chooses $x_{i-2} = x_{i-2}^{FB}$ and so on. Thus, in general, an agent $j+1 < i$ offers agent j a transfer of

$$
\begin{aligned}
t_j(x) = \sum_{i=1}^{n-1} C_i(x_i^{FB},\theta_i) + \sum_{i=1}^{n} D_i(q_i(x^{FB})) - \sum_{k=j+1}^{n} D_k(q_k(x_1,...,x_j,x_{j+1}^{FB},...,x_{n-1}^{FB})) \\
- \sum_{k=j+1}^{n-1} C_k(x_k^{FB},\theta_k) - \sum_{l=1}^{j} \bar{k}_l,
\end{aligned}
\tag{5.6}
$$

which ensures that agent j is set indifferent and agent $j+1$'s cooperation gain is maximized. At the last step of the upriver contracting, agent 1 receives a transfer of

$$
t_1(x) = \sum_{i=1}^{n-1} C_i(x_i^{FB},\theta_i) + \sum_{i=1}^{n} D_i(q_i(x^{FB})) - \bar{k}_1 - \sum_{j=2}^{n} D_j(q_j(x_1,x_2^{FB},...,x_{n-1}^{FB})) - \sum_{j=3}^{n-1} C_j(x_j^{FB},\theta_j).
$$

He thus solves the following minimization problem

$$min_{x_1} C_1(x_1, \theta_1) - \sum_{i=1}^{n-1} C_i(x_i^{FB}, \theta_i) - \sum_{i=1}^{n} D_i(q_i(x^{FB})) + \bar{k}_1$$
$$+ \sum_{j=2}^{n} D_j(q_j(x_1, x_2^{FB}, ..., x_{n-1}^{FB})) + \sum_{j=2}^{n-1} C_j(x_j^{FB}, \theta_j),$$

implying $x_1 = x_1^{FB}$. Let us now use backwards induction to show that indeed no agent $j < i$ has an incentive to refuse the contract offered. At the last step of the upriver contracting, agent 1 receives $t_1(.)$ as given in (5.6) and thus faces the following minimization problem

$$min_{x_1} C_1(x_1, \theta_1) - \sum_{i=1}^{n-1} C_i(x_i^{FB}, \theta_i) - \sum_{i=1}^{n} D_i(q_i(x^{FB})) + \sum_{j=2}^{n} D_j(q_j(x_1, x_2^{FB}, ..., x_{n-1}^{FB}))$$
$$+ \sum_{j=2}^{n-1} C_j(x_j^{FB}, \theta_j) + \sum_{j=1}^{1} \bar{k}_j.$$

Given that all other agents choose first best optimal abatement levels, the best agent 1 can do is to select $x_1 = x_1^{FB}$, leaving him with $\bar{k}_1 = 0$. Similarly, moving downriver to an agent $j < i$, we have

$$x_j^{FB} \in argmin_{x_j} C_j(x_j, \theta_j) + D_j(q_j(x^{FB})) - t_j(x) + t_{j-1}(x)$$

with $t_j(x), t_{j-1}(x)$ given in (5.6), i.e.

$$x_j^{FB} \in argmin_{x_j} \sum_{k=1}^{j-1} C_k(x_k^{FB}, \theta_k) + \sum_{k=2}^{j-1} D_k(q_k(x^{FB})) + C_j(x_j, \theta_j) + D_j(q_j(x^{FB})) - \sum_{i=1}^{n-1} C_i(x_i^{FB}, \theta_i) + \bar{k}_j$$
$$- \sum_{i=2}^{n} D_i(q_i(x^{FB})) + \sum_{k=j+1}^{n} D_k(q_k(x_1^{FB}, ..., x_{j-1}^{FB}, x_j, x_{j+1}^{FB}, ..., x_{n-1}^{FB})) + \sum_{k=j+1}^{n-1} C_k(x_k^{FB}, \theta_k).$$

As a consequence, all agents $j < i$ end up with their reservation costs $\bar{k}_j$.

Next, let us have a look at the downriver part of the contracting process with agent i proposing a contract to agent $i+1$. With the transfer t_i given in (3.3), which agent i demands from agent $i+1$, agent $i+1$'s minimization problem is

$$min_{x_{i+1}, ..., x_{n-1}, t_{i+1}} C_{i+1}(x_{i+1}, \theta_{i+1}) + D_{i+1}(q_{i+1}(x_1^{FB}, ..., x_i^{FB})) + \sum_{j=i+1}^{n} \bar{k}_j$$
$$- \sum_{j=i+1}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=1}^{n} D_j(q_j(x^{FB})) - t_{i+1}(x)$$

with the binding individual rationality constraint of agent $i+2$, i.e. with

$$t_{i+1}(x) = \sum_{j=i+2}^{n} \bar{k}_j - \sum_{j=i+2}^{n-1} C_j(x_j, \theta_j) - \sum_{j=i+2}^{n} D_j(q_j(x)).$$

Thus, agent $i+1$ offers agent $i+2$ an abatement level of x_{i+1}^{FB} and a transfer of

$$t_{i+1}(x) = \sum_{j=i+2}^{n} \bar{k}_j - \sum_{j=i+2}^{n-1} C_j(x_j^{FB}, \theta_j) + \sum_{j=i+2}^{n} D_j(q_j(x^{FB})).$$

Hence, in general, an agent $j > i$ will offer his direct downriver agent $j+1$ an abatement level of x_j^{FB} and a transfer of

$$t_j(x) = \sum_{l=j+1}^{n} \bar{k}_l - \sum_{k=j+1}^{n-1} C_k(x_k^{FB}, \theta_k) - \sum_{k=j+1}^{n} D_j(q_k(x^{FB})). \tag{5.7}$$

At the last stage of the downriver contracting, agent $n-1$ sets agent n indifferent, i.e.

$$t_{n-1} = \bar{k}_n - D_n(q_n(x^{FB})),$$

implying that agent $n-1$ will face the following minimization problem

$$\begin{aligned} min_{x_{n-1}} & \sum_{j=1}^{n-2} C_j(x_j^{FB}, \theta_j) + C_{n-1}(x_{n-1}, \theta_{n-1}) + \sum_{j=1}^{n-1} D_j(q_j(x^{FB})) \\ & + D_n(q_n(x_1^{FB}, ..., x_{n-2}^{FB}, x_{n-1})) - \sum_{j=1}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=1}^{n} D_j(q_j(x^{FB})) + \bar{k}_{n-1}, \end{aligned}$$

with the result that he will choose x_{n-1}^{FB}. To prove that indeed no agent has an incentive to decline these contracts, we use backwards induction. In the last stage of the game, by accepting the contract offered by agent $n-1$, agent n incurs

$$k_n(.) = D_n(q_n(x^{FB})) + \bar{k}_n - D_n(q_n(x^{FB})) = \bar{k}_n.$$

Hence, he is indifferent between accepting the contract or not and will thus accept. In the second last step, agent $n-1$ has to decide whether he accepts the offer by agent $n-2$ and whether choosing x_{n-1}^{FB} and t_{n-1} is optimal. With t_{n-2} given in (5.7),

$$\begin{aligned} x_{n-1}^{FB} \in argmin_{x_{n-1}} C_{n-1}(x_{n-1}, \theta_{n-1}) + D_{n-1}(q_{n-1}(x^{FB})) - \bar{k}_n + D_n(q_n(x_1^{FB}, ..., x_{n-2}^{FB}, x_{n-1})) \\ + \sum_{j=n-1}^{n} \bar{k}_j - \sum_{j=n-1}^{n-1} C_j(x_j^{FB}, \theta_j) - \sum_{j=n-1}^{n} D_j(q_j(x_1^{FB}, ..., x_{n-1}^{FB})) \end{aligned}$$

and agent $n-1$ will end up with $\bar{k}_{n-1}$. Similarly, moving upriver, all agents $j > i$ will end up with their reservation costs $\bar{k}_j$ as with t_j, t_{j-1} given in (5.7), we have

$$x_j^{FB} \in argmin_{x_j} C_j(x_j, \theta_j) + D_j(q_j(x^{FB})) - t_j(x^{FB}) + t_{j-1}(x^{FB}),$$

i.e.

$$x_j^{FB} \in argmin_{x_j} C_j(x_j,\theta_j) + D_j(q_j(x^{FB})) - \sum_{l=j+1}^{n} \bar{k}_l + \sum_{k=j+1}^{n-1} C_k(x_k^{FB},\theta_k) + \sum_{l=j}^{n} \bar{k}_l - \sum_{k=j}^{n-1} C_k(x_k^{FB},\theta_k)$$
$$+ \sum_{k=j+1}^{n} D_k(q_k(x_1^{FB},...,x_{j-1}^{FB},x_j,x_{j+1}^{FB},...,x_{n-1}^{FB})) - \sum_{k=j}^{n} D_k(q_k(x^{FB})).$$

As a consequence, all agents $j > i$ end up with their reservation costs $\bar{k}_j$.
In contrast, agent i, the principal, will end up with the entire cooperation gain, i.e.

$$\begin{aligned} k_i(\theta_i) &= C_i(x_i^{FB},\theta_i) + D_i(q_i(x^{FB})) - t_i(x^{FB}) + t_{i-1}(x^{FB}) \\ &= \sum_{j=1}^{n-1} C_j(x_j^{FB},\theta_j) + \sum_{j=2}^{n} D_j(q_j(x^{FB})) - \sum_{j=1,j\neq i}^{n} \bar{k}_j. \end{aligned}$$

with $t_i(.)$ as given in (5.7) and t_{i-1} as given in (5.6).

The transfers (5.6) for all $j < i$ and the transfers (5.7) for all $j > i$ are the best ones the agents can choose. First, the transfers lead to the implementation of the first best solution. This in turn results in the largest cooperation gain which is in the interest of all agents. Second, each agent sets his down-river respectively upriver contracting partner exactly indifferent between accepting or not and thus extracts the maximum for himself. Or in other words, at the moment of contracting, agent j maximizes his cooperation gain by choosing t_{j-1} if $j < i$ as given in (5.6) respectively $t_j(.)$ if $j > i$ as given in (5.7) . □

Proof. of proposition 5
From the standard principal agent model, each agent $j < i$ should at least get a transfer of

$$t_j(x) = C_j(x_j(\theta_j),\theta_j) + \frac{\partial C_j(x_j(\theta_j),\theta_j)}{\partial\theta_j}\frac{F(\theta_j)}{f(\theta_j)} + D_j(q_j(x)) - \bar{k}_j + t_{j-1}(x).$$

Henceforth, under any incentive compatible scheme, agent $j+1$ has to offer agent j at least

$$t_j^c(x) = \sum_{k=1}^{j} C_k(x_k(\hat{\theta}_k),\hat{\theta}_k) + \sum_{k=1}^{j} \frac{\partial C_k(x_k(\hat{\theta}_k),\hat{\theta}_k)}{\partial\hat{\theta}_k}\frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} + \sum_{k=1}^{j} D_k(q_k(x)) - \sum_{l=1}^{j} \bar{k}_l. \tag{5.8}$$

Similarly, for agent $j > i$ to report truthfully and to ensure his compliance, agent j has to offer agent $j+1$ at least

$$t_j(x) = \bar{k}_{j+1} - C_{j+1}(x_{j+1},\theta_{j+1}) - \frac{\partial C_{j+1}(x_{j+1},\hat{\theta}_{j+1})}{\partial\hat{\theta}_{j+1}}\frac{F(\hat{\theta}_{j+1})}{f(\hat{\theta}_{j+1})} - D_{j+1}(q_{j+1}(x)) - t_{j-1}(x).$$

Iterating over the transfers yields

$$t_j^c(x) = \sum_{l=j+1}^{n} \bar{k}_l - \sum_{k=j+1}^{n-1} C_k(x_k,\theta_k) - \sum_{k=j+1}^{n-1} \frac{\partial C_k(x_k,\hat{\theta}_k)}{\partial\hat{\theta}_k}\frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \sum_{k=j+1}^{n} D_k(q_k(x)). \tag{5.9}$$

Similar to the model with full information, agent i maximizes his cooperation gain under the assumption that he as to pay agent $i-1$ at least $t^c_{i-1}(x)$ as given in (5.8) and can demand at most a transfer of $t^c_i(.)$ as given in (5.9) from agent $i+1$ for an abatement level of x_i. Thus, he faces the following minimization problem

$$min_{x_1,...,x_{n-1}} E_{\theta_{-i}}[C_i(x_i,\theta_i) + D_i(q_i(x)) + \sum_{j=1,j\neq i}^{n-1} C_j(x_j(\hat{\theta}_j),\hat{\theta}_j) + \frac{\partial C_j(x_j(\hat{\theta}_j),\hat{\theta}_j)}{\partial \hat{\theta}_j}\frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)}$$
$$+ \sum_{j=1,j\neq i}^{n-1} D_j(q_j(x)) - \sum_{j=1,j\neq i}^{n-1} \bar{k}_j].$$

Agent i's objective is thus equivalent to the one he faces in the centralized principal agent model. However, in contrast to the centralized model, here agent i cannot choose all abatement levels himself. Agent i thus has to ensure that all intermediate agents $j \neq i$ will choose abatement levels, which are optimal from his point of view.

Let us first consider the upstream-oriented contracting part of the model. We claim that agent i offers agent $i-1$ a transfer of

$$t_{i-1}(x) =\omega(\hat{\theta}_{i-1}) - \sum_{j=i}^{n} D_j(q_j(x_1,...,x_i,x^{P_i}_{i+1},...,x^{P_i}_{n-1})) - \sum_{j=i}^{n-1} C_j(x^{P_i}_j,\hat{\theta}_j)$$
$$- \sum_{j=i+1}^{n-1} \frac{\partial C_j(x^{P_i}_j,\hat{\theta}_j)}{\partial \hat{\theta}_j}\frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} - \frac{\partial C_{i-1}(x_{i-1},\hat{\theta}_{i-1})}{\partial \hat{\theta}_{i-1}}\frac{F(\hat{\theta}_{i-1})}{f(\hat{\theta}_{i-1})},$$

where $\omega(\hat{\theta}_{i-1})$ is chosen in a way that agent $i-1$ reports his type truthfully and accepts the contract. Note that $\omega(\hat{\theta}_{i-1})$ depends only on $\hat{\theta}_{i-1}$ and not on any abatement levels. Next, agent $i-1$ has to pay agent $i-2$ at least $t^c_{i-2}(x)$ given in (5.8), implying the following minimization problem for agent $i-1$

$$min_{x_1,...,x_{i-1}} E_{\theta_1,...,\theta_{i-2}}[C_{i-1}(x_{i-1},\theta_{i-1}) + D_{i-1}(q_{n-1}(x)) + \sum_{j=i}^{n-1} C_j(x_j,\hat{\theta}_j) + \sum_{j=i}^{n} D_j(q_j(x))$$
$$+ \sum_{j=i+1}^{n-1} \frac{\partial C_j(x_j,\hat{\theta}_j)}{\partial \hat{\theta}_j}\frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} - \omega(\hat{\theta}_{i-1}) - \sum_{j=1}^{i-2} \bar{k}_j + \sum_{i=1}^{i-2} C_i(x_i,\hat{\theta}_i)$$
$$+ \sum_{j=1}^{i-2} D_j(q_j(x)) + \sum_{j=1}^{i-1} \frac{\partial C_j(x_j,\hat{\theta}_j)}{\partial \hat{\theta}_j}\frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)}].$$

Consequently, agent $i-1$ reports $\hat{\theta}_{i-1}$ and then chooses $x^{P_i}_{i-1}(\hat{\theta}_1,...,\hat{\theta}_{i-1},\theta_{i-1})$. In the next step, to maximize his cooperation gain, agent $i-1$ sets the following transfer

$$t_{i-2}(x) =\omega(\hat{\theta}_{i-2}) - \sum_{j=i-1}^{n} D_j(q_j(x_1,...,x_{i-2},x^{P_i}_{i-1},...,x^{P_i}_{n-1})) - \sum_{j=i-1}^{n-1} C_j(x^{P_i}_j,\hat{\theta}_j)$$
$$- \sum_{j=i-1}^{n-1} \frac{\partial C_j(x^{P_i}_j,\hat{\theta}_j)}{\partial \hat{\theta}_j}\frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} - \frac{\partial C_{i-2}(x_{i-2},\hat{\theta}_{i-2})}{\partial \hat{\theta}_{i-2}}\frac{F(\hat{\theta}_{i-2})}{f(\hat{\theta}_{i-2})}.$$

Agent $i-2$ accepts by reporting $\hat{\theta}_{i-2}$ and will consequently choose $x_{i-2}^{P_i}(\hat{\theta}_1, ..., \hat{\theta}_{i-1}, \theta_{i-2})$. Moving upstream, agent j receives a transfer of

$$\begin{aligned} t_j(x) =& \omega(\hat{\theta}_j) - \sum_{k=j+1}^{n} D_k(q_k(x_1, ..., x_j, x_{j+1}^{P_i}, ..., x_{n-1}^{P_i})) - \sum_{k=j+1}^{n-1} C_k(x_k^{P_i}, \hat{\theta}_k) \\ & - \sum_{k=j+1, k\neq i}^{n-1} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \frac{\partial C_j(x_j, \hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)}, \end{aligned} \tag{5.10}$$

with $\omega(\hat{\theta}_j)$ given in general by

$$\begin{aligned} \omega(\hat{\theta}_j) =& \sum_{k=1}^{n-1} C_k(x_k^{P_i}, \theta_k) + \sum_{k=2}^{n} D_k(q_k(x^{P_i})) + \sum_{k=j}^{n-1} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \sum_{l=1}^{j} \bar{k}_l \\ & + \sum_{k=1}^{j} \int_{\hat{\theta}_k}^{\bar{\theta}_k} \frac{\partial C_k(x_k(\theta_{-k}, s, s,), s)}{\partial \hat{\theta}_k} ds, \forall j, j < i \end{aligned}$$

and reports $\hat{\theta}_j$ with the result that he faces the same minimization problem as agent i, i.e.

$$\begin{aligned} & min_{x_1,...,x_j} E_{\theta_1,...,\theta_{j-1}} [\sum_{k=1}^{j} C_k(x_k, \theta_k) + D_k(q_k(x)) + \sum_{k=1}^{j} \frac{\partial C_k(x_k, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} + \sum_{k=j+1}^{n-1} C_k(x_k^{P_i}, \hat{\theta}_k) - \omega(\hat{\theta}_j) \\ & \quad + \sum_{k=j+1}^{n} D_k(q_k(x_1, ..., x_j, x_{j+1}^{P_i}, ..., x_{n-1}^{P_i})) + \sum_{k=j+1, k\neq i}^{n-1} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \sum_{l=1}^{j-1} \bar{k}_l] \\ & \quad := min_{x_1,...,x_j} M_j(\theta_j, \hat{\theta}_j), \end{aligned} \tag{5.11}$$

and therefore sets $x_j^{P_i}(\hat{\theta}_1, ..., \hat{\theta}_{n-1}, \theta_j)$. In the last step of the upstream-contracting part of the game, agent 1 receives the transfer $t_1(.)$ as given in (5.10) and solves the following minimization problem

$$\begin{aligned} & min_{x_1} C_1(x_1, \theta_1) + \sum_{j=2}^{n} D_j(q_j(x_1, x_2^{P_i}, ..., x_{n-1}^{P_i})) + \sum_{j=2}^{n-1} C_j(x_j^{P_i}, \hat{\theta}_j) \\ & + \frac{\partial C_1(x_1, \hat{\theta}_1)}{\partial \hat{\theta}_1} \frac{F(\hat{\theta}_1)}{f(\hat{\theta}_1)} + \sum_{j=2, j\neq i}^{n-1} \frac{\partial C_j(x_j^{P_i}, \hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} - \omega(\hat{\theta}_1) \\ & := min_{x_1} M_1(\theta_1, \hat{\theta}_1), \end{aligned}$$

resulting in the choice of $x_1^{P_i}(\hat{\theta}_1, ..., \hat{\theta}_{n-1}, \theta_1)$.
Next, let us consider the downwards-oriented part of contracting. We claim that agent i offers agent $i+1$ a contract consisting of $x_i^{P_i}(.)$ and the following transfer

$$t_i(x) = \sum_{j=1}^{i-1} \frac{\partial C_j(x_j^{P_i}, \hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} + \frac{\partial C_{i+1}(x_{i+1}, \hat{\theta}_{i+1})}{\partial \hat{\theta}_{i+1}} \frac{F(\hat{\theta}_{i+1})}{f(\hat{\theta}_{i+1})} - \varphi(\hat{\theta}_{i+1}),$$

with $\varphi(\hat{\theta}_{i+1})$ being chosen in a way so that agent $i+1$ is willing to accept the transfer and reports his type truthfully. Given this transfer, agent $i+1$'s minimization problem is

$$\begin{aligned} min_{x_{i+1},...,x_{n-1}} E_{\theta_{i+2},...,\theta_{n-1}}[C_{i+1}(x_{i+1},\theta_{i+1}) + D_{i+1}(q_{i+1}(x_1^{P_i},...,x_i^{P_i})) + \frac{\partial C_{i+1}(x_{i+1},\hat{\theta}_{i+1})}{\partial \hat{\theta}_{i+1}} \frac{F(\hat{\theta}_{i+1})}{f(\hat{\theta}_{i+1})} \\ - \varphi(\hat{\theta}_{i+1}) + \sum_{j=i+2}^{n-1} C_j(x_j,\theta_j) + \sum_{j=i+3}^{n-1} \frac{\partial C_j(x_j,\hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} + \sum_{j=i+3}^{n} D_j(q_j(x)) - \sum_{j=i+3}^{n} \bar{k}_j] \\ := min_{x_{i+1},...,x_{n-1}} M_{i+1}(\theta_{i+1},\hat{\theta}_{i+1}). \end{aligned}$$

Agent $i+1$ then chooses $x_{i+1}^{P_i}$ and sets

$$t_{i+1}(x) = \sum_{j=1,j\neq i}^{i+1} \frac{\partial C_j(x_j^{P_i},\hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} + \frac{\partial C_{i+2}(x_{i+2},\hat{\theta}_{i+2})}{\partial \hat{\theta}_{i+2}} \frac{F(\hat{\theta}_{i+2})}{f(\hat{\theta}_{i+2})} - \varphi(\hat{\theta}_{i+2})$$

and so on. Thus, in general, an agent $j > i$, agent j solves

$$\begin{aligned} min_{x_j,...,x_{n-1}} E_{\theta_{j+1},...,\theta_{n-1}}[C_{j+1}(x_{j+1},\theta_{j+1}) + D_{j+1}(q_{j+1}(x_1^{P_i},...,x_{j-1}^{P_i})) + \frac{\partial C_j(x_j,\hat{\theta}_j)}{\partial \hat{\theta}_j} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} \\ - \varphi(\hat{\theta}_j) + \sum_{k=j+1}^{n-1} C_k(x_k,\theta_k) + \sum_{k=j+2}^{n-1} \frac{\partial C_k(x_k,\hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} \\ + \sum_{k=j+2}^{n} D_k(q_k(x)) - \sum_{l=j+2}^{n} \bar{k}_l] \\ := min_{x_j,...,x_{n-1}} M_j(\theta_j,\hat{\theta}_j). \end{aligned}$$

and proposes to agent $j+1$ an abatement level of $x_j^{P_i}$ in exchange for the following transfer

$$t_j(x) = \sum_{k=1,k\neq i}^{j} \frac{\partial C_k(x_k^{P_i},\hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} + \frac{\partial C_{j+1}(x_{j+1},\hat{\theta}_{j+1})}{\partial \hat{\theta}_{j+1}} \frac{F(\hat{\theta}_{j+1})}{f(\hat{\theta}_{j+1})} - \varphi(\hat{\theta}_{j+1}). \tag{5.12}$$

with $\varphi(\hat{\theta}_{j+1})$ given in general by

$$\begin{aligned} \varphi(\hat{\theta}_{j+1}) = \sum_{k=j+1}^{n-1} C_k(x_k^{P_i},\theta_k) + \sum_{k=j+1}^{n} D_k(q_k(x^{P_i})) + \sum_{k=1,k\neq i}^{j+1} \frac{\partial C_k(x_k^{P_i},\hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} \\ + \sum_{k=j+1}^{n-1} \int_{\hat{\theta}_k}^{\bar{\theta}_k} \frac{\partial C_k(x_k(\theta_m,s,s),s)}{\partial \hat{\theta}_k} ds - \sum_{l=j+1}^{n} \bar{k}_l. \end{aligned}$$

Moving downriver, we reach agent $n-1$ with the following minimization problem

$$\begin{aligned} min_{x_{n-1}} C_{n-1}(x_{n-1},\theta_{n-1}) + D_{n-1}(q_{n-1}(x_1^{P_i},...,x_{n-2}^{P_i},x_{n-1})) + \frac{\partial C_j(x_{n-1},\hat{\theta}_{n-1})}{\partial \hat{\theta}_{n-1}} \frac{F(\hat{\theta}_{n-1})}{f(\hat{\theta}_{n-1})} \\ + D_n(q_n(x_1^{P_i},...,x_{n-2}^{P_i},x_{n-1})) - \varphi(\hat{\theta}_{n-1}) - \bar{k}_n \\ := min_{x_{n-1}} M_{n-1}(\theta_{n-1},\hat{\theta}_{n-1}) \end{aligned}$$

so that agent $n-1$ will propose $x_{n-1}^{P_i}$ and the transfer $t_{n-1} = \overline{k}_n - D_n(q_n(x^{P_i}))$ to agent n.

Next, let us prove that incentive compatibility holds for all agents $j \neq i$. To show incentive compatibility, we apply the following slightly altered lemmas from Mirrlees (1971) (Lemma 6.1, 6.3)

Lemma 1. Let $x_i^\star(\theta_i) \in argmin C(x_i, \theta_i) - t(x_i)$. Then,

$$\frac{dk_i(x_i(\theta_i), \theta_i)}{d\theta_i} = \frac{\partial C_i(x_i(\theta_i), \theta_i)}{\partial \theta_i} + \frac{\partial C_i(x_i(\theta_i), \theta_i)}{\partial x_i(\theta_i)} \cdot \frac{\partial x_i(\theta_i)}{\partial \theta_i} - \frac{\partial t_i(x_i(\theta_i))}{\partial x_i(\theta_i)} \cdot \frac{\partial x_i(\theta_i)}{\partial \theta_i}.$$

Applying the envelope theorem yields

$$\frac{dk_i(x_i^\star(\theta_i), \theta_i)}{d\theta_i} = \frac{\partial C_i(x_i^\star(\theta_i), \theta_i)}{\partial \theta_i}.$$

Thus,

$$k_i(x_i^\star(\theta_i), \theta_i) - k_i(x_i^\star(0), 0) = \int_0^{\theta_i} \frac{\partial C_i(x_i^\star(s), s)}{\partial \theta_i} ds.$$

Lemma 2. Let $k_i(\theta_i) = k_i(x_i^\star(\theta_i), \theta_i)$ and $k_i(\hat{\theta}_i) = k_i(x_i^\star(\hat{\theta}_i), \hat{\theta}_i)$. Furthermore, let

$$k_i(\theta_i) - k_i(\hat{\theta}_i) = \int_{\hat{\theta}_i}^{\theta_i} \frac{\partial C_i(x_i^\star(k), k)}{\partial \theta_i} dk \text{ if } \hat{\theta}_i < \theta_i$$
$$k_i(\theta_i) - k_i(\hat{\theta}_i) = -\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i^\star(k), k)}{\partial \theta_i} dk \text{ if } \hat{\theta}_i > \theta_i$$

It follows that if $\frac{\partial C_i(x_i^\star(\hat{\theta}_i), \theta_i)}{\partial \theta_i}$ is non-increasing in $\hat{\theta}_i$, i.e. $x_i^\star(\hat{\theta}_i)$ decreasing in $\hat{\theta}_i$, then $x_i(\theta_i)$ minimizes $k_i(x_i, \theta_i)$.

Proof. of lemma 2
Let $\hat{\theta}_i < \theta_i$. Then

$$\begin{aligned} k_i(x_i^\star(\theta_i), \theta_i) - k_i(x_i^\star(\hat{\theta}_i), \hat{\theta}_i) = \int_{\hat{\theta}_i}^{\theta_i} \frac{\partial C_i(x_i^\star(k), k)}{\partial \theta_i} dk &\leq \int_{\hat{\theta}_i}^{\theta_i} \frac{\partial C_i(x_i^\star(\hat{\theta}_i), k)}{\partial \theta_i} dk \\ &= k_i(x_i^\star(\hat{\theta}_i), \theta_i) - k_i(x_i^\star(\hat{\theta}_i), \hat{\theta}_i) \end{aligned}$$

Thus, $k_i(x_i^\star(\theta_i), \theta_i) \leq k_i(x_i^\star(\hat{\theta}_i), \theta_i)$, so that $x_i^\star(\theta_i)$ must be minimizing $k_i(x_i, \theta_i)$.
Let $\hat{\theta}_i > \theta_i$. Then

$$\begin{aligned} k_i(x_i^\star(\theta_i), \theta_i) - k_i(x_i^\star(\hat{\theta}_i), \hat{\theta}_i) = -\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i^\star(k), k)}{\partial \theta_i} dk &\leq -\int_{\theta_i}^{\hat{\theta}_i} \frac{\partial C_i(x_i^\star(\hat{\theta}_i), k)}{\partial \theta_i} dk \\ &= k_i(x_i^\star(\hat{\theta}_i), \theta_i) - k_i(x_i^\star(\hat{\theta}_i), \hat{\theta}_i) \end{aligned}$$

Thus, $k_i(x_i^\star(\theta_i), \theta_i) \leq k_i(x_i^\star(\hat{\theta}_i), \theta_i)$, so that $x_i^\star(\theta_i)$ must be minimizing $k_i(x_i, \theta_i)$.

According to lemma 2, for agent j to report truthfully it suffices to show that

$$\frac{\partial M_j(\theta_j, \hat{\theta}_j)}{\partial \theta_j} \text{ is weakly decreasing in } \hat{\theta}_j. \tag{5.13}$$

Let $x_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j)$, $j = 1, ..., n-1$, denote the minimizers of the minimization problem $min_{x_1,...,x_j} J_j(\theta_j, \hat{\theta}_j)$ given in (5.11). For condition (5.13) to hold, it suffices to show that

$$\frac{\partial M_j(\theta_j, \hat{\theta}_j)}{\partial \theta_j} = E_{\theta_1,...,\theta_{j-1}}[\frac{\partial C_j(x_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j), \theta_j)}{\partial \theta_j}] \text{ is decreasing in } \hat{\theta}_j, \text{ for j < i respectively}$$
$$\frac{\partial M_j(\theta_j, \hat{\theta}_j)}{\partial \theta_j} = E_{\theta_{j+1},...,\theta_{n-1}}[\frac{\partial C_j(x_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j), \theta_j)}{\partial \theta_j}] \text{ is decreasing in } \hat{\theta}_j, \text{ for j > i}$$

Clearly, this is the case as long as $x_j(\hat{\theta}_k, \theta_j, \hat{\theta}_j)$ is decreasing in $\hat{\theta}_j$. This holds true as

$$\frac{dx_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j)}{d\hat{\theta}_j} = -\frac{\frac{\partial^2 M_j(\theta_j, \hat{\theta}_j)}{\partial \theta_j \partial x_j}}{\frac{\partial^2 M_j(\theta_j, \hat{\theta}_j)}{\partial^2 x_j}}$$
$$= -\frac{\frac{\partial^3 C_j(x_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j), \hat{\theta}_j)}{\partial^2 \hat{\theta}_j \partial x_j(.)} \frac{F(\hat{\theta}_j)}{f(\hat{\theta}_j)} + \frac{\partial^2 C_j(x_j(\hat{\theta}_{-j}, \theta_j, \hat{\theta}_j), \hat{\theta}_j)}{\partial \hat{\theta}_j \partial x_j(.)} \frac{\partial \left(F(\hat{\theta}_j)/f(\hat{\theta}_j)\right)}{\partial \hat{\theta}_j}}{\frac{\partial^2 M_j(\theta_j, \hat{\theta}_j)}{\partial^2 x_j}} \leq 0,$$

with $\frac{\partial^2 M_j(\theta_j, \hat{\theta}_j)}{\partial^2 x_j} \geq 0$ as $x_j(\hat{\theta}_k, \theta_j, \hat{\theta}_j)$ is a minimum. The same argument holds for all agents $j \neq i$. It remains to be shown, that the individual rationality constraints are fulfilled for all agents $j \neq i$. For this to hold, it suffices that the individual rationality constraints are binding for the highest type, i.e. $J_j(\overline{\theta}_j, \overline{\theta}_j) = \overline{k}_j$. Recall

$$\omega(\hat{\theta}_j) = \sum_{k=1}^{n-1} C_k(x_k^{P_i}, \theta_k) + \sum_{k=2}^{n} D_k(q_k(x^{P_i})) + \sum_{k=j}^{n-1} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)} - \sum_{l=1}^{j} \overline{k}_l$$
$$+ \sum_{k=1}^{j} \int_{\hat{\theta}_k}^{\overline{\theta}_k} \frac{\partial C_k(x_k(\theta_{-k}, s, s,), s)}{\partial \hat{\theta}_k} ds$$

and

$$\varphi(\hat{\theta}_j) = \sum_{k=j}^{n-1} C_k(x_k^{P_i}, \theta_k) + \sum_{k=j}^{n} D_k(q_k(x^{P_i})) + \sum_{k=1, k \neq i}^{j} \frac{\partial C_k(x_k^{P_i}, \hat{\theta}_k)}{\partial \hat{\theta}_k} \frac{F(\hat{\theta}_k)}{f(\hat{\theta}_k)}$$
$$+ \sum_{k=j}^{n-1} \int_{\hat{\theta}_k}^{\overline{\theta}_k} \frac{\partial C_k(x_k(\theta_m, s, s), s)}{\partial \hat{\theta}_k} ds - \sum_{l=j}^{n} \overline{k}_l.$$

As

$$\int_{\hat{\theta}_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(\theta_{-j}, s, s), s)}{\partial \hat{\theta}_j} ds = C_j(x_j(\theta_{-j}, \overline{\theta}_j, \overline{\theta}_j), \overline{\theta}_j) - C_j(x_j(\theta_{-j}, \hat{\theta}_j, \hat{\theta}_j), \hat{\theta}_j)$$

is decreasing in $\hat{\theta}_j$, and $M_j(\theta_j, \theta_j)$ is increasing in θ_j, all individual rationality constraints are binding at the top. Given these transfers and according to the revenue equivalence theorem, i.e.

$$k_j(\theta_j) - k_j(\overline{\theta}_j) = -\int_{\theta_j}^{\overline{\theta}_j} \frac{\partial C_j(x_j(\theta_{-j}, s, s), s)}{\partial \theta_j} ds$$

all individual rationality constraints are fulfilled. Similar to the proof in the full information case, it is evident from backwards induction, that given the transfers t_j in (5.10) respectively (5.12), no agent j can do better by choosing an abatement level other than $x_j^{P_i}$. Furthermore, as each agent j sets the transfer in a way to maximize his cooperation gain, there is no other transfer that is preferable. □

Proof. of proposition 6

The difference Δ_i in expected costs for agent i between the delegated and centralized model P_i amounts to

$$E_{\theta_{-i}}[\Delta_i] = E_{\theta_{-i}}[\sum_{j=1,j\neq i}^{n-1} \int_{\underline{\theta}_j}^{\bar{\theta}_j} [\int_{\theta_j}^{\bar{\theta}_j} \frac{\partial C_j(x_j^{P_i}(\theta_{-j},s,s),s)}{\partial \theta_j} ds - \frac{\partial C_j(x_j^{P_i},\theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)}] f(\theta_j) d\theta_j].$$

Integration by parts yields

$$\int_{\underline{\theta}_j}^{\bar{\theta}_j} [\int_{\theta_j}^{\bar{\theta}_j} \frac{\partial C_j(x_j^{P_i}(\theta_{-j},s,s),s)}{\partial \theta_j} ds f(\theta_j) d\theta_j = \int_{\underline{\theta}_j}^{\bar{\theta}_j} \frac{\partial C_j(x_j^{P_i},\theta_j)}{\partial \theta_j} F(\theta_j) d\theta_j$$

and thus

$$\begin{aligned} E_{\theta_{-i}}[\Delta_i] &= E_{\theta_{-i}}[\sum_{j=1,j\neq i}^{n-1} \int_{\underline{\theta}_j}^{\bar{\theta}_j} [\frac{\partial C_j(x_j^{P_i}(.),\theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)} - \frac{\partial C_j(x_j^{P_i}(.),\theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)}] f(\theta_j) d\theta_j] \\ &= 0. \end{aligned}$$

□

Proof. of proposition 7

Let $x = (x_1, ..., x_{n-1})$ be the vector of abatement levels. The minimization problem of agent i in case of asymmetric information, i.e. in the model P_i, differs from the minimization problem in case of full information in the information rents that have to be paid. Thus, consider the parametrized minimization problem of agent i

$$min_x J_i(x,\kappa) = min_x \sum_{i=1}^{n-1} C_i(x_i,\theta_i) + \sum_{i=1}^{n} D_n(q_n(x)) + \kappa \sum_{j=1,j\neq i}^{n-1} \frac{\partial C_j(x_j,\theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)}, \tag{5.14}$$

with $\kappa = 1$ in P_i with asymmetric information and $\kappa = 0$ in case of full information. We need to prove that the solution x to the minimization problem of agent i in (5.14) is decreasing in κ, implying $x_j^{P_i} \geq x_i^{FB}$, for all i.

Let $\kappa'' > \kappa'$ and $x'' \in argmin_x J_i(x,\kappa'')$ and $x' \in argmin_x J_i(x,\kappa')$. Then, by revealed preferences

$$\begin{aligned} J_i(x',\kappa') &\leq J_i(x'',\kappa'), \\ J_i(x'',\kappa'') &\leq J_i(x',\kappa''). \end{aligned}$$

Adding up the above inequalities and rearranging implies

$$J_i(x',\kappa'') - J_i(x',\kappa'') \geq J_i(x'',\kappa'') - J_i(x'',\kappa').$$

Rewriting the above expression yields

$$\int_{\kappa'}^{\kappa''} \frac{\partial J_i(x'', \kappa)}{\partial \kappa} d\kappa \leq \int_{\kappa'}^{\kappa''} \frac{\partial J_i(x', \kappa)}{\partial \kappa} d\kappa, \tag{5.15}$$

Note that

$$\frac{\partial J_i(x, \kappa)}{\partial \kappa} = \sum_{j=1, j\neq i}^{n-1} \frac{\partial C_j(x_j, \theta_j)}{\partial \theta_j} \frac{F(\theta_j)}{f(\theta_j)}.$$

Thus, inequality (5.15) can be rewritten as

$$\sum_{j=1, j\neq i} \int_{\kappa'}^{\kappa''} \int_{x_j''}^{x_j'} \frac{\partial^2 J_i(x, \kappa)}{\partial x_j \partial \kappa} d\kappa dx = \int_{\kappa'}^{\kappa''} \int_{x_j''}^{x_j'} \sum_{j=1, j\neq i}^{n-1} \frac{\partial^2 C_j(x_j, \theta_j)}{\partial \theta_j \partial x_j} \frac{F(\theta_j)}{f(\theta_j)} \geq 0.$$

Due to the single crossing property, the above inequality is satisfied, whenever

$$(\kappa'' - \kappa')(x_j' - x_j'') \geq 0, \forall j.$$

Thus, given that $\kappa'' > \kappa'$, we need $x_j' > x_j''$ for the above inequality to hold. This requires that x_j is decreasing in κ. Hence,

$$x_j^{FB}(\text{ if } \kappa = 0) \geq x_j^{P_i}(\text{ if } \kappa = 1), \forall i, j, j \neq i.$$

□

Proof. of proposition 8
Consider two potential principals i and k. Let

$$E_\theta[f(x)] = E_\theta[\sum_{i=1}^{n-1} C_i(x_i, \theta_i) + \sum_{i=2}^{n} D_i(q_i(x)) + \sum_{j=1, j\neq i,k}^{n} \frac{\partial C_j(x_j, \theta_j)}{\partial \theta_J} \frac{F(\theta_j)}{f(\theta_j)}].$$

From linear approximation

$$E_\theta[f(x^{P_i}) - f(x^{P_k})] \approx E_\theta[\sum_{i=1}^{n} f'_{x_i}(x^{P_k})(x_i^{P_i} - x_i^{P_k})].$$

This implies

$$\begin{aligned} E_\theta[f(x^{P_i}) - f(x^{P_k})] \approx & E_\theta[\sum_{j=1}^{n-1} \frac{C_j(x_j^{P_j}, \theta_j)}{\partial x_j}(x_j^{P_i} - x_j^{P_k}) + \sum_{k=j+1}^{n} \frac{\partial D_k(q_k(x^{P_k}))}{\partial x_j}(x_j^{P_i} - x_j^{P_k}) \\ & + \sum_{j=1, j\neq i,k} \frac{\partial^2 C_j(x_j^{P_k}, \theta_j)}{\partial \theta_j \partial x_j} \frac{F(\theta_j)}{f(\theta_j)}(x_j^{P_i} - x_j^{P_k})]. \end{aligned}$$

This can be rewritten as

$$E_\theta[f(x^{P_i})-f(x^{P_k})] \approx E_\theta[\sum_{j=1}^{n-1}\frac{C_j(x_j^{P_k},\theta_j)}{\partial x_j}(x_j^{P_i}-x_j^{P_k})+\sum_{k=j+1}^{n}\frac{\partial D_k(q_k(x^{P_k}))}{\partial x_j}(x_j^{P_i}-x_j^{P_k})$$
$$+\sum_{j=1,j\neq i}\frac{\partial^2 C_j(x_j^{P_k},\theta_j)}{\partial\theta_j\partial x_j}\frac{F(\theta_j)}{f(\theta_j)}(x_j^{P_i}-x_j^{P_k})$$
$$+\frac{\partial^2 C_i(x_i^{P_k},\theta_j)}{\partial\theta_i\partial x_i}\frac{F(\theta_i)}{f(\theta_i)}(x_i^{P_i}-x_i^{P_k})-\frac{\partial^2 C_i(x_i^{P_k},\theta_i)}{\partial\theta_i\partial x_i}\frac{F(\theta_i)}{f(\theta_i)}(x_i^{P_i}-x_i^{P_k})]. \quad (5.16)$$

Applying the envelope theorem to the right hand side of expression (5.16), we attain

$$E_\theta[f(x^{P_i})-f(x^{P_k})] \approx E_\theta[-\frac{C_i(x_i^{P_k},\theta_i)}{\partial x_i\partial\theta_i}\frac{F(\theta_i)}{f(\theta_i)}(x_i^{P_i}-x_i^{P_k})].$$

As a consequence,

$$E_\theta[K^{P_i}(.)-K^{P_k}(.)] \approx E_\theta[\frac{\partial C_k(x_k^{P_i},\theta_k)}{\partial\theta_k}\frac{F(\theta_k)}{f(\theta_k)}-\frac{\partial C_i(x_i^{P_k},\theta_i)}{\partial\theta_i}\frac{F(\theta_i)}{f(\theta_i)}-\frac{\partial^2 C_i(x_i^{P_n},\theta_i)}{\partial\theta_i\partial x_i}\frac{F(\theta_i)}{f(\theta_i)}(x_i^{P_i}-x_i^{P_k})].$$

Thus, $E_\theta[K^{P_i}(.)] \leq E_\theta[K^{P_k}(.)]$ as long as

$$E_\theta[\frac{\partial C_k(x_k^{P_i},\theta_k)}{\partial\theta_k}-\frac{\partial C_i(x_i^{P_k},\theta_i)}{\partial\theta_i}] \leq E_\theta[\frac{\partial^2 C_i(x_i^{P_k},\theta_i)}{\partial\theta_i\partial x_i}(x_i^{P_i}-x_i^{P_k})].$$

□

Proof. of corollary 1
Let us first establish the following relationship for the abatement levels of an agent i.

Lemma 2. Agent i abates more in P_i than in P_n, i.e.

$$x_i^{P_i} \geq x_i^{P_n}, i=1,...,n-1$$

Proof. of lemma 2
Agent i's minimization problem in P_i differs from the minimization problem of agent n in the additional information rent that agent n has to pay to agent i. Thus, consider the parametrized minimization problem

$$J_i(x,\kappa)=\sum_{i=1}^{n-1}C_i(x_i,\theta_i)+\sum_{i=1}^{n}D_i(q_i(x))+\sum_{j=1,j\neq i}^{n-1}\frac{\partial C_j(x_j,\theta_j)}{\partial\theta_j}\frac{F(\theta_j)}{f(\theta_j)}+\kappa\frac{\partial C_i(x_i,\theta_i)}{\partial\theta_i}\frac{F(\theta_i)}{f(\theta_i)}$$

with $\kappa=1$ in P_n and $\kappa=0$ in P_i. Showing that the cost minimizing x_i is decreasing in κ, implies $x_i^{P_i} \geq x_i^{P_n}$.

Let $\kappa'' > \kappa'$ and $x_i''(x_j'') \in argmin_{x_i} J_i(x_i,x_j,\kappa'')$ and $x_i'(x_i') \in argmin_{x_i} J_i(x_i,x_j,\kappa')$, where x_j stands for all abatement levels except x_i. Then, by revealed preferences

$$\begin{aligned} J_i(x_i'(x_j'), x_j', \kappa') &\leq J_i(x_i''(x_j''), x_j'', \kappa'), \\ J_i(x_i''(x_j''), x_j'', \kappa'') &\leq J_i(x_i'(x_j'), x_j', \kappa''), \forall i, j, i \neq j. \end{aligned}$$

Adding up the above inequalities and rearranging yields

$$J_i(x_i'(x_j'), x_j', \kappa'') - J_i(x_i'(x_j'), x_j', \kappa') \geq J_i(x_i''(x_j''), x_j'', \kappa'') - J_i(x_i''(x_j''), x_j'', \kappa'), \forall i, j, i \neq j.$$

Rewriting the above expression yields

$$\int_{\kappa'}^{\kappa''} \frac{\partial J_i(x_i''(x_j''), x_j'', \kappa)}{\partial \kappa} d\kappa \leq \int_{\kappa'}^{\kappa''} \frac{\partial J_i(x_i'(x_j'), x_j', \kappa)}{\partial \kappa} d\kappa, \tag{5.17}$$

Note that

$$\frac{\partial J_i(x_i, x_j, \kappa)}{\partial \kappa} = \frac{\partial C_i(x_i, \theta_i)}{\partial \theta_i} \frac{F(\theta_i)}{f(\theta_i)},$$

Hence, inequality (5.17) can be rewritten as

$$\int_{\kappa'}^{\kappa''} \int_{x_i''(x_j'')}^{x_i'(x_j')} \frac{\partial^2 J_i(x_i, x_j, \kappa)}{\partial x_i \partial \kappa} \geq 0, \forall i, j, i \neq j.$$

Due to the single crossing property, the above inequality is satisfied, when

$$(\kappa'' - \kappa')(x_i'(x_j') - x_i''(x_j'')) > 0.$$

Thus, given our assumption that $\kappa'' > \kappa'$, we need $x_i'(x_j') > x_i''(x_j'')$. Thus, $x_i(.)$ must be decreasing in κ. Hence,

$$x_i^{P_i}(\text{ if } \kappa = 0) \geq x_i^{P_n}(\text{ if } \kappa = 1).$$

According to proposition 8, agent i is better than agent n if

$$E_\theta[K^{P_i}(.) - K^{P_n}(.)] \approx E_\theta[\frac{\partial C_n(x_n^{P_i}, \theta_n)}{\partial \theta_n} \frac{F(\theta_n)}{f(\theta_n)} - \frac{\partial C_i(x_i^{P_n}, \theta_i)}{\partial \theta_i} \frac{F(\theta_i)}{f(\theta_i)} - \frac{\partial^2 C_i(x_i^{P_n}, \theta_i)}{\partial \theta_i \partial x_i} \frac{F(\theta_i)}{f(\theta_i)} (x_i^{P_i} - x_i^{P_n})] \leq 0. \tag{5.18}$$

Agent n does not abate, i.e. $x_n^{P_i} = 0$ and thus $\frac{\partial C_n(x_n^{P_i}, \theta_n)}{\partial \theta_n} \frac{F(\theta_n)}{f(\theta_n)} = 0$. In addition, according to lemma 2, $x_i^{P_i} \geq x_i^{P_n}$. Combined with $\frac{\partial C_i(x_i^{P_n}, \theta_i)}{\partial \theta_i} \geq 0$, inequality (5.18) is fulfilled.

According to the minimization problem (3.1) of any principal i, the following first order conditions must hold for all agents $j \neq n$

$$\frac{\partial C_j(x_j^{P_i}, \theta_j)}{\partial x_j} + \frac{\partial^2 C_j(x_j^{P_i}, \theta_j)}{\partial \theta_j \partial x_j} \frac{F(\theta_j)}{f(\theta_j)} + \sum_{k=j+1}^{n} \frac{\partial D_k(q_k(x^{P_i}))}{\partial x_j} = 0, \forall j.$$

Subtracting the first order condition for agent j in P_n from the first order condition in P_i as given above yields

$$\frac{\partial C_j(x_j^{P_i},\theta_j)}{\partial x_j}-\frac{\partial C_j(x_j^{P_n},\theta_j)}{\partial x_j}+\frac{\partial^2 C_j(x_j^{P_i},\theta_j)}{\partial\theta_j\partial x_j}\frac{F(\theta_j)}{f(\theta_j)}-\frac{\partial^2 C_j(x_j^{P_n},\theta_j)}{\partial\theta_j\partial x_j}\frac{F(\theta_j)}{f(\theta_j)}$$
$$+\sum_{k=j+1}^{n}\frac{\partial D_k(q_k(x^{P_i}))}{\partial x_j}-\sum_{k=j+1}^{n}\frac{\partial D_k(q_k(x^{P_n}))}{\partial x_j}=0. \tag{5.19}$$

As $x_j^{P_i}\leq x_j^{P_n}$ according to lemma 2 and $\frac{\partial C_j(x_j,\theta_j)}{\partial x_j}\geq 0$ as well as $\frac{\partial^2 C_j(x_j,\theta_j)}{\partial\theta_j\partial x_j}\geq 0$, the first two differences in (5.19) are negative. Thus, for equality (5.19) to be satisfied,

$$\sum_{k=j+1}^{n}\frac{\partial D_k(q_k(x^{P_i}))}{\partial x_j}-\sum_{k=j+1}^{n}\frac{\partial D_k(q_k(x^{P_n}))}{\partial x_j}\geq 0 \tag{5.20}$$

has to hold. Due to $\frac{\partial D_k(q_k(x))}{\partial x_j}<0$, the necessary condition for (5.20) to be satisfied is $q_k(x^{P_i})\leq q_k(x^{P_n}),\forall k$. It follows that $\sum_{j=1}^{n-1}x_j^{P_i}\geq\sum_{j=1}^{n-1}x_j^{P_n}$. □

Proof. of proposition 9
From the minimization problem (3.1), we attain the following abatement levels

$$x_i^{P_i}=x_i^{FB}=c_i'^{-1}\left(\frac{\sum_{j=i+1}^{n}\beta_j}{\theta_i}\right)$$

and

$$x_j^{P_i}=c_j'^{-1}\left(\frac{\sum_{k=j+1}^{n}\beta_k}{\theta_j+\frac{F(\theta_j)}{f(\theta_j)}}\right)\leq x_j^{FB}$$

with $c_j'^{-1}(.)$ being the inverse function of $c_j'(.), j=1,...,n-1$. According to proposition 8,

$$E_\theta[K^{P_i}(.)]\leq E_\theta[K^{P_j}(.)]\Leftrightarrow\int_{E_\theta[x_i^{P_k}]}^{E_\theta[x_k^{P_i}]}\frac{\partial^2 C(x_i,E[\theta])}{\partial E[\theta]\partial x_i}dx_i-\frac{\partial^2 C(E_\theta[x_i^{P_k}],E[\theta])}{\partial E[\theta]\partial x_i}(E_\theta[x_i^{P_i}]-E_\theta[x_i^{P_k}])\leq 0.$$

Due to $\frac{\partial^2 C(x,\theta)}{\partial\theta\partial x}>0$, the above inequality is satisfied if $E_\theta[x_k^{P_i}]\leq E_\theta[x_i^{P_k}]$ and $E_\theta[x_i^{P_k}]\leq E_\theta[x_i^{P_i}]$. This is the case for all agents i upriver of agent k due to $\sum_{j=i+1}^{n}\beta_j>\sum_{j=k+1}^{n}\beta_j, k>i$ and $c_j'^{-1}(.)$ being an increasing function. The bigger $E_\theta[x_k^{P_i}]$ relative to $E_\theta[x_i^{P_k}]$ as well as $E_\theta[x_i^{P_k}]$ to $E_\theta[x_i^{P_i}]$, the bigger the difference between expected total costs in P_i and P_k. Hence, with agent 1 as the principal, the expected difference in total costs is the largest. □

Proof. of lemma 1
The optimal abatement levels $x_i^{P_i}$ are the solution to the following minimization problem

$$min_{x_1,x_2}K^{P_i}(x_1,x_2,\theta_1,\theta_2,\kappa_1,\kappa_2)=min_{x_1,x_2}[D_3(e_1+e_2-x_1-x_2)+C_1(x_1,\theta_1)+\kappa_1\frac{\partial C_1(x_1,\theta_1)}{\partial\theta_1}\frac{F(\theta_1)}{f(\theta_1)}$$
$$+C_2(x_2,\theta_2)+\kappa_2\frac{\partial C_2(x_2,\theta_2)}{\partial\theta_2}\frac{F(\theta_2)}{f(\theta_2)}+D_2(e_1-x_1)-D_2(e_1)]$$

with $\kappa_i = 0, \kappa_j = 1$ for structure $P_i, i, j = 1, 2, j \neq i$ and $\kappa_1 = 1$ and $\kappa_j = 1$ for structure P_3. Set $\frac{F(\theta_i)}{f(\theta_i)} = h(\theta_i)$. Solving the above minimization problem implies

$$x_1^{P_i} = \frac{(\beta_2 e_1 + \beta_3(e_1 + e_2))(\beta_3 + \kappa_2 h(\theta_2) + \theta_2) - \beta_3^2(e_1 + e_2)}{(\beta_2 + \beta_3 + \kappa_1 h(\theta_1) + \theta_1)(\beta_3 + \kappa_2 h(\theta_2) + \theta_2) - \beta_3^2}, \forall i$$

and

$$x_2^{P_i} = \frac{\beta_3(\beta_2 e_2 + (e_1 + e_2)(\theta_1 + \kappa_1 h(\theta_1)))}{(\theta_1 + \kappa_1 h(\theta_1))(\kappa_2 h(\theta_2) + \theta_2) + \beta_2(\beta_3 + \kappa_2 h(\theta_2) + \theta_2) + \beta_3(\kappa_1 h(\theta_1) + \kappa_2 h(\theta_2) + \theta_1 + \theta_2)}, \forall i$$

Let $e_1 = \alpha e_2$ and

$$f(\alpha) := E_{\theta_1,\theta_2}[K^{P_1}(\alpha) - K^{P_2}(\alpha)].$$

Set $\tilde{h} = E[h(\theta)]$, $m = \tilde{h} + E[\theta]$ and $b = \tilde{h} + 2E[\theta]$, then

$$f(\alpha) = -\frac{\beta_2 \tilde{h}(\beta_2(\beta_3^2(-e_2 + e_2\alpha)(e_2 + e_2\alpha) + e_2^2\alpha^2 E[\theta]m + \beta_3 e_2^2 \alpha^2 b) + \beta_3(e_2 + e_2\alpha)(2e_2\alpha E[\theta]m + \beta_3(-e_2 + e_2\alpha)b))}{2(\beta_3(\beta_2 + \tilde{h}) + (\beta_2 + 2\beta_3 + \tilde{h})E[\theta] + E[\theta]^2)(E[\theta]m + \beta_2(\beta_3 + m) + \beta_3 b)}$$

Clearly, as long as $-e_2 + e_2\alpha \geq 0$, i.e. $\alpha \geq 1$, we get $f(\alpha) \leq 0$. There exist $\alpha^{id_j}, j = 1, 2$ for which we have $E_{\theta_1,\theta_2}[K^{P_1}(.)] = E_{\theta_1,\theta_2}[K^{P_2}(.)]$, namly,

$$\alpha^{id_1} = \frac{\beta_3^2(\beta_2 + b)}{\beta_3 m E[\theta] + \sqrt{\beta_3^2(\beta_3(\beta_2 + b)(\beta_3 b + \beta_2(\beta_3 + b)) + (\beta_2 + 2\beta_3)m(\beta_2 + b)E[\theta] + m^2 E[\theta]^2)}}$$

and

$$\alpha^{id_2} = -\frac{\beta_3 m E[\theta] + \sqrt{\beta_3^2(\beta_3(\beta_2 + b)(\beta_3 b + \beta_2(\beta_3 + b)) + (\beta_2 + 2\beta_3)m(\beta_2 + b)E[\theta] + m^2 t^2)}}{(\beta_3(\beta_3 b + \beta_2(\beta_3 + b)) + (\beta_2 + 2\beta_3)mE[\theta])}$$

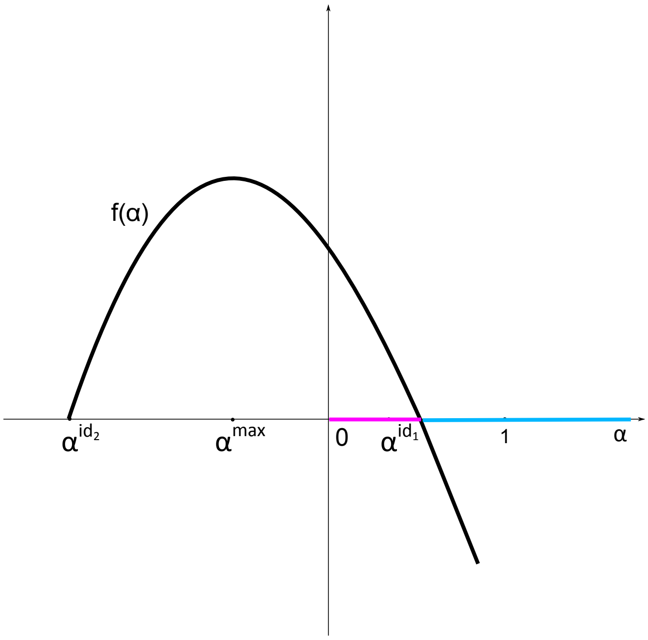
with $\alpha^{id_1} > 0, \alpha^{id_2} < 0$. Furthermore, $f(\alpha)$ has a maximum at $\alpha^{max} < 0$, i.e. $f'(\alpha) = 0 \Leftrightarrow$

$$\alpha^{max} = -\frac{(\beta_3 m E[\theta])}{(\beta_3(\beta_3 b + \beta_2(\beta_3 + b)) + (\beta_2 + 2\beta_3)mE[\theta])}$$

with $f''(\alpha^{max}) < 0$ and $f(\alpha^{max}) > 0$. Thus, as we only consider $\alpha > 0$, for all $\alpha \in (0, \alpha^{id_1})$ with $\alpha^{id_1} < 1$ we get that choosing agent 2 as the principal is better than agent 1 and for all $\alpha > \alpha^{id_1}$, choosing agent 1 is better.

□

Figure 3: In the pink area, $E_{\theta_1,\theta_2}[K^{P_1}(\alpha)] > E_{\theta_1,\theta_2}[K^{P_2}(\alpha)]$. In the blue area, $E_{\theta_1,\theta_2}[K^{P_1}(\alpha)] < E_{\theta_1,\theta_2}[K^{P_2}(\alpha)]$.



Chapter 3

A Second-Best Optimal Solution for Pollution Abatement in Multi-Polluter Networks

Abstract

We propose a second-best optimal solution to the problem of pollution abatement in a multi-polluter network with heterogeneously dispersed pollution. Instead of taking an exogenously given and predetermined pollution cap in a cap-and-trade system, the pollution cap is endogenized so that it is determined by the total cost-minimizing equilibrium of a cap-and-trade system. We show that with quadratic abatement costs and linear damage costs, the first-best optimal pollution cap implements the second-best cost-minimizing equilibrium of the cap-and-trade system for any network. However, the second-best optimal abatement allocation differs from the first-best optimal abatement allocation, implying higher second-best optimal total costs than first-best optimal. In particular, second-best optimal total abatement costs fall short of first-best total abatement costs, while second-best optimal total damage costs exceed first-best optimal damage costs. These findings hold for two different cap-and-trade systems considered, the emission permit market and the ambient pollution market.

1 Introduction

The regulation of uniformly distributed pollution with multiple sources and receptors of pollution has been well studied (e.g. Baumol and Oates (1988), Tietenberg (1995)). The first-best solution may be implemented by a Pigou tax or a cap-and-trade system. However, controlling heterogeneously dispersed pollution in multi-polluter networks is more complex, because emissions released at the sources and the damage-inducing ambient pollution levels accumulated at the receptors may not coincide. Because emissions from different sources induce different damage costs for the receptors, and because the marginal damage costs may vary across receptors, the first-best optimal abatement allocation can only be implemented by source-dependent taxes or prices of the tradable pollution rights in cap-and-trade systems. It is, however, questionable whether such source-dependent regulating instruments could be implemented.

In this chapter, we propose a second-best optimal solution to the problem of pollution abatement in a multi-polluter network with heterogeneously dispersed pollution. Instead of taking an exogenously given and predetermined pollution cap in a cap-and-trade system, we endogenize the pollution cap. The endogenized pollution cap, termed the second-best pollution cap, is determined by the total cost-minimizing equilibrium of the cap-and-trade system. We establish that, when facing linear damages and quadratic abatement costs to reduce emissions, the first-best optimal pollution cap implements the second-best cost-minimizing equilibrium of the cap-and-trade system for any network. However, the second-best optimal abatement allocation implied by the second-best pollution cap is not equivalent to the first-best optimal abatement allocation, indicating higher second-best total costs for the network than the first-best optimal. In particular, second-best total abatement costs in the cap-and-trade system falls short of first-best total abatement costs, while second-best total damage costs exceed first-best optimal damage costs.

These findings hold for two different cap-and-trade systems, for the emission permit market and ambient pollution market. The main distinction between these two markets is that an emission permit gives a source a right to emit a unit of emission, whereas an ambient pollution permit is defined as a permission to deposit a unit of emission at a specific receptor (Montgomery (1972)). Thus, ambient pollution markets incorporate the externality structure of the network, while emission markets ignore it. Yet, counter-intuitively, we find that an ambient pollution market is not always the better choice as an optimal second-best policy instrument. Furthermore, for other functional forms of abatement and damage cost functions, first-best optimal pollution caps do not implement the cost-minimizing equilibrium of the cap-and-trade system. Interestingly, second-best pollution caps may either fall short of or exceed the first-best pollution caps. The resulting policy implication for the regulation of heterogeneously distributed pollution in multi-polluter networks is to implement a cap-and-trade system that fixes the cap at the first-best optimal pollution level whenever real-life damages can be approximated to be linear and abatement cost functions to be quadratic. However, in general, when facing other functional forms of damage and abatement costs, this policy guideline should not be adopted.

It is well established that tradable pollution permits are a cost-efficient and cost-efficient policy instrument for uniformly distributed pollutants with multiple sources and receptors of pollution. They provide incentives for the greatest reductions in pollution by those that can achieve these reductions most cheaply and thus allowing any desired level of pollution to be realized at least costs to a network (Baumol and Oates (1971)). However, the literature on the optimal regulation of spatially heterogeneous externalities where the emissions from one source affect receptors differently is rather limited. Montgomery (1972) was the first who showed that a cost-effective, marketable permits system must be spatially differentiated and proposed ambient pollution permits. He argued that as long as the authorities can specify a vector of transfer coefficients for each emitter, linking emissions at each location with concentrations at each of the predefined receptor locations, specific trades can be defined which cost-effectively allocate the responsibility. Later, Krupnick et al. (1983) proposed the approach of pollution offsets allowing trading among sources as long as it does not violate ambient air quality standards at any receptor point. New emitters must acquire permits from existing sources to completely 'offset' the effects of the new emissions on pollutant concentrations at receptor points so that, in effect, exchange rates are endogenously given. This procedure provides for ambient quality goals to be attained at least costs. More recently, Klaassen et al. (1994) proposed a hybrid instrument that combines emission trading with a command-and-control policy, termed the exchange-rate emission trading system. In this exchange-rate emission trading system, the environmental authority first calculates and sets exchange rates ex-ante equal to the ratios of the sources' marginal abatement costs in the first-best solution. Sources then trade with each other according to these exogenous exchange rates. It was established that generally, this system will not achieve the least cost solution and does not guarantee that environmental deposition constraints are upheld, although total abatement costs are always reduced. Another market-oriented approach was proposed by Hung and Shaw (2005). They designed a trading-ratio system of tradable discharge permits for water pollution control. They show that their cap-and-trade system is a cost-effective instrument that meets predetermined environmental standards with the least aggregate abatement costs. These proposed regulation instruments are cost effective, but provide second-best policy instruments because none of them is cost-efficient. However, these instruments do not implement the second-best cost minimum. To the best of our knowledge, no one has thus far studied second-best optimality for a market-based policy instrument for a multi-polluter network with spatially distributed pollution.

The rest of the chapter is organized as follows. In Section 2, we introduce the multi-polluter network model and the first-best optimal solution to the problem of pollution abatement. In section 3, we discuss cap-and-trade systems as second-best solution concepts. In section 3, we present the second-best optimal solution concept and address which cap-and-trade regime should be chosen regarding second-best optimality. Section 5 concludes the chapter.

2 Multi-Polluter Network with non-uniformly distributed Pollution

Consider a multi-polluter network G consisting of n sources of pollution represented by a set of agents $N = 1, ..., n$. The agents are arranged in the network G according to an exogenously given geographical structure g. Each agent i produces gross emissions in exogenously given amount e_i, whereby $e_i \geq 0$ for all i and $e_j > 0$ for at least one j. Gross emissions of agent i may pose negative and spatially heterogeneous externalities on other agents $j \neq i$ in the network. Each multi-polluter network exhibits a specific externality structure Γ with elements $\gamma_{ij} \in [0, 1]$. For any pair of agents i, j, $\gamma_{ij} > 0$ states that agent j is negatively affected by agent i's emissions and the size of γ_{ij} specifies the percentage of the emissions released by agent i that reach agent j. In contrast, $\gamma_{ij} = 0$ indicates that the emissions of agent i do not affect agent j. Furthermore, we assume that $\gamma_{ii} = 0$ for all agents, so that pollution is an external effect. To clarify, externalities only go in one direction, so that $\gamma_{ij} \neq 0$ does not necessarily imply $\gamma_{ji} \neq 0$. Examples of such multi-polluter networks with spatially heterogeneously distributed pollution may encompass heterogeneously distributed global air pollutants, noise pollution and river pollution.

All agents i may choose to abate pollution in the amount x_i with $0 \leq x_i \leq e_i$ by incurring abatement costs $c_i(x_i)$. The abatement costs faced are strictly increasing, differentiable and strictly convex functions $c_i(.)$, i.e. $c_i'(.) > 0$, $c_i''(.) > 0$ and $c_i(0) = 0$. Let $x = (x_1, \ldots, x_n)$ be the vector of abatement efforts. Net emissions $e_i - x_i$ are released at each agent i, and are assumed to accumulate in the network, so that the ambient pollution level q_i at agent i of the network G is given by the sum of net emissions by the agents $j \neq i$ with $\gamma_{ji} > 0$, i.e.

$$q_i(x) = \sum_{j=1}^{n} \gamma_{ji}(e_j - x_j). \tag{2.1}$$

Intuitively, γ_{ji} measures by how much the ambient pollution level $q_i(x)$ at agent i increases, if net emissions of agent j increase by one unit. The ambient pollution level $q_i(x)$ imposes costs on agent i in form of damages, which are differentiable, convex and strictly increasing functions $d_i(q_i(x))$ of the

ambient pollution level $q_i(x)$ given in (2.1), i.e. $d_i'(.) > 0$ and $d_i''(.) \geq 0$. Without loss of generality, $d_i(0) = 0$, no damages are incurred if there is no pollution.
Total costs of an agent i are denoted by $k_i(x_i, q_i(x))$ and are the sum of his damage and abatement costs, i.e.

$$k_i(x_i, q_i(x)) = d_i(q_i(x)) + c_i(x_i).$$

As the vector of abatement efforts $x = (x_1, ..., x_n)$ together with the vector of exogenously given emissions $e = (e_1, ..., e_n)$ fully characterize the vector of ambient pollution levels $q(x) = (q_1(x), ..., q_n(n))$, total individual costs $k_i(x_i, q_i(x))$ of an agent i are fully determined by the abatement allocation x.

To sum up, a multi-polluter network G with heterogeneously dispersed pollution is characterized by (N, e, c, d, Γ), where N is the number of agents, and $c = (c_1, ..., c_n)$ respectively $d = (d_1, ..., d_n)$ denote the vectors of abatement and damage cost functions. We assume perfect information in all variables and functions (N, e, c, d, Γ).

There exists a unique and optimal abatement allocation minimizing overall costs in the multi-polluter network, i.e. the sum of all individual abatement and damage costs.

Proposition 1. *There exists a unique abatement vector $x^\star$ which is the solution to the following constrained minimization problem*

$$min_{x_1,...,x_n} \sum_{i=1}^{n} c_i(x_i) + d_i(q_i),$$

subject to

$$q_i(x) = \sum_{j=1}^{n} \gamma_{ji}(e_j - x_j)$$

and

$$0 \leq x_i \leq e_i.$$

Proof. of proposition 1
Existence and uniqueness follow directly from the strict convexity of the individual total cost functions $k_i(x_i, q_i(x))$. □

As we restrict our attention to interior solutions, the necessary conditions for the total cost minimum are

$$c_i'(x_i) = \sum_{j=1}^{n} \gamma_{ij} d_j'(q_j), \forall i, j, \tag{2.2}$$

which follow directly from the corresponding Lagrangian

$$\mathcal{L} = \sum_{i=1}^{n} c_i(x_i) + d_i(q_i) + \sum_{j=1}^{n} \lambda_j [\sum_{i=1}^{n} \gamma_{ij}(e_i - x_i) - q_j],$$

where λ_j is the Lagrange-multiplier of each pollution level q_j, and the combination of the resulting first order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_i} &= c_i'(x_i) - \sum_{j=1}^{n} \lambda_j \gamma_{ij} = 0, \forall i = 1, ..., n; j = 1, ..., n \\ \frac{\partial \mathcal{L}}{\partial q_i} &= d_i'(q_i) - \lambda_i = 0, \forall i = 1, ..., n.\end{aligned}$$

Due to pollution being an external effect and because abatement is costly, agents do not have any incentive to abate.

Proposition 2. *If the multi-polluter network G is considered to be a non-cooperative game among the agents, each agent's dominant strategy is not to abate,* $x_i = 0, \forall i$.

As a consequence of proposition 2, the first-best optimal abatement allocation will not be implemented by the agents voluntarily. Thus, external regulation is deemed necessary. Such regulatory instruments include, for example, command-and-control regulation and market-based approaches such as cap-and-trade systems. However, in contrast to multi-polluter networks with either uniformly mixed pollutants or pollution impact measured at a single receptor, multi-polluter networks with non-uniformly mixed pollutants and many affected receptors would require many different permit trading markets or source specific emission taxes to implement the first-best solution. Yet, in reality implementing many such markets or taxes may be difficult to be administered. Thus, in this chapter we focus on establishing only one cap-and-trade system charging the same price for the pollution rights for all emitters with the drawback that only second-best outcomes may be achieved in the case of spatially distributed pollution.

In the next section, we introduce two different cap-and-trade systems, the emission permit market and the ambient pollution permit market. We illustrate that these two markets indeed do not implement the first-best solution in our setting in general, except for the conditions established.

3 A second-best Solution: Cap-and-Trade Systems

In cap-and-trade systems, an external authority chooses a pollution cap $\overline{z}$ for the whole network. Pollution permits, representing the right to emit one unit of pollution, are then issued in this amount and allocated to the agents at no costs so that each agent receives $\overline{z_i}$ permits. Agents are allowed to trade the permits they are equipped with at a price of p. Let z_i be the net emissions of agent i. If $\overline{z_i}$ exceeds z_i, an agent i may sell the excess $(\overline{z_i} - z_i)$ at the price p in exchange for money. Similarly, he may buy $\overline{z_i} - z_i$ if the pollution released exceeds the number of permits held by the agent i. A buyer of permits is thus paying a charge for polluting, while a seller is being rewarded for having reduced his pollution. Thus, in theory, those who can reduce pollution most cheaply will do so, reaching the pollution target at the lowest cost. Clearly, if the external authority sets the pollution cap too high, the price of one permit is too low, reducing the incentives for the agents to cut back emissions and damage costs will be high. On the other hand, setting the pollution cap too low might lead to a high permit price, implying high abatement costs. The initial allocation of permits has no consequences for overall costs in a permit market, as we have perfect information, perfect competition and no transaction costs (Montgomery 1972). In addition, we assume that agents are not budget restricted. In the following, two different permit market regimes are introduced, the emission market, where the externality structure of the network is ignored, and the ambient pollution market, where it is incorporated.

Emission market

In an emission market, emission permits are traded. An emission permit gives an agent the right to discharge one unit of pollution, regardless of how many and by how much other agents are affected by this unit of pollution. Let $\overline{z}^E$ be the pollution cap set by an external authority for the emission market and $z = (z_1, ..., z_n)$ be the vector of net emissions. Each agent i will choose the least-costly way to comply with the pollution regulation, i.e.

$$min_{z_i} c_i(e_i - z_i) + d_i(q_i(z)) + p \cdot (z_i - \overline{z_i})$$
$$\text{subject to } 0 \leq z_i \leq e_i \text{ and } q_i(z) = \sum_{j=1}^{n} \gamma_{ji} z_j,$$

where $p \cdot (z_i - \overline{z_i})$ with a positive (negative) sign represents agent $i's$ permit trading expenditure (revenue). The associated first order conditions are

$$c_i'(e_i - z_i) = p, \forall i = 1, ..., n, \tag{3.1}$$

stating that an agent i adjusts his abatement level until the marginal abatement costs are equal to the permit price. Let $c_i^{-1}(.)$ denote the inverse function of $c_i(.)$. From the first oder conditions, we attain the individual permit demand functions

$$z_i^E(p) = e_i - c_i'^{-1}(p), \forall i. \tag{3.2}$$

As $c_i(.)$ is increasing and convex, $c_i^{-1}(.)$ is increasing and concave. Thus, the permit demand functions $z_i^E(p)$ are decreasing functions of the permit price p. Market clearing demands the sum of all permit demands to be equal to the permit supply, i.e.

$$\overline{z}^E = \sum_{i=1}^{n} z_i^E(p). \tag{3.3}$$

From this the equilibrium price p^E can be attained. Let $z^E = (z_1^E(p^E), ..., z_n^E(p^E))$ be the resulting vector of permit demands for an emission market given $\overline{z}^E$. Total costs for an agent i add up to

$$k_i^E(p^E, z^E) = c_i(e_i - z_i^E(p^E)) + d_i(q_i(z^E)) + p^E \cdot (z_i^E(p^E) - \overline{z_i})$$

whereby $q_i(z^E) = \sum_{j=1}^{n} \gamma_{ji} z_j^E(p^E)$.

Summing up over all agents individual cost levels $k_i^E(p^E, z^E)$ yields a total cost level of T^E, i.e.

$$T^E = \sum_{i=1}^{n} k_i^E(p^E, z^E) = \sum_{i=1}^{n} c_i(e_i - z_i^E) + d_i(q_i(z^E)),$$

where individual revenues and expenditures on permits add up to zero.

Ambient Pollution Market

Next, we consider an ambient pollution market. An ambient pollution permit gives agent i the permission to deposit a unit of emissions at a specific agent. Thus, in this market, the externality structure Γ is integrated in that each agent i has to hold permits in the amount of his induced ambient pollution. Let $\overline{z}^I$ be the ambient pollution target level set by the coordinator. Similar to the emission market, the cost-minimizing behaviour of the agents $i = 1, ..., n$ implies the following minimization problem

$$min_{z_i} c_i(e_i - z_i) + d_i(q_i(z)) + p \cdot (\sum_{j=1}^{n} \gamma_{ij} z_i - \overline{z_i})$$
$$\text{subject to } 0 \leq z_i \leq e_i \text{ and } q_i(z) = \sum_{j=1}^{n} \gamma_{ji} z_j.$$

Solving the minimization problem yields the first order conditions

$$c_i'(e_i - z_i) = p \cdot \sum_{j=1}^{n} \gamma_{ij}, \forall i = 1, ..., n. \tag{3.4}$$

Analogously to the emission market, the costs to abate one more unit of pollution have to be equal to the costs to emit one more unit of pollution. However, in contrast to the emission market, the costs to emit one more unit of pollution are composed of the price of a permit times the sum of the increases in the ambient pollution levels at the receptors caused by the additional unit of pollution. From (3.4), the *net emission function* $z_i^I(p) = e_i - c_i'^{-1}(p \cdot \sum_{j=1}^{n} \gamma_{ij})$ of agent i can be attained. Individual permit demand is then given by $\sum_{j=1}^{n} \gamma_{ij} z_i^I(p)$, which is, analogously to the emission market, decreasing in the permit price p. Market clearing demands

$$\overline{z}^I = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} z_i^I(p^I), \tag{3.5}$$

from which we obtain the equilibrium price p^I and the vector of net emission functions $z_i^I = (z_1^I(p^I), ..., z_n^I(p^I))$. The resulting individual total costs for an agent i are

$$k_i^I(p^I, z^I) = c_i(e_i - z_i^I(p^I)) + d_i(q_i(z^I)) + p^I \cdot (\sum_{j=1}^{n} \gamma_{ij} z_i^I(p^I) - \overline{z_i})$$

with $q_i(z^I) = \sum_{j=1}^{n} \gamma_{ji} z_j^I(p^I)$. Adding up all individual cost levels yields total costs in an ambient pollution market of

$$T^I = \sum_{i=1}^{n} k_i^I(p^I, z^I) = \sum_{i=1}^{n} c_i(e_i - z_i^I) + d_i(q_i(z^I)),$$

whereby total expenditures and revenues of the permit trades add up to zero.

Both permit market regimes generally fail to implement the least cost minimum given any predetermined pollution cap in networks with non-uniformly distributed pollution. In the following proposition, we establish under which specific conditions permit markets are able to implement the first-best solution.

Proposition 3. *Emission markets are first-best optimal iff*

$$\sum_{j=1}^{n} \gamma_{ij} d_j'(q_j) = \sum_{j=1}^{n} \gamma_{kj} d_j'(q_j), \forall k, i,$$

i.e. a unit-increase in net emissions of each agent $i = 1, ..., n$ needs to have exactly the same total negative effect on all other agents $j \neq i$.
Ambient pollution markets are first-best optimal iff

$$d_i'(q_i) = d_j'(q_j), \forall i, j,$$

i.e. marginal damages have to be equalized across all agents i.

The proof of proposition 3 may be found in the appendix.

Instead of setting an arbitrary pollution level, it is reasonable to assume that a coordinator will set the pollution cap to be first-best optimal, i.e. to the first-best optimal total level of pollution in case of an emission market or the first-best optimal total level of ambient pollution in case of an ambient pollution market. The *first-best pollution cap* $\overline{z}^{\star E}$ for an emission market is determined by the sum of all endogenously given gross emissions e_i minus the first-best abatement levels $x^\star$, i.e.

$$\overline{z}^{\star E} = \sum_{i=1}^{n} e_i - x_i^\star. \tag{3.6}$$

Similarly, the *first-best pollution cap* $\overline{z}^{\star I}$ for the ambient pollution market is given by the sum of each agent's first-best optimal ambient pollution level $q_i^\star$, i.e.

$$\overline{z}^{\star I} = \sum_{i=1}^{n} q_i^\star(x) = \sum_{i=1}^{n} (\sum_{j=1}^{n} \gamma_{ji}(e_j - x_j^\star)). \tag{3.7}$$

In the next section, we propose a second-best optimal solution for the pollution abatement problem in networks with non-uniformly dispersed pollution.

4 A Second-best Optimal Solution

As established in the previous section, whenever we face heterogeneously distributed pollution in multi-polluter networks, cap-and-trade systems fail to implement the first-best optimal solution given any predetermined pollution cap. Cap-and-trade systems are cost-effective, in that a predetermined pollution cap is met at least costs, however, these least costs might not correspond to the cost minimum of the cap-and-trade system. Thus, we propose the following second-best optimal solution for the problem of emission abatement in multi-polluter networks: Instead of taking an exogenously given pollution cap, we endogenize the pollution cap. The endogenized pollution cap, we call the *second-best pollution cap*, is determined by the total cost minimizing equilibrium of the cap-and-trade system considered.

Formally, let $\overline{z}^w$ be a given pollution cap for the two markets $w = I, E$, where I stands for an ambient pollution and E for an emission market. Total costs in a permit market in dependence of the pollution cap $\overline{z}^w$, denoted by $T^w(\overline{z}^w)$, are uniquely determined by $\overline{z}^w$ and amount to

$$T^w(\overline{z}^w) = \sum_{i=1}^{n} k_i(p^w(\overline{z}^w), \overline{z}^w) = \sum_{i=1}^{n} d_i(\sum_{j=1}^{n} \gamma_{ji} z_i^w(p^w(\overline{z}^w))) + \sum_{i=1}^{n} c_i(e_i - z_i^w(p^w(\overline{z}^w))), w = I, E. \quad (4.1)$$

The individual permit demands $z_i^w(p^w(\overline{z}^w))$ follow from the cost minimization conditions (3.1) and (3.4) for the emission and ambient pollution market respectively and $p^w(\overline{z}^w)$ is attained from the market clearing conditions for the emission market (3.3) respectively ambient pollution market (3.5).

Proposition 4. *For both permit markets $w = I, E$, there exists a pollution cap $\overline{z}^{\star\star w}$ which implements the total cost minimizing equilibrium of the permit market if $c_i'''(e_i - z_i^w(.)) \leq 0$.*

The pollution cap $\overline{z}^{\star\star w}$ solving $\frac{\partial T^w(\overline{z}^w)}{\partial \overline{z}^w} = 0$ is a global minimum as long as $T^w(\overline{z}^w)$ is convex in $\overline{z}^w$, which is shown to be the case whenever $c_i'''(e_i - z_i^w(.)) \leq 0$. The proof of proposition 4 may be found in the appendix. Examples for abatement functions with $c_i'''(e_i - z_i^w(.)) \leq 0$ are of the form $c_i(x_i) = x_i^n$ with $n > 1$ and $c_i(x_i) = 1/k \cdot exp(x_i) - 1/k$.

4.1 Second-best Pollution Caps

The second-best pollution cap $\overline{z}^{\star\star w}$ minimizes total costs $T^w(\overline{z}^w)$ in the corresponding permit market, $w = I, E$. Thus, differentiating $T^w(\overline{z}^w)$ given in (4.1) with respect to $\overline{z}^w$ and setting equal to zero yields

$$\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial d_i(\sum_{j=1}^{n} \gamma_{ji} z_j^w(p^w(\overline{z}^w)))}{\partial z_j^w(p^w(\overline{z}^w))} \cdot \frac{\partial z_j^w(p^w(\overline{z}^w))}{\partial p^w(\overline{z}^w)} \right) \cdot \frac{\partial p^w(\overline{z}^w)}{\partial \overline{z}^w} + \left(\sum_{j=1}^{n} \frac{\partial c_j(e_j - z_j^w(p^w(\overline{z}^w)))}{\partial z_j^w(p^w(\overline{z}^w))} \cdot \frac{\partial z_j^w(p^w(\overline{z}^w))}{\partial p^w(\overline{z}^w)} \right) \cdot \frac{\partial p^w(\overline{z}^w)}{\partial \overline{z}^w} = 0.$$

Applying the envelope theorem implies

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial d_i(\sum_{j=1}^{n} \gamma_{ji} z_j^w(p^w(\overline{z}^w)))}{\partial z_j^w(p^w(\overline{z}^w))} \frac{\partial z_j^w(p^w(\overline{z}^w))}{\partial p^w(\overline{z}^w)} |_{\overline{z}^w = \overline{z}^{\star\star w}} + \sum_{j=1}^{n} \frac{\partial c_j(e_j - z_j^w(p^w(\overline{z}^w)))}{\partial z_j^w(p^w(\overline{z}^w))} \frac{\partial z_j^w(p^w(\overline{z}^w))}{\partial p^w(\overline{z}^w)} |_{\overline{z}^w = \overline{z}^{\star\star w}} = 0. \quad (4.2)$$

In proposition 4, we state that there always exits a $\overline{z}^{\star\star w}$ for which (4.2) holds. Furthermore, $\overline{z}^{\star\star w}$ constitutes a global minimum, because total costs $T(\overline{z}^w)$ are convex.

Do second-best pollution caps differ from first-best pollution caps and if yes, do they exceed or fall short of first-best pollution caps? Intuitively, second-best pollution caps should deceed first-best pollution caps. Even though externalities are better incorporated in the second-best optimal concept compared to traditional cap-and-trade systems, permit markets are primarily cost effective, leading to least total abatement costs possible. Due to this, it is reasonable to assume that second-best optimal caps should fall short of first-best caps. Surprisingly, we find that when facing linear damages and quadratic abatement costs, the first-best pollution caps coincide with the second-best pollution caps.

Proposition 5. *Suppose agent i faces quadratic abatement costs represented by*

$$c_i(x_i) = \frac{1}{2}\alpha_i \cdot x_i^2, \text{ with } \alpha_i > 0, i = 1, ..., n,$$

and linear damage functions $d_i(.)$ taking the following form

$$d_i(q_i(x)) = \beta_i \cdot q_i(x), \text{ with } \beta_i > 0, q_i(x) = \sum_{j=1}^{n} \gamma_{ji}(e_j - x_j), i = 1, ..., n.$$

Then, the first-best optimal pollution cap $\overline{z}^{\star w}$ implements the second-best cost minimizing equilibrium of the cap-and-trade system $w = I, E$ for any network.

The appendix provides the proof of proposition 5. Proposition 5 states that if facing linear damages and quadratic abatement costs, establishing a cap-and-trade system and choosing a pollution cap corresponding to the first-best optimal pollution level implements the second-best cost minimizing equilibrium of the cap-and-trade system for any network with non-uniformly distributed pollution and spatially distributed sources and receptors. However, we show that the second-best optimal abatement allocation implied by the second-best pollution cap generally differs from the first-best optimal abatement allocation for at least one agent expect for the conditions stated in proposition 3, indicating higher second-best minimal total costs for the network than first-best optimal. In other words, the dead weight loss, defined as the difference between the second-best cost minimum and the first-best cost minimum, is weakly positive. Furthermore, we establish that in the second-best cost minimizing equilibrium, total abatement costs fall short of the first-best total abatement costs, whereas total damage costs exceed first-best total damage costs for both permit market regimes.

Proposition 6. *If facing linear damage and quadratic abatement cost functions, for both market regimes it holds that*

$$\Delta C^w = C^w - C^\star \leq 0, w = I, E$$

and

$$\Delta D^w = D^w - D^\star \geq 0, w = I, E$$

with C^w and D^w being the second-best minimal total abatement costs respectively damage costs in the market regime w and $C^\star$ and $D^\star$ being the total abatement and damage costs in the first-best solution respectively. Moreover, the dead weight loss is positive

$$DWL^w = \Delta C^w + \Delta D^w \geq 0, w = I, E.$$

The proof of proposition 6 may be found in the appendix. In conclusion, the cost minimizing equilibrium in a cap-and-trade system differs from the total cost minimum in the multi-polluter network. Although total abatement effort is the same as in first-best solution, it is not optimally distributed among the agents because the externalities are still not perfectly internalized in the second-best optimal solution and the cost-effective property of the cap-and-trade systems seems to be predominant.

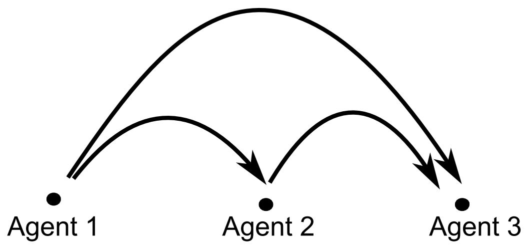
By facing other functional forms for the abatement and damage cost functions, the first-best optimal pollution cap does not implement the second-best cost minimizing equilibrium of the cap-and-trade system. Second-best pollution caps may either exceed or fall short of first-best pollution caps. This finding is illustrated for quadratic damage costs and quadratic abatement costs for a specific network.

Example 1. Let us consider the problem of efficient emission abatement in an international river. Agents are located along the river and pollute the water body by discharging waste water from commercial and industrial waste in the amount of e_i. Net emissions $e_i - x_i$ accumulate while moving downriver and we assume the river does not have any assimilative capacity. Furthermore, suppose the agents $i = 1, ..., n$ are linearly ordered along the river from up to down, so that $j > i$ indicates that agent j is located downriver of agent i. The externality structure Γ for the river sharing problem is characterized by

$$\gamma_{ji} = \begin{cases} 0, & \forall j \geq i \\ 1, & \forall j < i \end{cases}$$

In words, an agent i's net emissions affect all his downstream agents $j > i$ equally and fully. Thus, this specific network has a linear hierarchical structure with downstream oriented externalities.

Figure 4: River pollution network with $n = 3$, where the arrows represent the externalities between the agents



Suppose, we have quadratic damage functions given by

$$d_i(q_i(x)) = 1/2\beta_i q_i^2(x), i = 1, ..., n$$

with $\beta_i > 0$, so that the marginal damage costs $d_i'(q_i)$ are positive and increasing in q_i, i.e. $d_i'(q_i) > 0, d_i''(q_i) > 0, \forall i$. As before, the functional form of the quadratic abatement functions is given by $c_i(x_i) = 1/2\alpha_i x_i^2$. The first order conditions (2.2) for the river network indicate that the marginal abatement costs of an agent i have to be equal to the sum of the marginal damage costs of all his followers $j > i$ along the river. The main consequence of choosing non-linear damage functions is that the first order conditions (2.2) of the minimization problem in proposition 1 constitute a n-dimensional system of equations as the marginal damage costs are not constant. Thus, for simplicity, we consider a river shared by only three agents.

The first order conditions (2.2) translate into

$$\begin{aligned} c_1'(x_1) &= d_2'(q_2(x)) + d_3'(q_3(x)) \\ c_2'(x_2) &= d_3'(q_3(x)) \\ c_3'(x_3) &= 0. \end{aligned}$$

Given the functional forms of the abatement and damage cost functions, the system of equations can be rewritten as

$$\begin{aligned} \alpha_1 x_1 &= \beta_2(e_1 - x_1) + \beta_3(e_1 + e_2 - x_1 - x_2) \\ \alpha_2 x_2 &= \beta_3(e_2 + e_1 - x_1 - x_2) \\ \alpha_3 x_3 &= 0. \end{aligned}$$

Solving the system of equations, yields the following first-best abatement levels

$$\begin{aligned} x_1^\star &= \frac{e_1(\beta_2\beta_3 + \alpha_2\beta_2) + \alpha_2\beta_3(e_1 + e_2)}{\alpha_2(\alpha_1 + \beta_2) + \beta_3(\alpha_1 + \alpha_2 + \beta_2)} \\ x_2^\star &= \frac{\beta_2\beta_3 e_2 + \alpha_1\beta_2(e_1 + e_2)}{\alpha_2(\alpha_1 + \beta_2) + \beta_3(\alpha_1 + \alpha_2 + \beta_2)} \\ x_3^\star &= 0. \end{aligned}$$

As the emissions of agent 3 do not harm any other agents, it would be inefficient for agent 3 to abate. Therefore, it is reasonable to assume that agent 3 is not required to hold any permits and no permits are allocated to agent 3 initially. Hence, the first-best pollution cap for the emission market amounts to

$$\overline{z}^{\star E} = \sum_{i=1}^{2} e_i - x_i^\star = \frac{\alpha_2\beta_2 e_2 + \alpha_1\alpha_2(e_1 + e_2)}{\alpha_2(\alpha_1 + \beta_2) + \beta_3(\alpha_1 + \alpha_2 + \beta_2)}$$

and for the ambient pollution market

$$\overline{z}^{\star I} = \sum_{i=1}^{2}\sum_{j=1}^{3} \gamma_{ij}(e_i - x_i^\star) = \frac{\alpha_1 e_1(2\alpha_2 + \beta_3) + \alpha_2 e_2(\alpha_1 + \beta_2 - \beta_3)}{\alpha_2(\alpha_1 + \beta_2) + \beta_3(\alpha_1 + \alpha_2 + \beta_2)}$$

with $\sum_{j=1}^{3} \gamma_{1j} = 2$ and $\sum_{j=1}^{3} \gamma_{2j} = 1$, indicating that agent 1's emissions at the source of the river impact the two downriver agents 2 and 3 whereas agent 2's emissions only affect agent 3.
Next, we calculate the second-best pollution cap. In an emission market, agent i minimizes

$$\begin{aligned} & min_{z_i} k_i(z_i, p) = min_{z_i} 1/2\alpha_i(e_i - z_i)^2 + 1/2\beta_i q_i^2(z) + p(z_i - \overline{z_i}) \\ & \text{so that } q_i(z) = \sum_{j=1}^{2} \gamma_{ji} z_j, \end{aligned}$$

implying $z_i^E(p) = e_i - \frac{p}{\alpha_i}, i = 1,2$. Market clearing demands

$$\sum_{i=1}^{2} z_i = e_1 - \frac{p}{\alpha_1} + e_2 - \frac{p}{\alpha_2} = \overline{z}^E,$$

from which we extract

$$p^E(\overline{z}^E) = \frac{e_1 + e_2 - \overline{z}^E}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2}}.$$

Inserting $p^E(\overline{z}^E)$ into $z_1^E(p)$ and $z_2^E(p)$ yields the individual permit demands of

$$z_i^E(p^E(\overline{z}^E)) = e_i - \frac{1}{\alpha_i}\frac{e_1 + e_2 - \overline{z}^E}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2}}, i = 1,2.$$

Total costs in the emission market in dependence of the pollution cap $\overline{z}^E$ amount to

$$\begin{aligned} T^E(\overline{z}^E) &= c_1(e_1 - z_1^E(p(\overline{z}^E))) + c_2(e_2 - z_2^E(p(\overline{z}^E))) + d_2(z_1^E(p(\overline{z}^E))) + d_3(\overline{z}^E) \\ &= 1/2\left(\frac{(e_1 + e_2 - \overline{z}^E)^2}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2}} + \beta_2\left(e_1 - \frac{e_1 + e_2 - \overline{z}^E}{\alpha_1(\frac{1}{\alpha_1} + \frac{1}{\alpha_2})}\right)^2 + \beta_3(\overline{z}^E)^2\right). \end{aligned}$$

Total costs $T^E(\overline{z}^E)$ reach their minimum at the second-best pollution cap $\overline{z}^{\star\star E}$, which can be expressed as

$$\overline{z}^{\star\star E} = \frac{\alpha_2(\alpha_1 e_1(\alpha_1 + \alpha_2 - \beta_2) + e_2(\alpha_1^2 + \alpha_1\alpha_2 + \alpha_2\beta_2))}{\alpha_1^2(\alpha_2 + \beta_3) + \alpha_2^2(\beta_2 + \beta_3) + \alpha_1\alpha_2(\alpha_2 + 2\beta_3)}.$$

Similarly, in an ambient pollution market, agent i minimizes

$$min_{z_i} k_i(z_i, p) = min_{z_i} 1/2\alpha_1(e_i - z_i)^2 + 1/2\beta_i q_i^2(z) + p(\sum_{j=1}^{3} \gamma_{ij} z_i - \overline{z_i})$$

$$\text{so that } q_i(z) = \sum_{j=1}^{2} \gamma_{ji} z_j,$$

with $\sum_{j=1}^{3} \gamma_{1j} = 2$ and $\sum_{j=1}^{3} \gamma_{2j} = 1$ for agent 1 and agent 2 respectively, implying net emission functions of $z_1^I(p) = e_1 - \frac{2p}{\alpha_1}$ and $z_2^I(p) = e_2 - \frac{p}{\alpha_2}$. Market clearing demands

$$\sum_{i=1}^{2}\sum_{j=1}^{3} \gamma_{ij} z_i^I(p) = 2(e_1 - \frac{2p}{\alpha_1}) + e_2 - \frac{p}{\alpha_2} \quad = \quad \overline{z}^I$$

from which we extract

$$p^I(\overline{z}^I) = \frac{2e_1 + e_2 - \overline{z}^I}{\frac{4}{\alpha_1} + \frac{1}{\alpha_2}}.$$

Plugging $p^I(\overline{z}^I)$ into $z_1^I(p)$ and $z_2(p)$ yields the individual net emission functions of

$$z_i^I(p^I(\overline{z}^I)) = e_i - \frac{\sum_{j=1}^{3}\gamma_{ij}}{\alpha_i}\left(\frac{2e_1+e_2-\overline{z}^I}{\frac{4}{\alpha_1}+\frac{1}{\alpha_2}}\right), i = 1, 2.$$

Total costs in a permit market in dependence of $\overline{z}^I$ are the sum of the individual cost levels

$$\begin{aligned} T^I(\overline{z}^I) &= c_1(e_1 - z_1^I(p^I(\overline{z}^I))) + c_2(e_2 - z_2^I(p^I(\overline{z}^I))) + d_2(z_1^I(p^I(\overline{z}^I))) + d_3(z_1^I(p^I(\overline{z}^I)) + z_2^I(p^I(\overline{z}^I))) \\ &= 1/2\left(\alpha_1\left(\frac{2}{\alpha_1}\frac{2e_1+e_2-\overline{z}^I}{\frac{4}{\alpha_1}+\frac{1}{\alpha_2}}\right)^2 + \alpha_2\left(\frac{1}{\alpha_2}\frac{2e_1+e_2-\overline{z}^I}{\frac{4}{\alpha_1}+\frac{1}{\alpha_2}}\right)^2\right) + 1/2\beta_2\left(e_1 - \frac{2}{\alpha_1}\frac{2e_1+e_2-\overline{z}^I}{(\frac{4}{\alpha_1}+\frac{1}{\alpha_2})}\right)^2 \\ &+ 1/2\beta_3\left(e_1 - \frac{2}{\alpha_1}\frac{2e_1+e_2-\overline{z}^I}{(\frac{4}{\alpha_1}+\frac{1}{\alpha_2})} + e_2 - \frac{1}{\alpha_2}\frac{2e_1+e_2-\overline{z}^I}{(\frac{4}{\alpha_1}+\frac{1}{\alpha_2})}\right)^2. \end{aligned}$$

Minimizing $T^I(\overline{z}^I)$ with respect to $\overline{z}^I$ yields the following second-best pollution cap

$$\overline{z}^{\star\star I} = \frac{\alpha_1 e_1(\alpha_1(2\alpha_2+\beta_3) + \alpha_2(8\alpha_2 - 2\beta_2 + 2\beta_3)) + \alpha_2 e_2(\alpha_1^2 + 4\alpha_1\alpha_2 + 4\alpha_2\beta_2 - 2\alpha_1\beta_3 - 4\alpha_2\beta_3)}{(\alpha_2+\beta_3)(\alpha_1^2 + 4\alpha_1\alpha_2) + 4\alpha_2^2(\beta_2+\beta_3)}.$$

For the case of linear damages, we established that first and second-best pollution caps coincide. By assuming specific abatement and damage cost parameters, it can be easily demonstrated that this equivalence does not hold in the case of quadratic damages. In particular, the divergence may go in both directions in that second-best pollution caps may exceed or deceed first-best pollution caps. For simplicity, let us consider the case of full symmetry, i.e. $\alpha_i = \alpha, \beta_i = \beta$ and $e_i = e$. In this case, the first-best pollution caps for the two permit market regimes simplify to

$$\overline{z}^{\star E} = \frac{\alpha e(2\alpha+\beta)}{\alpha^2 + 3\alpha\beta + \beta^2}$$

and

$$\overline{z}^{\star I} = \frac{\alpha^2 e + \alpha e(2\alpha+\beta)}{\alpha^2 + 3\alpha\beta + \beta^2}.$$

The second-best pollution caps reduce to

$$\overline{z}^{\star\star E} = \frac{4\alpha e}{2\alpha + 5\beta}$$

and

$$\overline{z}^{\star\star I} = \frac{(15\alpha - \beta)e}{5\alpha + 13\beta}.$$

Thus, given full symmetry, first-best exceed second-best pollution caps as

$$\overline{z}^{\star\star E} - \overline{z}^{\star E} = -\frac{\alpha\beta^2 e}{(2\alpha+5\beta)(\alpha^2+3\alpha\beta+\beta^2)} < 0$$

and

$$\overline{z}^{\star\star I} - \overline{z}^{\star I} = -\frac{\beta^2 e(\alpha+\beta)}{(5\alpha+13\beta)(\alpha^2+3\alpha\beta+\beta^2)} < 0.$$

Yet, by deviating slightly from full symmetry, for example by setting $\beta_2 = 1/3\beta_3$ in an emission market and $\beta_2 = 2\beta_3$ in an ambient pollution market, first-best pollution caps fall short of second-best pollution caps.

Example 1 illustrates that the first-best pollution cap does not implement the cost minimizing equilibrium in the two permit markets considered if damage costs are non-linear. In order to attain the second-best least costs in cap-and-trade systems, the second-best pollution cap has to be derived formally for each specific network.

4.2 Second-best Optimal Choice of Permit Market Regime

Should the external authority choose an ambient pollution market or an emission market? Intuitively, ambient pollution markets seem to be the better choice as they incorporate the externality structure and therefore look at ambient pollution levels not just emission levels. However, against intuition, this is not always the case. To see this, assume without loss of generality, that we face linear damage costs and quadratic abatement costs and let us compare the dead weight losses created by the two market regimes.

Let $m_i = \sum_{j=1}^{n} \gamma_{ij}\beta_j$. Intuitively, m_i measures the total marginal effect of an increase in agent i's net emissions $e_i - x_i$ on the damages of all agents j, with $\gamma_{ij} > 0$. Applying lemma 3 and using slightly rewritten expressions for ΔC^E and ΔD^I derived in (5.11) and (5.15) respectively in the proof of proposition 6, implies

$$DWL^E = 1/2\left(\sum_{i=1}^{n}\frac{m_i^2}{\alpha_i} - \frac{\left(\sum_{i=1}^{n}\frac{m_i}{\alpha_i}\right)^2}{\sum_{i=1}^{n}\frac{1}{\alpha_i}}\right) \tag{4.3}$$

and

$$DWL^I = 1/2\left(\sum_{i=1}^{n}\frac{m_i^2}{\alpha_i} - \frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\frac{m_i}{\alpha_i}\right)^2}{\sum_{i=1}^{n}\frac{1}{\alpha_i}(\sum_{j=1}^{n}\gamma_{ij})^2}\right). \tag{4.4}$$

By taking the difference

$$DWL^E - DWL^I = 1/2\left(\frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\frac{m_i}{\alpha_i}\right)^2}{\sum_{i=1}^{n}\frac{1}{\alpha_i}(\sum_{j=1}^{n}\gamma_{ij})^2} - \frac{\left(\sum_{i=1}^{n}\frac{m_i}{\alpha_i}\right)^2}{\sum_{i=1}^{n}\frac{1}{\alpha_i}}\right)$$

we observe that as

$$\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\frac{m_i}{\alpha_i}\right)^2 \geq \left(\sum_{i=1}^{n}\frac{m_i}{\alpha_i}\right)^2$$

as well as

$$\sum_{i=1}^{n}\frac{1}{\alpha_i}(\sum_{j=1}^{n}\gamma_{ij})^2 \geq \sum_{i=1}^{n}\frac{1}{\alpha_i},$$

we cannot draw the conclusion, that the ambient pollution market incorporating the externality structure, should always be chosen as a second-best optimal policy instrument. On the contrary, as illustrated in the following example, the second-best cost minimizing equilibrium of an emission market may beat the one from an ambient pollution market.

Example 2. Recall the river pollution network introduced in example 1. To illustrate that the second-best total cost minimum of an ambient pollution market may exceed the second-best least costs of an emission market, we consider a river with 3 agents and compare the dead weight losses created by establishing a second-best optimal permit market.
Recall the formulas of the dead weight losses for an emission respectively ambient pollution market derived in (4.3) respectively (4.4). For the river network, $m_i = \sum_{j=i+1}^{n}\beta_j$. Thus, the dead weight loss for a second-best optimal emission market amounts to

$$DWL^E = \frac{1/2\beta_2^2}{\alpha_1 + \alpha_2}$$

and similarly, for an ambient pollution market we attain

$$DWL^I = \frac{1/2(\beta_2 - \beta_3)^2}{\alpha_1 + 4\alpha_2}.$$

Let $\alpha_i = \alpha$ and β_i vary across agents $i = 1,2,3$. Then,

$$DWL^I = \frac{(\beta_2 - \beta_3)^2}{10\alpha} > DWL^E = \frac{\beta_2^2}{4\alpha} \Leftrightarrow (\beta_2 - \beta_3)^2 > 5/2\beta_2^2.$$

Clearly, there exist values for β_2, β_3 for which this inequality is satisfied, for example for $\beta_3 = 3\beta_2$.

5 Conclusion

In this chapter, we propose a second-best optimal solution to the problem of pollution abatement in a setting where the multiple sources and receptors of pollution are spatially distributed and pollution is non-uniformly dispersed. Instead of taking an exogenously given and pre-set pollution cap in a cap-and-trade system, the pollution cap is endogenized in such a way that it results from the total cost minimizing equilibrium of the cap-and-trade system. We show that setting the pollution cap at the first-best optimal pollution level implements the second-best cost minimizing equilibrium for any network if damage costs are linear and abatement costs are quadratic. However, the second-best cost minimum exceeds the first-best total cost minimum, with total second-best optimal abatement costs falling short of first-best total abatement costs and total second-best optimal damage costs exceeding total first-best damage costs, indicating that the second-best abatement allocation differs from the first-best abatement allocation for at least one agent. Thus, we provide the following policy implication for the environmental regulation of pollution in networks with heterogeneously dispersed pollution: whenever real life damages are approximately linear and abatement costs quadratic, and a cap-and-trade system was established to regulate the pollution, then the pollution cap should be set at the first-best optimal level to attain least total costs possible. Yet, for other functional forms of damage and abatement cost functions, no such policy implication can be made. In these cases, first-best pollution caps are not second-best optimal and second-best pollution caps have to be calculated for each specific network structure.

One drawback of the second-best optimal solution concept proposed in this chapter and cap-and-trade systems in general is that they require perfect information of the abatement and damage cost functions as well as the exogenously given pollution levels. Abatement costs may be especially difficult to monitor and agents may have an incentive to overstate their true abatement costs. Furthermore, if sources and receptors are assumed to be nations, no international government exists that can impose a cap-and-trade system. Hence, establishing a cap-and-trade system would require cooperation among the sovereign nations. Yet, they can only be expected to cooperate if it makes them better or at least not worse off. Thus, the initial distribution of second-best optimal amount of permits should be used as an instrument for compensating the potential losers of an international policy instrument. As a final point, clearly there exists an isomorphism between a second-best pollution cap and a second-best Pigou tax. By endogenizing the Pigou tax so that it is determined by the second-best cost minimum of the multi-polluter network given a tax regime, the resulting second-best tax implies a total pollution level equivalent to the first-best optimal pollution level.

Appendix

Proof. of proposition 3
Let us first consider emission markets. In an emission market equilibrium, marginal abatement costs have to be equalized over all agents as all agents face the same price p^E, i.e.

$$c_i'(e_i - z_i) = c_j'(e_j - z_j), \forall i, j. \tag{5.1}$$

In order for the emission market to be first-best optimal, the cost minimizing conditions (2.2) have to hold. Combined with (5.1), this implies that

$$\sum_{j=1}^{n} \gamma_{ij} d_j'(q_j) = \sum_{j=1}^{n} \gamma_{kj} d_j'(q_j), \forall k, i, j = 1, ..., n.$$

Similarly, for an ambient pollution market, the necessary cost minimizing conditions (3.4) combined with p^I having to be equal for all agents leads to

$$\frac{c_i'(e_i - z_i)}{\sum_{m=1}^{n} \gamma_{im}} = \frac{c_j'(e_j - z_j)}{\sum_{m=1}^{n} \gamma_{jm}}, \forall i, j.$$

Together with the first order conditions for the first-best solution (2.2), an ambient pollution market is first-best optimal whenever

$$\frac{\sum_{j=1}^{n} \gamma_{ij} d_j'(q_j)}{\sum_{m=1}^{n} \gamma_{im}} = \frac{\sum_{j=1}^{n} \gamma_{kj} d_j'(q_j)}{\sum_{m=1}^{n} \gamma_{km}}, \forall i, k.$$

Rearranging implies

$$\begin{aligned}
\sum_{m=1}^{n} \gamma_{km} \sum_{j=1}^{n} \gamma_{ij} d_j'(q_j) &= \sum_{m=1}^{n} \gamma_{im} \sum_{j=1}^{n} \gamma_{kj} d_j'(q_j), \forall i, k, \Leftrightarrow \\
(\gamma_{i1} + ... + \gamma_{in})(\gamma_{k1} d_1'(q_1) + ... + \gamma_{kn} d_n'(q_n)) &= (\gamma_{k1} + ... + \gamma_{kn})(\gamma_{i1} d_1'(q_1) + ... + \gamma_{in} d_n'(q_n)) \Leftrightarrow \\
\sum_{j=1}^{n} \sum_{m \neq j}^{n} \gamma_{ij} \gamma_{km} (d_m'(q_m) - d_j'(q_j)) &= 0 \Leftrightarrow \\
d_j'(q_j) &= d_m'(q_m), \forall j, m.
\end{aligned}$$

□

Proof. of proposition 4
We have to prove that there exists a $\bar{z}^{\star\star w}$ that is the solution to $\frac{\partial T^w(\bar{z}^w)}{\partial \bar{z}^w} = 0$ and that this local extremum is a global minimum. For this, $T^w(\bar{z}^w)$ needs to be convex in $\bar{z}^w$, which is the case whenever $c_i'''(e_i - z_i^w(.)) \leq 0$.

Lemma 1. Total costs $T^w(\bar{z}^w)$ are convex in $\bar{z}^w$ if $c_i'''(e_i - z_i^w(.)) \leq 0$.

Proof. of lemma 1
To show that total costs are convex, a few properties of the price and permit demand functions need to hold, as established in the subsequent lemma.

Lemma 2. For both permit markets $w = I, E$, the price $p^w(\bar{z}^w)$ is decreasing in $\bar{z}^w$ and convex whenever $c_i'''(e_i - z_i^w(.)) \geq 0$ and concave whenever $c_i'''(e_i - z_i^w(.)) \leq 0$. The permit demands $z_i^w(p(\bar{z}^w))$ are decreasing in $p^w(\bar{z}^w)$ and increasing in $\bar{z}^w$. Furthermore, the permit demands are concave functions of $\bar{z}^w$ whenever $c_i'''(e_i - z_i^w(.)) \geq 0$ and convex whenever $c_i'''(e_i - z_i^w(.)) \leq 0$.

Proof. of lemma 2
Let us first show that the demand for permits decreases with an increase in the permit price. Following from the necessary conditions of the emission (3.2) and ambient pollution market (3.4), $z_i(.) = e_i - c_i'^{-1}(.)$. Thus,

$$\frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial p^w(\bar{z}^w)} = -\frac{1}{c_i''(e_i - z_i^w(p^w(\bar{z}^w)))} < 0, w = I, E,$$

as $c_i''(e_i - z_i^w(p^w(\bar{z}^w)) > 0$. Next, let us establish that $\frac{\partial p^w(\bar{z}^w)}{\partial \bar{z}^w} < 0$ and $\frac{\partial^2 p^w(\bar{z}^w)}{\partial^2 \bar{z}^w} \geq 0$, $w = I, E$. The proofs for the emission and ambient pollution market differ, but only slightly. For completeness, both proofs will be presented. From the market clearing condition for an emission market, we obtain

$$\bar{z}^E = \sum_{i=1}^{n} e_i - c_i'^{-1}(p).$$

Thus,

$$\sum_{i=1}^{n} c_i'^{-1}(p^E(\bar{z}^E)) - \sum_{i=1}^{n} e_i + \bar{z}^E = 0.$$

Applying the implicit function theorem yields

$$\sum_{i=1}^{n} \frac{\partial c_i'^{-1}(p^E(\bar{z}^E))}{\partial p^E(\bar{z}^E)} \frac{\partial p^E(\bar{z}^E)}{\partial \bar{z}^E} + 1 = 0.$$

Solving for $\frac{\partial p^E(\bar{z}^e)}{\partial \bar{z}^E}$ gives

$$\frac{\partial p^E(\bar{z}^E)}{\partial \bar{z}^E} = -\frac{1}{\sum_{i=1}^{n} c_i'^{-1}(p^E(\bar{z}^E))'} = -\frac{1}{\sum_{i=1}^{n} \frac{1}{c_i''(e_i - z_i)}} < 0$$

as $c_i''(e_i - z_i(.)) > 0$, indicating a decrease in the permit price if the pollution cap increases. Analogously, for the ambient pollution market,

$$\bar{z}^I = \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}(e_i - c_i'^{-1}(p \sum_{j=1}^{n} \gamma_{ij}))$$

and by the implicit function theorem,

$$\frac{\partial \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} c_i'^{-1}(p^I(\bar{z}^I) \sum_{j=1}^{n} \gamma_{ij})}{\partial \bar{z}^I} \frac{\partial p^I(\bar{z}^I)}{\partial \bar{z}^I} - 1 = 0.$$

From this we obtain

$$\frac{\partial p^I(\bar{z}^I)}{\partial \bar{z}^I} = -\frac{1}{\frac{\partial \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} c_i'^{-1}(p^I(\bar{z}^I) \sum_{j=1}^n \gamma_{ij})}{\partial \bar{z}^I}} = -\frac{1}{\sum_{i=1}^n \frac{\sum_{j=1}^n \gamma_{ij}}{c_i''(e_i - z_i^I(p^I(\bar{z}^I)))}} < 0.$$

Taking the second order derivative of $p^w(\bar{z}^w)$ with respect to $\bar{z}^w$ yields

$$\frac{\partial^2 p^w(\bar{z}^w)}{\partial^2 \bar{z}^w} = -\frac{1}{\sum_{i=1}^n \frac{g_i}{-(c_i''(e_i - z_i^w(p^w(\bar{z}^w))))^2} \cdot c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) \cdot \frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial p^w(\bar{z}^w)} \frac{\partial p^w(\bar{z}^w)}{\partial \bar{z}^w}},$$

with $g_i = 1$ for the emission market and $g_i = \sum_{j=1}^n \gamma_{ij}$ for the ambient pollution market. Thus,

$$\frac{\partial^2 p^w(\bar{z}^w)}{\partial^2 \bar{z}^w} \begin{cases} > 0 & \text{if } c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) > 0 \\ < 0 & \text{if } c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) < 0. \end{cases}$$

Additionally, let us analyse the curvature of $z_i^w(p^w(\bar{z}^w))$, i.e.

$$\frac{\partial^2 z_i^w(p^w(\bar{z})^w)}{\partial^2 p^w(\bar{z}^w)} = \frac{1}{(c_i''(e_i - z_i^w(p^w(\bar{z}^w))))^2} \cdot c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) \cdot \frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial p^w(\bar{z}^w)}$$

so that

$$\frac{\partial^2 z_i^w(p^w(\bar{z}^w))}{\partial^2 p^w(\bar{z}^w)} \begin{cases} < 0 & \text{if } c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) > 0 \\ > 0 & \text{if } c_i'''(e_i - z_i^w(p^w(\bar{z}^w))) < 0. \end{cases}$$

For both permit markets

$$\frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial \bar{z}^w} = \underbrace{\frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial p^w(\bar{z}^w)}}_{<0} \cdot \underbrace{\frac{\partial p^w(\bar{z}^w)}{\partial \bar{z}^w}}_{<0} > 0.$$

Finally, with

$$\frac{\partial^2 z_i^w(p^w(\bar{z}^w))}{\partial^2 \bar{z}^w} = \frac{\partial^2 z_i^w(p^w(\bar{z}^w))}{\partial^2 p^w(\bar{z}^w)} \left(\frac{\partial p^w(\bar{z}^w)}{\partial \bar{z}^w}\right)^2 + \frac{\partial z_i^w(p^w(\bar{z}^w))}{\partial p^w(\bar{z}^w)} \frac{\partial^2 p^w(\bar{z}^w)}{\partial^2 \bar{z}^w}$$

and given the properties of $p^w(\bar{z}^w)$ and $z_i^w(p^w(\bar{z}^w))$, we attain

$$\frac{\partial^2 z_i^w(p^w(\bar{z}^w))}{\partial^2 \bar{z}^w} \begin{cases} < 0 & \text{if } c_i'''(e_i - z_i^w(.)) > 0 \\ > 0 & \text{if } c_i'''(e_i - z_i^w(.)) < 0. \end{cases}$$

Thus, $z_i^w(p^w(\bar{z}^w))$ is a convex function of $\bar{z}^w$ whenever $c_i'''(e_i - z_i^w(.)) < 0$. □

Let $D(\bar{z}^w) = \sum_{i=1}^n d_i(\sum_{j=1}^n \gamma_{ji} z_j^w(p(\bar{z}^w)))$ and $C(\bar{z}^w) = \sum_{i=1}^n c_i(e_i - z_i^w(p(\bar{z}^w)))$, with $T(\bar{z}^w) = D(\bar{z}^w) + C(\bar{z}^w)$. From convex function calculus, we know that when f is convex and g is convex and monotonically increasing, then $g \circ f$ is convex. Given that $c_i'''(e_i - z_i^w(.)) \leq 0$, $z_i^w(p^w(\bar{z}^w))$ is convex in

$\bar{z}^w$. As $d_i(.)$ as well as $c_i(.)$ are convex and monotonically increasing, we have that $D(\bar{z}^w)$ and $C(\bar{z}^w)$ are convex functions in $\bar{z}^w$. Hence, $T^w(\bar{z}^w) = D(\bar{z}^w) + C(\bar{z}^w)$ being the sum of two convex functions must be convex as well. Because $T^w(\bar{z}^w)$ is a convex function of $\bar{z}^w$ in the case of $c_i'''(e_i - z_i^w(.)) < 0$, $T^w(\bar{z}^w)$ has a global minimum at $\bar{z}^{\star\star w}$, which however must not be unique as we do not have strictly convex functions. □

Proof. of proposition 5

Let us first characterize the first-best optimal abatement levels $x^\star$. According to proposition 1, the first-best optimal abatement allocation has to satisfy the following first order conditions

$$c_i'(x_i) = \sum_{j=1}^{n} \gamma_{ij} d_j'(q_j).$$

To simplify our analysis, let $m_i = \sum_{j=1}^{n} \gamma_{ij} d_j'(q_j)$. Intuitively, m_i measures the total marginal effect of an increase in agent i's net emissions $e_i - x_i$ on the damages of all agents j, with $\gamma_{ij} > 0$. As the marginal damages $d_j'(q_j)$ are constants for all agents j, the first-best abatement levels take the following form

$$x_i^\star = c_i'^{-1}(m_i), i = 1, ..., n.$$

Given the functional form of $c_i(.)$, the first-best solutions can be expressed as

$$x_i^\star = \frac{m_i}{\alpha_i}, i = 1, ..., n, \tag{5.2}$$

indicating that $x_i^\star$ is inversely proportional to α_i. As per (3.6), the first-best pollution cap for an emission market amounts to

$$\bar{z}^{\star E} = \sum_{j=1}^{n} (e_j - x_j^\star) = \sum_{j=1}^{n} e_j - \frac{m_j}{\alpha_j} \tag{5.3}$$

and similarly, in compliance with (3.7), the first-best pollution cap for an ambient pollution market is

$$\bar{z}^{\star I} = \sum_{j=1}^{n} \sum_{i=1}^{n} \gamma_{ji}(e_j - x_j^\star) = \sum_j \sum_i \gamma_{ji}(e_j - \frac{m_j}{\alpha_j}). \tag{5.4}$$

Next, we determine the second-best pollution caps, first for an emission market and then for an ambient pollution market. Given the functional form of $c_i(.)$ and the individual cost minimizing conditions for the permit demands (3.2), we attain

$$z_i^E(p^E(\bar{z}^E)) = e_i - \frac{p^E(\bar{z}^E)}{\alpha_i} \tag{5.5}$$

implying

$$\frac{\partial z_i^E(p^E(\bar{z}^E))}{\partial p^E(\bar{z}^E)} = -\frac{1}{\alpha_i}. \tag{5.6}$$

Furthermore, $c_i'(e_i - z_i) = \alpha_i(z_i - e_i)$. The second-best pollution cap $\overline{z}^{\star\star E}$ has to fulfill equation (4.2). Inserting the expressions derived in (5.5) and (5.6) into equation (4.2) yields

$$\sum_{i=1}^{n}\sum_{j=1}^{n} m_j \cdot \left(\frac{-1}{\alpha_j}\right) - \sum_{j=1}^{n} \alpha_j(e_j - z_j^E(p^E(\overline{z}^{\star\star E}))) \cdot \left(\frac{-1}{\alpha_j}\right) = 0.$$

Rearranging implies

$$\sum_{j=1}^{n} z_j^E(p(\overline{z}^{\star\star E})) = \sum_{j=1}^{n} e_j - \frac{m_j}{\alpha_j}. \tag{5.7}$$

By definition,

$$\sum_{j=1}^{n} z_j^E(p(\overline{z}^{\star\star E})) = \overline{z}^{\star\star E}.$$

Thus, according to (5.7), we get

$$\overline{z}^{\star\star E} = \sum_{j=1}^{n} e_j - \frac{m_j}{\alpha_j}.$$

Given the first-best pollution cap (5.3), we conclude

$$\overline{z}^{\star\star E} = \overline{z}^{\star E}.$$

Analogously, for the ambient pollution market, given the functional form of $c_i(.)$ and the individual cost minimizing conditions for the permit demands (3.4), we attain

$$z_j^I(p^I(\overline{z}^I)) = e_j - \frac{p^I(\overline{z}^I)\sum_{k=1}^{n}\gamma_{jk}}{\alpha_j}$$

and thus

$$\frac{\partial z_j^I(p^I(\overline{z}^I))}{\partial p^I(\overline{z}^I)} = -\frac{\sum_{k=1}^{n}\gamma_{jk}}{\alpha_j}.$$

The second-best pollution cap $\overline{z}^{\star\star I}$ has to fulfill (4.2). Inserting the expressions derived above into (4.2) implies

$$\sum_{j=1}^{n} m_j \cdot \left(\frac{-\sum_{k=1}^{n}\gamma_{jk}}{\alpha_j}\right) - \sum_{j=1}^{n} \alpha_j(e_j - z_j(p(\overline{z}^{\star\star I}))) \cdot \left(-\frac{\sum_{k=1}^{n}\gamma_{jk}}{\alpha_j}\right) = 0$$

Rearranging yields

$$\sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{jk} z_j^I(p^I(\overline{z}^{I\star\star})) = \sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{jk} e_j - \sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{jk}\frac{m_j}{\alpha_j}.$$

By definition

$$\overline{z}^{\star\star I} = \sum_{j=1}^{n}\sum_{k=1}^{n} \gamma_{jk} z_j^I(p^I(\overline{z}^{I\star\star})),$$

which according to (5.4) implies

$$\overline{z}^{\star\star I} = \overline{z}^{\star I}.$$

□

Proof. of proposition 6
The first-best allocation derived in (5.2) determines first-best total abatement costs, i.e.

$$C^\star = \sum_{i=1}^{n} c_i(x_i^\star) = 1/2 \cdot \left(\sum_{i=1}^{n} \frac{m_i^2}{\alpha_i} \right) \tag{5.8}$$

and first-best total damage costs, i.e.

$$D^\star = \sum_{i=1}^{n} d_i(q_i(x^\star)) = \sum_{i=1}^{n} m_i(e_i - \frac{m_i}{\alpha_i}). \tag{5.9}$$

To attain C^w and D^w, the second-best optimal permit demands have to be acquired. With the functional forms of the damage and abatement functions and the cost minimizing conditions of the emission market (3.1), we obtain $z_i^E(p) = e_i - \frac{p}{\alpha_i}$. In the second-best cost minimum, market clearing demands

$$\sum_{i=1}^{n} z_i^E(p(\overline{z}^{\star\star E})) = \sum_{i=1}^{n} e_i - \frac{p(\overline{z}^{\star\star E})}{\alpha_i} = \overline{z}^{\star\star E}.$$

Solving for $p(\overline{z}^{\star\star E})$ yields

$$p^E(\overline{z}^{\star\star E}) = \frac{\sum_{i=1}^n e_i - \overline{z}^{\star\star E}}{\sum_{i=1}^n \frac{1}{\alpha_i}}.$$

Inserting $p^E(\overline{z}^{\star\star E})$ into $z_i^E(p) = e_i - \frac{p}{\alpha_i}$, yields permit demands

$$z_i^E(p(\overline{z}^{\star\star E})) = e_i - \frac{\sum_{i=1}^n e_i - \overline{z}^{\star\star E}}{\alpha_i \sum_{i=1}^n \frac{1}{\alpha_i}}. \tag{5.10}$$

According to proposition 5, second-best and first-best permit pollution caps coincide. Thus, inserting $\overline{z}^{\star E}$ derived in (5.3) into the above expression (5.10) leads to the second-best optimal permit demands, i.e.

$$z_i^E(p(\overline{z}^{\star\star E})) = e_i - \frac{\sum_i \frac{m_i}{\alpha_i}}{\alpha_i \sum_i \frac{1}{\alpha_i}}.$$

Thus, total abatement costs in the second-best cost minimum amount to

$$C^E = \sum_{i=1}^{n} c_i(e_i - z_i^E(p^E(\overline{z}^{\star\star E}))) = 1/2 \left(\frac{(\sum_{i=1}^n \frac{m_i}{\alpha_i})^2}{\sum_{i=1}^n \frac{1}{\alpha_i}} \right) \tag{5.11}$$

and total damage costs to

$$D^E = \sum_{i=1} d_i(\sum_{j=1}^{n} \gamma_{ji} z_j^E(p^E(\overline{z}^{\star\star E}))) = \sum_{i=1}^{n} \beta_i \sum_{j=1}^{n} \gamma_{ji} \left(e_j - \frac{1}{\alpha_j} \frac{\sum_{i=1}^{n} \frac{m_i}{\alpha_i}}{\sum_{i=1}^{n} \frac{1}{\alpha_i}} \right). \tag{5.12}$$

Given the expressions of $C^\star$ in (5.8) and C^E in (5.11), we attain

$$\Delta C^E = C^E - C^\star = 1/2 \left(\frac{(\sum_{i=1}^{n} \frac{m_i}{\alpha_i})^2}{\sum_{i=1}^{n} \frac{1}{\alpha_i}} - \sum_{i=1}^{n} \frac{m_i^2}{\alpha_i} \right). \tag{5.13}$$

And analogously, with $D^\star$ derived in (5.9) and D^E given in (5.12), we attain

$$\begin{aligned} \Delta D^E &= D^E - D^\star = \sum_{i=1}^{n} \beta_i \sum_{j=1}^{n} \gamma_{ji} \left(e_j - \frac{1}{\alpha_j} \frac{\sum_{i=1}^{n} \frac{m_i}{\alpha_i}}{\sum_{i=1}^{n} \frac{1}{\alpha_i}} \right) - \sum_{i=1}^{n} m_i (e_i - \frac{m_i}{\alpha_i}) \\ &= \sum_{j=1}^{n} \frac{m_j}{\alpha_j} \left(m_j - \frac{\sum_{i=1}^{n} \frac{1}{\alpha_i} m_i}{\sum_{i=1}^{n} \frac{1}{\alpha_i}} \right). \end{aligned}$$

Equivalently, for an ambient pollution market, the net emission functions resulting from the individual cost minimizing conditions (3.4) amount to $z_i^I(p) = e_i - \frac{p \sum_{j=1}^{n} \gamma_{ij}}{\alpha_i}$. Combined with market clearing, we receive

$$p^I(\overline{z}^{\star\star I}) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} e_i - \overline{z}^{\star\star I}}{\sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\sum_{j=1}^{n} \gamma_{ij} \right)^2}$$

so that

$$z_i^I(p^I(\overline{z}^{\star\star I})) = e_i - \frac{1}{\alpha_i} \cdot \sum_{j=1}^{n} \gamma_{ij} \left(\frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} e_i - \overline{z}^{\star\star I}}{\sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\sum_{j=1}^{n} \gamma_{ij} \right)^2} \right).$$

Inserting the first-best pollution cap derived in (5.4) into the above expression, yields second-best net emission functions of

$$z_i^I(p(\overline{z}^{\star\star I})) = e_i - \frac{1}{\alpha_i} \cdot \sum_{j=1}^{n} \gamma_{ij} \left(\frac{\sum_{j=1}^{n} \frac{m_j}{\alpha_j} \sum_{k=1}^{n} \gamma_{jk}}{\sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\sum_{j=1}^{n} \gamma_{ij} \right)^2} \right).$$

Thus, the total abatement cost in the second-best total cost minimum amount to

$$C^I = \sum_{i=1}^{n} c_i(e_i - z_i^I(p(\overline{z}^{\star\star I}))) = 1/2 \cdot \left(\frac{(\sum_{j=1}^{n} \frac{m_j}{\alpha_j} \sum_{k=1}^{n} \gamma_{jk})^2}{\sum_{i=1}^{n} \frac{1}{\alpha_i} \left(\sum_{j=1}^{n} \gamma_{ij} \right)^2} \right) \tag{5.14}$$

and similarly, total damage costs to

$$D^I = \sum_{i=1} d_i(\sum_{j=1}^{n} \gamma_{ji} z_j^I(p(\overline{z}^{\star\star I}))) = \sum_{i=1}^{n} \beta_i \sum_{k=1}^{n} \gamma_{ki} \left(e_k - \frac{1}{\alpha_k} \sum_{j=1}^{n} \gamma_{kj} \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\gamma_{ji}}{\alpha_j} \sum_{k=1}^{n} \gamma_{jk} \beta_k}{\sum_{i=1}^{n} \frac{1}{\alpha_i} (\sum_{j=1}^{n} \gamma_{ij})^2} \right). \tag{5.15}$$

Given the expressions of $C^\star$ in (5.8) and C^I in (5.14), we conclude that

$$\Delta C^I \quad = \quad C^I - C^\star = 1/2 \cdot \left(\frac{(\sum_{j=1}^n \frac{m_j}{\alpha_j} \sum_{k=1}^n \gamma_{jk})^2}{\sum_{i=1}^n \frac{1}{\alpha_i} \left(\sum_{j=1}^n \gamma_{ij}\right)^2} - \sum_{i=1}^n \frac{(m_i)^2}{\alpha_i} \right). \tag{5.16}$$

Similarly, according to $D^\star$ derived in (5.9) and D^I given (5.15), the difference in total damage costs amounts to

$$\begin{aligned}
\Delta D^I &= D^I - D^\star \\
&= \sum_{i=1}^n \beta_i \sum_{k=1}^n \gamma_{ki} \left(e_k - \frac{1}{\alpha_k} \sum_{j=1}^n \gamma_{kj} \frac{\sum_{i=1}^n \sum_{j=1}^n \frac{\gamma_{ji}}{\alpha_j} \sum_{k=1}^n \gamma_{jk}\beta_k}{\sum_{i=1}^n \frac{1}{\alpha_i} (\sum_{j=1}^n \gamma_{ij})^2} \right) - \sum_{i=1}^n m_i(e_i - \frac{m_i}{\alpha_i}) \\
&= \sum_{k=1}^n \frac{m_k^2}{\alpha_k} - \sum_{k=1}^n \frac{m_k}{\alpha_k} \left(\sum_{j=1}^n \gamma_{kj} \frac{\sum_{i=1}^n \sum_{j=1}^n \frac{m_j \gamma_{ji}}{\alpha_j}}{\sum_{i=1}^n \frac{1}{\alpha_i} (\sum_{j=1}^n \gamma_{ij})^2} \right).
\end{aligned}$$

Lemma 3. The following relationship between ΔC^w and ΔD^w holds

$$\Delta D^w = -2 \cdot \Delta C^w, w = I, E.$$

Proof. of lemma 3
Given the definitions of ΔC^E and ΔC^I in (5.13) respectively (5.16), we have

$$\begin{aligned}
-2\Delta C^E &= -2 \cdot \left(1/2 \left(\frac{(\sum_{i=1}^n \frac{m_i}{\alpha_i})^2}{\sum_{i=1}^n \frac{1}{\alpha_i}} - \sum_{i=1}^n \frac{m_i^2}{\alpha_i} \right) \right) \\
&= \sum_{i=1}^n \frac{m_i^2}{\alpha_i} - \frac{(\sum_{i=1}^n \frac{m_i}{\alpha_i})^2}{\sum_{i=1}^n \frac{1}{\alpha_i}} \\
&= \sum_{i=1}^n \frac{m_i}{\alpha_i} \left(m_i - \frac{\sum_{i=1}^n \frac{m_i}{\alpha_i}}{\sum_{i=1}^n \frac{1}{\alpha_i}} \right). \\
&= \Delta D^E.
\end{aligned}$$

Similarly,

$$\begin{aligned}
-2\Delta C^I &= -2 \left(1/2 \cdot \left(\frac{(\sum_{j=1}^n \frac{m_j}{\alpha_j} \sum_{k=1}^n \gamma_{jk})^2}{\sum_{i=1}^n \frac{1}{\alpha_i} \left(\sum_{j=1}^n \gamma_{ij}\right)^2} - \sum_{i=1}^n \frac{(m_i)^2}{\alpha_i} \right) \right) \\
&= \sum_{i=1}^n \frac{m_i^2}{\alpha_i} - \frac{(\sum_{j=1}^n \sum_{k=1}^n \frac{\gamma_{jk} m_j}{\alpha_j})^2}{\sum_{i=1}^n \frac{1}{\alpha_i} (\sum_{j=1}^n \gamma_{ij})^2} \\
&= \sum_{i=1}^n \frac{m_i^2}{\alpha_i} - \frac{(\sum_{j=1}^n \sum_{k=1}^n \frac{\gamma_{jk} m_j}{\alpha_j})}{\sum_{i=1}^n \frac{1}{\alpha_i} (\sum_{j=1}^n \gamma_{ij})^2} \cdot (\sum_{j=1}^n \sum_{k=1}^n \frac{\gamma_{jk} m_j}{\alpha_j}) \\
&= \Delta D^I.
\end{aligned}$$

□

Let us now prove that $\Delta C^w \leq 0$ and $\Delta D^w \geq 0$. By contradiction, let $\Delta C^E > 0$. Then,

$$\begin{aligned}
\Delta C^E > 0 \quad &\Leftrightarrow \quad 1/2\left(\frac{(\sum_{i=1}^n \frac{m_i}{\alpha_i})^2}{\sum_{i=1}^n \frac{1}{\alpha_i}} - \sum_{i=1}^n \frac{1}{\alpha_i}(m_i)^2\right) > 0 \\
&\Leftrightarrow \quad \sum_{i=1}^n \frac{1}{\alpha_i} m_i^2 \sum_{i=1}^n \frac{1}{\alpha_i} - \left(\sum_{i=1}^n \frac{1}{\alpha_i} m_i\right)^2 < 0 \\
&\Leftrightarrow \quad \sum_{i=1}^n \sum_{j=1, j\neq i}^n \frac{1}{\alpha_i \alpha_j}(m_i - m_j)^2 < 0.
\end{aligned}$$

The last inequality does not hold due to $(m_i - m_j)^2$ always being positive. Thus, $\Delta C^E > 0$ cannot be satisifed. Given $\Delta C^E \leq 0$ and applying lemma 3, implies $\Delta D^E \geq 0$.

Equivalently, for the ambient pollution market, assume by contradiction that $\Delta D^I < 0$. Then,

$$\begin{aligned}
\Delta D^I < 0 \quad &\Leftrightarrow \quad \sum_{k=1}^n \frac{(m_k)^2}{\alpha_k} - \sum_{k=1}^n \frac{m_k}{\alpha_k}\left(\sum_{j=1}^n \gamma_{kj} \frac{\sum_{i=1}^n \sum_{j=1}^n \frac{\gamma_{ji}}{\alpha_j} m_j}{\sum_{i=1}^n \frac{1}{\alpha_i}(\sum_{j=1}^n \gamma_{ij})^2}\right) < 0 \\
&\Leftrightarrow \quad (\sum_{i=1}^n \frac{1}{\alpha_i}(\sum_{j=1}^n \gamma_{ij})^2)(\sum_{i=1}^n \frac{m_i^2}{\alpha_i}) - \left(\sum_{i=1}^n \frac{m_i}{\alpha_i}\sum_{j=1}^n \gamma_{ij}\right)^2 < 0 \\
&\Leftrightarrow \quad 1/2 \sum_{i=1}^n \sum_{j=1}^n \frac{\left(m_i \sum_{k=1}^n \gamma_{ik} - m_j \sum_{k=1}^n \gamma_{jk}\right)^2}{\alpha_i \alpha_j} < 0.
\end{aligned}$$

The last inequality does not hold true due to $\left(m_i \sum_{k=1}^n \gamma_{ik} - m_j \sum_{k=1}^n \gamma_{jk}\right)^2$ always being positive. Hence, $\Delta D^I \geq$ and it follows from lemma 3, that $\Delta^I C \leq 0$.

Finally, we show that $DWL^w \geq 0$. By definition of DWL^w and as a consequence of lemma 3, we obtain

$$DWL^w = \Delta D^w + \Delta C^w = -2\Delta C^w + \Delta C^w = -\Delta C^w, w = I, E.$$

Given $\Delta C^w \leq 0$, we conclude $DWL^w \geq 0$. □

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"Ich erkläre hiermit, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen benutzt habe. Alle Koautorenschaften sowie alle Stellen, die wörtlich oder sinngemäss aus Quellen entnommen wurden, habe ich als solche gekennzeichnet. Mir ist bekannt, dass andernfalls der Senat gemäss Artikel 36 Absatz 1 Buchstabe o des Gesetzes vom 5. September 1996 über die Universität zum Entzug des aufgrund dieser Arbeit verliehenen Titels berechtigt ist."

Datum: 24.11.2015 Sarina Steinmann